Minimum-entropy phase adjustment for ISAR

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Abstract: A new technique is developed for phase adjustment in ISAR imaging. The adjustment phase is found by iteratively solving an equation, which is derived by minimising the entropy of the image. This technique can be used to estimate adjustment phases of any form. Moreover, the optimisation method used in this technique is computationally more efficient than trial-and-error methods.

1 Introduction

Inverse synthetic aperture radar (ISAR) utilises the Fourier transform to resolve the scatterers in azimuth. Before taking the Fourier transform, translation compensation is used to remove the effect of the translation between the radar and the target in range. Translation compensation consists of range alignment, which aligns the signals from the same scatterer in range, and phase adjustment, which removes the translational Doppler phase.

Typical methods for phase adjustment include the dominant-scatterer method [1], the scattering-centroid method [2], the phase-gradient method [3, 4], the time–frequency method [5], the maximum-contrast method [6] and the minimum-entropy method [7, 8]. These methods apply even if no prior knowledge is available about the translation.

The minimum-entropy method is a promising technique for phase adjustment [7, 8]. It uses the principle that when the image is focused, the entropy of the image is a minimum. Originally, this technique is implemented using a parametric method [7]. A parametric model is used for the adjustment phase, and the parameters of this model are estimated. However, depending on the translation between the radar and the target in range, the adjustment phase may take any form. The parametric method does not work well if the adjustment phase does not fit the assumed model.

In order to remove this limitation, a nonparametric method is presented to implement the minimum-entropy phase adjustment [8]. This method does not assume any parametric model of the adjustment phase and thus applies universally. However, in this method, the optimisation problem is solved using a trial-and-error method, which is computationally inefficient.

We develop a new technique for phase adjustment in ISAR imaging. The adjustment phase is found by iteratively solving an equation, which is derived by minimising the entropy of the image. This technique can be used to estimate adjustment phases of any form. Moreover, the optimisation method used in this technique is computationally more efficient than trial-and-error methods.

Some results of this work and related work have been given in [9–11].

2 Fundamentals of ISAR

ISAR uses the relative motion between the radar and the target to obtain the image of the target [12]. In ISAR imaging, the radar can be ground-based, airborne or spaceborne. The platform can be stationary or moving. The beam tracks moving targets of interest. The targets range from man-made objects like ships, airplanes and satellites to natural objects like moons and planets. This paper treats ISAR. The ideas and methods discussed here, however, may also apply to some modes of synthetic aperture radar (SAR).

Figure 1 gives a simplified geometry of ISAR. The co-ordinate system is chosen such that the origin is fixed at a point on the target and the y-axis is directed from the radar to this point. This co-ordinate system is also chosen as the frame of reference. Thus, no matter how the radar and the target move, only the rotation of the target around the origin and the translation of the radar along the y-axis are observed. We assume that the target is rigid and each scatterer has a constant scattering coefficient. It should be clarified that the target discussed here is actually the projection of the physical target on the imaging plane.

We only consider the range–Doppler algorithm, which applies to small rotational angles [13]. For large rotational angles, typical algorithms include the subaperture algorithm, the subpatch algorithm, the polar-format algorithm and the back-projection algorithm [13].

2.1 Rotation-induced imaging

First consider the rotation of the target around the origin only. The translation between the radar and the target in range is ignored in this analysis.

The scatterers with different ranges are resolved using different time delays. A wideband technique such as the matched-filter technique, the stretch technique or the stepped-frequency technique can be used to improve range resolution [2, 12].

When the rotational angle of the target is small, it is assumed that the signals from the same scatterer are centred at the same range bin in different echoes (see Fig. 2; t and τ are called slow time and fast time, respectively, and for clarity only the amplitude of the signal is shown).

In each range bin (Fig. 2), the scatterers with different azimuths are resolved using different Doppler frequencies. The most widely used method is the Fourier transform. Other methods include modern spectral estimation [14] and time–frequency representation [15].
Typical methods include the peak method [1], the maximum-correlation method [1], the frequency-domain method [1], the Hough-transform method [16], the minimum-entropy method [8] and the global method [17].

### 2.4 Phase adjustment

The other effect of the translation is that a time-varying translational Doppler phase is contained in the phase of the signal. Usually, the Doppler frequency corresponding to a scatterer is not a constant any more due to the translational Doppler phase. This means that the image will be blurred if the Fourier transform is directly used for azimuth imaging. Thus, before taking the Fourier transform, the translational Doppler phase should be removed, converted to a constant or linearised so that the Doppler frequency corresponding to a scatterer can be converted into a constant. This is called phase adjustment. Typical methods for phase adjustment include the dominant-scatterer method [1], the scattering-centroid method [2], the phase–gradient method [3, 4], the time–frequency method [5], the maximum-contrast method [6] and the minimum-entropy method [7, 8]. These methods apply even if no prior knowledge is available about the translation.

### 3 Minimum-entropy phase adjustment

Phase adjustment and azimuth imaging can be formulated as

\[ g(k, n) = \sum_{m=0}^{M-1} f(m, n) \exp[j\phi(m)] \exp\left(-j \frac{2\pi}{M} km \right) \quad (1) \]

where \( m, n \) and \( k \) are the indices of echoes, range bins and Doppler frequency, respectively, \( f(m, n) \) is the signal resolved and aligned in range, \( \phi(m) \) is the adjustment phase and \( g(k, n) \) is the complex image. In (1), phase adjustment is first carried out by multiplying \( f(m, n) \) by \( \exp[j\phi(m)] \) and azimuth imaging is then carried out by taking the Fourier transform of \( f(m, n) \exp[j\phi(m)] \) with respect to \( m \). The key to phase adjustment is the estimation of \( \phi(m) \). In the minimum-entropy phase adjustment, \( \phi(m) \) is designed to minimize the entropy of \( |g(k, n)|^2 \).

#### 3.1 Principle

The entropy of \( |g(k, n)|^2 \) is defined as

\[ e[|g(k, n)|^2] = \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} \frac{|g(k, n)|^2 \ln \frac{S}{|g(k, n)|^2}}{S} \quad (2) \]

where

\[ S = \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k, n)|^2 \quad (3) \]

Entropy can be used to measure the smoothness of a distribution function. The smoother a distribution function is, the larger it entropy. Owing to this property, in ISAR imaging, entropy can be used to measure the focus quality of
an image \([7–11]\). Better focus results in a sharper image and thus smaller entropy. Thus, in phase adjustment, the adjustment phase can be designed to minimise the entropy of the image. Equation (2) can be written as

\[
e^2[g(k,n)^2] = \ln S - \frac{1}{k} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} [g(k,n)^2 \ln |g(k,n)|]^2
\]  

(4)

Since \(S\) is a constant in ISAR imaging, entropy can be redefined as

\[
e^2'[g(k,n)^2] = - \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2 \ln |g(k,n)|^2
\]

(5)

Thus, the minimum-entropy phase adjustment can be formulated as finding the adjustment phase that minimises (5).

### 3.2 Equation of adjustment phase

The \(\phi(m)\) that minimises \(e^2'[g(k,n)^2]\) satisfies

\[
\frac{\partial e^2'[g(k,n)^2]}{\partial \phi(m)} = 0
\]

(6)

The derivative of \(e^2'[g(k,n)^2]\) with respect to \(\phi(m)\) is obtained from (5), i.e.

\[
\frac{\partial e^2'[g(k,n)^2]}{\partial \phi(m)} = - \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} [1 + \ln |g(k,n)|^2] \frac{\partial |g(k,n)|^2}{\partial \phi(m)}
\]

(7)

Since \(|g(k,n)|^2 = g(k,n)g^*(k,n)\)

\[
\frac{\partial |g(k,n)|^2}{\partial \phi(m)} = 2\text{Re}\left[ g^*(k,n) \frac{\partial g(k,n)}{\partial \phi(m)} \right]
\]

(8)

Substituting (8) into (7), one obtains

\[
\frac{\partial e^2'[g(k,n)^2]}{\partial \phi(m)} = -2\text{Re}\left\{ \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} [1 + \ln |g(k,n)|^2] g^*(k,n) \frac{\partial g(k,n)}{\partial \phi(m)} \right\}
\]

(9)

Substituting \(m'\) for \(m\) in (1), one obtains

\[
g(k,n) = \sum_{m'=0}^{M-1} f(m',n) \exp[j\phi(m')] \exp\left(-j\frac{2\pi}{M}km'\right)
\]

(10)

The derivative of \(g(k,n)\) with respect to \(\phi(m)\) is obtained from (10), i.e.

\[
\frac{\partial g(k,n)}{\partial \phi(m)} = jf(m,n) \exp[j\phi(m)] \exp\left(-j\frac{2\pi}{M}km\right)
\]

(11)

In order to obtain (11), it should be noted that \(m\) is only a sample of \(m'\) here. Substituting (11) into (9), one obtains

\[
\frac{\partial e^2'[g(k,n)^2]}{\partial \phi(m)} = 2M \text{Im}\{\exp[j\phi(m)]a^*(m)\}
\]

(12)

where

\[
a(m) = \sum_{n=0}^{N-1} f^*(m,n) \frac{1}{M} \sum_{k=0}^{M-1} [1 + \ln |g(k,n)|^2] g(k,n) \exp\left(\frac{j2\pi km}{M}\right)
\]

(13)

Substituting (12) into (6), one obtains

\[
\phi(m) = \ln + j\phi(m)
\]

(14)

where \(l\) is an arbitrary integer and \(j\phi(m)\) denotes the phase of \(a(m)\). The simplest form of (14), which works very well in our test, is

\[
\phi(m) = j\phi(m)
\]

(15)

### 3.3 Algorithm

Equation (15) is an implicit expression of \(\phi(m)\) because calculating \(a(m)\) requires \(\phi(m)\) (see (13) and (1)). This equation can be solved using the fix-point iteration algorithm in Fig. 4.

\(\phi(m)\) is initialised as zero or the phase obtained by a simple phase-adjustment method like the dominant-scatterer method \([1]\). Our tests show that this can usually guarantee the convergence of \(\phi(m)\) to the desired optimal value. Our tests involve about twenty sets of field data and simulated data. For each data set, \(\phi(m)\) is initialised as zero and the phase obtained by the dominant-scatterer method, respectively. \(\phi(m)\) converges to the desired optimal value in each of these tests. The convergence is faster when \(\phi(m)\) is initialised as the phase obtained by the dominant-scatterer method.

The iteration is carried out until the following condition is satisfied:

\[
\max_{m=0}^{M-1}\left| \exp[j\phi(m)] - \exp[j\phi_{-1}(m)] \right| \leq \mu
\]

(16)

where \(\phi(m)\) and \(\phi_{-1}(m)\) are the values of \(\phi(m)\) after the current iteration and the previous iteration, respectively, and \(\mu\) is the threshold of accuracy. Equation (16) is also written as

\[
\max_{m=0}^{M-1}\left| \sin \left(\frac{\phi(m) - \phi_{-1}(m)}{2}\right) \right| \leq \frac{\mu}{2}
\]

(17)

If a \(\phi(m)\) minimises \(e^2'[g(k,n)^2]\), then this \(\phi(m)\) plus a linear phase can also minimise \(e^2'[g(k,n)^2]\). The reason is that this linear phase only results in a circular shift of the image in azimuth, but does not change \(e^2'[g(k,n)^2]\). Thus, there is an infinite number of \(\phi(m)\) that can minimise
The contrast of implemented using a parametric method\cite{7}. A parametric total number of imaging processes will be $2^j$ minimises the entropy of the image, where $D_j$ phase adjustment \cite{8}. First, a method is presented to implement the minimum-entropy adjustment phase may take any form. The parametric translation between the radar and the target in range, the of this model are estimated. However, depending on the contrast of the image is a maximum when the image is maximum-contrast method \cite{6}, a similar method, needs to when the minimum-entropy method is discussed, the adjustment phase is optimal only for the range bins in use but not for all the range bins.

3.4 Comparison

Originally, the minimum-entropy phase adjustment is implemented using a parametric method \cite{8}. A parametric model is used for the adjustment phase, and the parameters of this model are estimated. However, depending on the translation between the radar and the target in range, the adjustment phase may take any form. The parametric method does not work well if the adjustment phase does not fit the assumed model.

In order to remove this limitation, a nonparametric method is presented to implement the minimum-entropy phase adjustment \cite{8}. First, $\varphi(0)$ is kept unchanged, decreased by $\Delta$ or increased by $\Delta$ based on which way minimises the entropy of the image, where $\Delta$ is the initial step size. Then, $\varphi(1), \varphi(2), \ldots, \varphi(M - 1)$ are treated in the same way to complete one round of adjustment. More rounds of adjustment are carried out until the entropy of the image is minimised. Next, the above process is repeated for step sizes $\Delta/2, \Delta/4$ and so on. This method does not assume any parametric model of the adjustment phase and thus applies universally. This trial-and-error method, however, is computationally inefficient. Each time when a sample of $\varphi(m)$ is adjusted, two imaging processes are required. Each round of adjustment thus requires $2M$ imaging processes. If there are $L$ rounds of adjustments in all, the total number of imaging processes will be $2ML$. The computation of an imaging process is given in (1).

Our method is also nonparametric. Like the nonparametric method in \cite{8}, it does not assume any parametric model of the adjustment phase and thus applies universally. In addition, our optimisation method is computationally more efficient than the trial-and-error method in \cite{8}. In our optimisation method, the computation in each round of adjustment, which is mainly the computation in (1) and (13), is approximately equivalent to that of two imaging processes. If there are $L'$ rounds of adjustments in all, the total number of imaging processes will be $2L'$. Generally, $2L'$ is much smaller than $2ML$, the total number of imaging processes required by the trial-and-error method in \cite{8}.

4 Maximum-contrast phase adjustment

When the minimum-entropy method is discussed, the maximum-contrast method \cite{6}, a similar method, needs to be mentioned. This method uses the principle that the contrast of the image is a maximum when the image is focused.

4.1 Principle

The contrast of $|g(k,n)|^2$ is defined as

$$c'[|g(k,n)|^2] = \frac{\sigma[|g(k,n)|^2]}{E[|g(k,n)|^2]}$$

(18)

$E[|g(k,n)|^2]$ is the mean of $|g(k,n)|^2$ with respect to $k$ and $n$, i.e.

$$E[|g(k,n)|^2] = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2$$

(19)

$\sigma[|g(k,n)|^2]$ is the standard deviation of $|g(k,n)|^2$ with respect to $k$ and $n$, i.e.

$$\sigma[|g(k,n)|^2] = \sqrt{\frac{1}{MN} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} (|g(k,n)|^2 - \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2)^2}$$

(20)

which is also written as

$$\sigma[|g(k,n)|^2] = \sqrt{\frac{1}{MN} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^4 - \left(\frac{1}{MN} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2\right)^2}$$

(21)

Contrast can be used to measure the sharpness of a distribution function. The sharper a distribution function is, the larger its contrast. Due to this property, in ISAR imaging, contrast can be used to measure the focus quality of an image \cite{6}. Better focus results in a sharper image and thus larger contrast. Thus, in phase adjustment, the adjustment phase can be designed to maximise the contrast of the image.

Substituting (19) and (21) into (18), one obtains

$$c'[|g(k,n)|^2] = \sqrt{\frac{MN}{2^L} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^4} - 1$$

(22)

Since $S$ is a constant in ISAR imaging, contrast can be redefined as

$$c'[|g(k,n)|^2] = \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^4$$

(23)

Thus, the maximum-contrast phase adjustment can be formulated as finding the adjustment phase that maximises (23).

4.2 Algorithm

The maximum-contrast phase adjustment can be implemented by a parametric method \cite{6}. That is, a parametric model is used for the adjustment phase, and the parameters of this model are estimated. The parametric method fails if the adjustment phase does not fit the assumed model.

Inspired by our minimum-entropy method (Section 3), we develop a new technique to carry out the maximum-contrast phase adjustment. The adjustment phase is found by iteratively solving an equation, which is derived by maximising the contrast of the image. This technique can be used to estimate adjustment phases of any form.

The $\varphi(m)$ that maximises $c'[|g(k,n)|^2]$ satisfies

$$\frac{\partial c'[|g(k,n)|^2]}{\partial \varphi(m)} = 0$$

(24)

Using the method in Section 3.2, we can derive the equation of the adjustment phase, i.e.
\[ \phi(m) = \beta b(m) \]  

(25)

where

\[ b(m) = \sum_{n=0}^{N-1} f^*(m, n) \frac{1}{M} \sum_{k=0}^{M-1} |g(k, n)|^2 g(k, n) \exp \left( j \frac{2\pi}{M} km \right) \]  

(26)

Then, an algorithm, which is similar to that in Section 3.3, can be developed for the maximum-contrast phase adjustment.

4.3 Comparison

The maximum-contrast method converges faster than the minimum-entropy method. Under the same threshold of accuracy, the maximum-contrast method requires a smaller number of iterations than the minimum-entropy method. However, we notice that when the target has a dominant scatterer, the maximum-contrast method cares about the dominant scatterer too much. In the image obtained by the maximum-contrast method, most scatterers may not be as well focused as the dominant scatterer. The minimum-entropy method, however, avoids this problem. It can attain a good compromise among all the scatterers and result in a globally good image.

5 Results

The field data of a Boeing-727 aircraft [18], provided by Prof. B. D. Steinberg of the University of Pennsylvania, are used to evaluate our method. The aircraft was 2.7 km away from the radar and flew at a speed of 147 m/s. The radar transmitted short pulses at a wavelength of 3.123 cm and a width of 7 ns, and the echoes were sampled at an interval of 5 ns. The pulse repetition frequency was 400 Hz. 512 echoes with 120 range bins each were recorded. The 512 echoes are divided into four equal segments, and each segment is processed individually. In all the imaging processes, range alignment is carried out by the global method [17].

Figure 5 shows the images obtained by the minimum-entropy method. The adjustment phase is initialised as zero and the threshold of accuracy is chosen as 0.01. Under these conditions, the numbers of iterations are 108, 90, 64 and 94 for the four data segments, respectively. When the adjustment phase is initialised as the phase obtained by the dominant-scatterer method [1], the numbers of iterations are reduced to 37, 26, 20 and 11 for the four data segments, respectively.

Figure 6 shows the images obtained by the dominant-scatterer method [1]. Due to the existence of a dominant scatterer, the dominant-scatterer method works well for these data. Nevertheless, as we see, its focus quality is inferior to that of the minimum-entropy method.

Figure 7 shows the images obtained by the maximum-contrast method presented in Section 4. The adjustment phase is initialised as zero and the threshold of accuracy is chosen as 0.01. Under these conditions, the numbers of
iterations are 9, 17, 7 and 7 for the four data segments, respectively. This shows that the maximum-contrast method converges faster than the minimum-entropy method. However, as we see, the focus quality of the maximum-contrast method is inferior to that of the minimum-entropy method. When the numbers of iterations are further increased, no matter how big the numbers of iterations are, the images essentially keep unchanged and the focus quality is still inferior to that of the minimum-entropy method.

In the above analysis, we use simple visual inspection to evaluate the performance of the minimum-entropy method, the dominant-scatterer method and the maximum-contrast method. It is applicable to these data. Further investigation needs to be carried out on the quantitative evaluation of these methods.

A sinusoidal translational Doppler phase is added to the phase of the original signal by simulation. Figure 8 shows the images obtained by the minimum-entropy method from these simulated data. We can see that the images are still focused. In fact, this algorithm is applied to about twenty sets of field data and simulated data and always works well. It does not assume any parametric model of the adjustment phase and applies universally.

Figure 9 shows the images obtained by the minimum-entropy method when $\mu$ in (16), the threshold of accuracy, is chosen as 0.0123. The corresponding error of the adjustment phase, according to (17), is $\pi/256$. Figure 10 shows the images obtained by the minimum-entropy method in [8]. Here, the smallest step size of the adjustment phase is chosen as $\pi/256$. Under the same accuracy, for the four data segments, our method takes $1/98$, $1/81$, $1/120$ and $1/64$ as much time as the method in [8], respectively. Evidently, our method is computationally more efficient than the method in [8].

6 Conclusions

The minimum-entropy method is effective and efficient for phase adjustment in ISAR imaging. The adjustment phase is found by iteratively solving an equation, which is derived by minimising the entropy of the image. This technique can be used to estimate adjustment phases of any form. Moreover, the optimisation method used in this technique is computationally more efficient than trial-and-error methods.

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8 References