Linear Relaying Scheme for MIMO Relay System With QoS Requirements

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Abstract—In this letter, we address the problem of fulfilling quality-of-service (QoS) requirements in a multiple-input multiple-output (MIMO) relay system, where a set of target signal-to-noise ratios (SNRs) should be attained on different substreams. By solving a two-step optimization problem, we obtain a power-efficient relaying scheme that can make the SNR requirements asymptotically fulfilled. A simple relay selection method is also proposed such that the relay-power consumption can be largely reduced when there is a large number of relays.

Index Terms—Multiple-input multiple-output (MIMO) relay, power consumption, quality-of-service (QoS).

I. INTRODUCTION

THE application of multiple-input multiple-output (MIMO) technology in a wireless relay network is designed to provide extended radio coverage and improved spectral efficiency [1]. The capacity of a MIMO relay system has been well studied in several papers [2], [3], where [3] revealed the log-increasing tendency of channel capacity with respect to the number of relay nodes. When channel state information (CSI) is available, advanced signal processing techniques can be applied at the relay nodes to further enhance the system performance. For example, [4] and [5] attempt to improve the instantaneous channel capacity by performing linear processing at the relay node. Other relaying strategies can be found in [6] and the references therein.

Note that in the above works [4]–[6], the authors all consider performance optimization with constrained power resources. However, in certain wireless network with quality-of-service (QoS) requirements, to satisfy the predefined performance measure is more important than power constraints. Take multimedia application as an example, where several types of information such as video and audio have to be sent simultaneously on different substreams, it is necessary to make each subchannel attain certain signal-to-noise ratio (SNR) for successful transmission [7]. Motivated by the above problem, our objective in this work is to design a linear relaying scheme such that a set of target SNRs can be attained on different substreams. By solving a two-step optimization problem, we derive a power-efficient relaying scheme in closed form, and we demonstrate that the

SNR requirements can be asymptotically fulfilled with lower relay-power as the number of relays increases. Although a similar problem in the single-antenna relay network has been well studied [8], the MIMO relay scenario involves the additional issue of high spatial dimensions.

The notations used in this letter are defined as follows: Boldface lowercase letter $\mathbf{a}$ and boldface uppercase letter $\mathbf{A}$ represent vector and matrix, respectively. We will denote an $N \times N$ identity matrix by $\mathbf{I}_N$ and use i.i.d. for independent and identically distributed. x^* is the complex conjugate of x. Finally, $\mathrm{tr}(\cdot)$ and $(\cdot)^H$ stand for trace and Hermitian transpose of a matrix, respectively.

II. SYSTEM MODEL

Consider a MIMO relay system where a single source-destination pair with N_s antennas communicates through N relays, each using N_r $(\geqslant N_s)$ antennas. Due to long distance, there is no direct link between the source and destination. Moreover, only half-duplex relaying is considered throughout this letter. In the first time slot, the source broadcasts to all the relay nodes through backward channel. The received signal at the kth relay is

$$\mathbf{y}_{r,k} = \mathbf{H}_{1,k}\mathbf{x} + \mathbf{n}_{1,k}. \tag{1}$$

Here $\mathbf{x}$ is the input symbol vector chosen from PSK constellation with $E\{\mathbf{x}\mathbf{x}^H\} = \sigma_x^2 \mathbf{I}_{N_s}$. $\mathbf{n}_{1,k}$ is zero-mean complex Gaussian noise vector with $E\left\{\mathbf{n}_{1,k}\mathbf{n}_{1,k}^H\right\} = \sigma_1^2 \mathbf{I}_{N_r}$. For simplicity, all the backward channel matrices $\{\mathbf{H}_{1,k}\}$ are supposed to have i.i.d. Gaussian entries with zero mean and variance $\sigma_{h,1}^2$. In the second time slot, the relays forward their data simultaneously to the destination after performing linear processing on the received signal. Thus, the destination observes

$$\mathbf{y}_d = \underbrace{\sum_{k=1}^{N} \mathbf{H}_{2,k}\mathbf{Q}_k(\mathbf{H}_{1,k}\mathbf{x} + \mathbf{n}_{1,k})}_{\widetilde{\mathbf{x}}} + \mathbf{z} = \widetilde{\mathbf{x}} + \mathbf{z} \tag{2}$$

where $\mathbf{Q}_k$ is the linear processing matrix at the kth relay, and $\widetilde{\mathbf{x}}$ is the signal component that can be manipulated by $\{\mathbf{Q}_k\}$. In a similar fashion, the elements of $\mathbf{H}_{2,k}$ can be modeled as i.i.d. zero-mean complex Gaussian random variables with variance $\sigma_{h,2}^2$, and $\mathbf{z}$ is zero-mean complex Gaussian noise vector on the forward channel with $E\{\mathbf{z}\mathbf{z}^H\} = \sigma_2^2 \mathbf{I}_{N_s}$. To reduce implementation complexity, we further assume that no channel equalization is performed at the destination, and the original symbol $\mathbf{x}$ is estimated through phase detection on $\mathbf{y}_d$.

III. RELAYING SCHEME DESIGN

In this section, we will present how to design the relay processing matrices $\{\mathbf{Q}_k\}$. Our objective is to obtain a good estimation for $\mathbf{x}$ with certain QoS requirements. Specifically, a set of target SNRs should be attained on different substreams. For this purpose, we propose solving the following problem:

$$\{\widehat{\mathbf{Q}}_k\} = \arg \min_{\{\mathbf{Q}_k\}} J \qquad (3)$$

where

$$J = E\{\|\widetilde{\mathbf{x}} - \mathbf{\Lambda}\mathbf{x}\|^2\} \qquad (4)$$

is the mean-squared error (MSE) between $\widetilde{\mathbf{x}}$ and the weighted version of $\mathbf{x}$, and $\mathbf{\Lambda} = diag(\Lambda_1, \Lambda_2, \ldots, \Lambda_{N_s})$ is a real-valued diagonal matrix under design. As we show later, the optimal solution to (3) will lead to $J_{\min} \to 0$ when there are multiple relays. Thus, the received signal at the destination can be well approximated by

$$\mathbf{y}_d \approx \mathbf{\Lambda}\mathbf{x} + \mathbf{z}. \qquad (5)$$

Now the MIMO relay channel is decomposed into a set of parallel Gaussian channels, and the SNR of the kth substream becomes $SNR_k \approx \Lambda_k^2 \sigma_x^2 / \sigma_2^2$. According to QoS requirements, SNR_k should be no less than the kth target SNR. Thus, we can set

$$\Lambda_k = \sqrt{\frac{\rho_k \sigma_2^2}{\sigma_x^2}} \qquad (6)$$

with ρ_k denoting the target SNR of the kth substream. Upon solving (3) with this parameter set, the predefined SNRs can be consequently attained.

The problem (3) can be solved straightforwardly. By differentiating J with respect to $\mathbf{Q}_k$ and setting it to zero, we have

$$\frac{\partial J}{\partial \mathbf{Q}_k} = \sigma_x^2 \mathbf{H}_{1,k} \left(\sum_{k=1}^{N} \mathbf{H}_{2,k} \mathbf{Q}_k \mathbf{H}_{1,k} - \mathbf{\Lambda} \right)^H \mathbf{H}_{2,k}$$
$$+ \sigma_1^2 (\mathbf{H}_{2,k} \mathbf{Q}_k)^H \mathbf{H}_{2,k} = \mathbf{0}. \qquad (7)$$

Since $\mathbf{H}_{2,k}$ has full row rank, it follows

$$(\mathbf{H}_{2,k} \mathbf{Q}_k)^H = \mathbf{H}_{1,k} \mathbf{\Omega}^H \qquad (8)$$

where

$$\mathbf{\Omega} = \frac{\sigma_x^2}{\sigma_1^2} \left(\mathbf{\Lambda} - \sum_{k=1}^{N} \mathbf{H}_{2,k} \mathbf{Q} \mathbf{H}_{1,k} \right). \qquad (9)$$

The above expression is undesirable since $\mathbf{\Omega}$ depends on all the relay matrices $\{\mathbf{Q}_k\}$. This prevents us from obtaining a closed-form solution for $\mathbf{Q}_k$. However, such trouble can be efficiently resolved by exploiting the particular structure of $\mathbf{\Omega}$. Pre-multiplying both sides of (8) by $\mathbf{H}_{1,k}^H$ and making summation for $k = 1, 2, \ldots, N$, we will obtain

$$\mathbf{\Lambda} - \frac{\sigma_1^2}{\sigma_x^2} \mathbf{\Omega}^H = \sum_{k=1}^{N} \mathbf{H}_{1,k}^H \mathbf{H}_{1,k} \mathbf{\Omega}^H. \qquad (10)$$

By (10), we can derive

$$\mathbf{\Omega} = \mathbf{\Lambda} \left(\sum_{k=1}^{N} \mathbf{H}_{1,k}^H \mathbf{H}_{1,k} + \frac{\sigma_1^2}{\sigma_x^2} \mathbf{I}_{N_s} \right)^{-1}. \qquad (11)$$

Note that $\mathbf{\Omega}$ is constant when channel realizations are given. Thus, each $\mathbf{Q}_k$ can be obtained independently from (8).

However, we cannot yet uniquely determine the optimum $\widehat{\mathbf{Q}}_k$ that minimizes J. In fact, (8) can be translated into $N_s \times N_r$ linear equations, while for each $\mathbf{Q}_k$, we have $N_r \times N_r$ unknown variables at hand. Therefore, the solution is not unique when $N_r > N_s$. Although all the solutions to (8) lead to the same MSE performance, they usually require different relay-power. The next step is to find the specific solution that requires the minimum relay-power by solving another optimization problem, i.e.,

$$\widehat{\mathbf{Q}}_k = \arg \min_{\mathbf{Q}_k} p_k \qquad (12)$$

$$\text{s.t.} \quad p_k = tr \left\{ \mathbf{Q}_k \left(\sigma_x^2 \mathbf{H}_{1,k} \mathbf{H}_{1,k}^H + \sigma_1^2 \mathbf{I}_{N_r} \right) \mathbf{Q}_k^H \right\} \qquad (13)$$

$$\mathbf{H}_{2,k} \mathbf{Q}_k = \mathbf{\Omega} \mathbf{H}_{1,k}^H \qquad (14)$$

where p_k denotes local power at the kth relay node. From Karush–Kuhn–Tucker (KKT) conditions [9], the optimum $\widehat{\mathbf{Q}}_k$ must satisfy

$$\widehat{\mathbf{Q}}_k = \mathbf{H}_{2,k}^H \mathbf{\Psi}^H (\sigma_x^2 \mathbf{H}_{1,k} \mathbf{H}_{1,k}^H + \sigma_1^2 \mathbf{I}_{N_r})^{-1} \qquad (15)$$

where $\mathbf{\Psi}$ is the Lagrange multiplier. Substituting (15) into (14) yields

$$\mathbf{\Psi}^H \left(\sigma_x^2 \mathbf{H}_{1,k} \mathbf{H}_{1,k}^H + \sigma_1^2 \mathbf{I}_{N_r} \right)^{-1} = \left(\mathbf{H}_{2,k} \mathbf{H}_{2,k}^H \right)^{-1} \mathbf{\Omega} \mathbf{H}_{1,k}^H. \qquad (16)$$

Finally, combining (16) and (15) leads to

$$\widehat{\mathbf{Q}}_k = \mathbf{H}_{2,k}^H \left(\mathbf{H}_{2,k} \mathbf{H}_{2,k}^H \right)^{-1} \mathbf{\Omega} \mathbf{H}_{1,k}^H. \qquad (17)$$

From (8), it is clear that the above solution is also valid when $N_r = N_s$.

Note that apart from local CSI, each $\widehat{\mathbf{Q}}_k$ depends on the common matrix $\mathbf{\Omega}$ that is determined by all the backward channel matrices $\{\mathbf{H}_{1,k}\}$. Therefore, there should be a central node which collects the whole backward channel information and then computes and broadcasts $\mathbf{\Omega}$ to all the relays. However, since there are no direct CSI exchanges among the relays, the feedback overhead is largely reduced. Another interesting observation is that when $N_s = N_r = 1$, (17) will reduce to the scalar

$$\hat{q}_k = \frac{h_{1,k}^* \Lambda}{\left(\sum\limits_{k=1}^{N} |h_{1,k}|^2 + \frac{\sigma_1^2}{\sigma_x^2} \right) h_{2,k}}. \qquad (18)$$

This is consistent with the results obtained in [8] for the single-antenna relay system. Obviously, our results are more general and can be regarded as an extension to the MIMO scenario.

IV. PERFORMANCE ANALYSIS

A. MSE Behavior

We are now ready to examine how well the target SNRs can be attained in terms of MSE performance. By substituting (17) into (4), it follows

$$J_{\min} = \sigma_1^2 tr \left\{ \mathbf{\Lambda} \left(\sum_{k=1}^{N} \mathbf{H}_{1,k}^H \mathbf{H}_{1,k} + \frac{\sigma_1^2}{\sigma_x^2} \mathbf{I}_{N_s} \right)^{-1} \mathbf{\Lambda} \right\}. \quad (19)$$

To give a deeper insight, we use the following approximation for large N:

$$\sum_{k=1}^{N} \mathbf{H}_{1,k}^H \mathbf{H}_{1,k} \approx N N_r \sigma_{h,1}^2 \mathbf{I}_{N_s}. \quad (20)$$

By (20), we can obtain

$$J_{\min} \approx \frac{\sigma_1^2}{N N_r \sigma_{h,1}^2 + \frac{\sigma_1^2}{\sigma_x^2}} tr \left\{ \mathbf{\Lambda}^2 \right\} = O \left(\frac{1}{N} \right). \quad (21)$$

As it stands, $J_{\min}$ will decrease to zero as N increases. This implies that the approximation (5) will become more accurate, and the target SNRs can be asymptotically attained. However, if the QoS requirements have to be fulfilled exactly in the case of finite N, $\{\rho_k\}$ should be set slightly above the target SNRs in (6) to compensate for the potential performance loss.

B. Relay-Power Consumption

In the previous design, although relay-power consumption is not our main concern, we will show that such expenses can be efficiently controlled when there are a large number of relays. To see this, we substitute (17) into (13) and get

$$p_k = tr \left\{ \left(\mathbf{H}_{2,k} \mathbf{H}_{2,k}^H \right)^{-1} \mathbf{\Omega} \mathbf{H}_{1,k}^H \right.$$
$$\left. \cdot \left(\sigma_x^2 \mathbf{H}_{1,k} \mathbf{H}_{1,k}^H + \sigma_1^2 \mathbf{I}_{N_r} \right) \mathbf{H}_{1,k} \mathbf{\Omega}^H \right\}. \quad (22)$$

Using (20) for large N again, it follows

$$p_k \approx \frac{1}{(N N_r \sigma_{h,1}^2 + \frac{\sigma_1^2}{\sigma_x^2})^2} tr \left\{ \mathbf{T}_{2,k} \mathbf{\Lambda} \mathbf{T}_{1,k} \mathbf{\Lambda} \right\}$$
$$\leqslant \frac{1}{(N N_r \sigma_{h,1}^2 + \frac{\sigma_1^2}{\sigma_x^2})^2} tr \left\{ \mathbf{T}_{2,k} \right\} tr \left\{ \mathbf{T}_{1,k} \right\} tr^2 \left\{ \mathbf{\Lambda} \right\} \quad (23)$$

where

$$\mathbf{T}_{1,k} = \mathbf{H}_{1,k}^H \left(\sigma_x^2 \mathbf{H}_{1,k} \mathbf{H}_{1,k}^H + \sigma_1^2 \mathbf{I}_{N_r} \right) \mathbf{H}_{1,k}$$
$$\mathbf{T}_{2,k} = \left(\mathbf{H}_{2,k} \mathbf{H}_{2,k}^H \right)^{-1}.$$

The inequality comes from the fact that for two positive semi-definite matrices $\mathbf{A}$ and $\mathbf{B}$, it follows [10]

$$tr \left\{ \mathbf{AB} \right\} \leqslant tr \left\{ \mathbf{A} \right\} tr \left\{ \mathbf{B} \right\}. \quad (24)$$
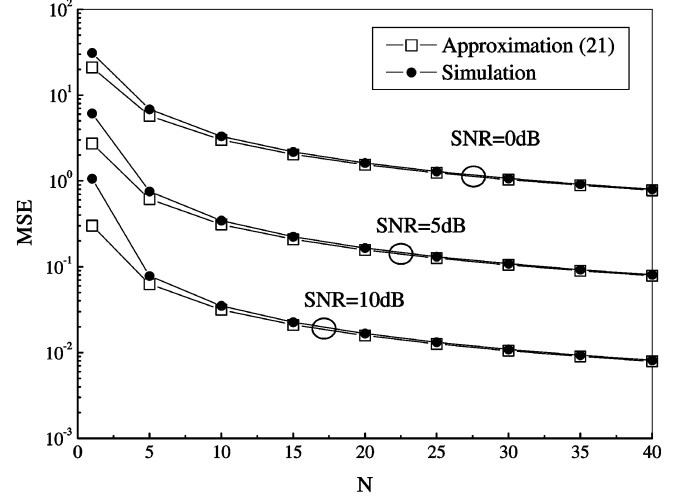

Fig. 1. Comparison of MSE performances between simulation and large N approximation (21).

In (23), we have obtained the upper bound of local relay-power by separating the influences of backward channel and forward channel. Through simulation, we find that p_k is sometimes large when the channel matrices are bad-conditioned. However, as suggested in [8], simple relay selection method can be applied to remove such nodes requiring large power. To be consistent with our analysis, we propose activating the relay nodes with $tr \{ \mathbf{T}_{2,k} \} \leqslant \gamma$ and $tr \{ \mathbf{T}_{1,k} \} \leqslant \beta$, where γ and β are thresholds that measure local power consumption. Thus, either forward channel information or backward channel information can be utilized to remove the bad relays.

The upper bound (23) now becomes

$$p_k \leqslant \frac{\gamma \beta}{\left(N N_r \sigma_{h,1}^2 + \frac{\sigma_1^2}{\sigma_x^2} \right)^2} tr^2 \left\{ \mathbf{\Lambda} \right\} = O \left(\frac{1}{N^2} \right) \quad (25)$$

and total relay-power can be approximated by

$$P_{total} = \sum_{k=1}^{N} p_k \leqslant \frac{\gamma \beta N}{\left(N N_r \sigma_{h,1}^2 + \frac{\sigma_1^2}{\sigma_x^2} \right)^2} tr^2 \left\{ \mathbf{\Lambda} \right\} = O \left(\frac{1}{N} \right). \quad (26)$$

These results are meaningful in practice because as N increases, the relay-power consumption can be largely reduced with no performance penalty.

V. NUMERICAL RESULTS

In this section, numerical results are presented to demonstrate the performance of the proposed relaying scheme. The target SNRs on all substreams are set to be 15 dB, and we assume $\sigma_1^2 = \sigma_2^2 = \sigma_n^2$ and $\sigma_{h,1}^2 = \sigma_{h,2}^2 = \sigma_h^2$. Data symbols are obtained from QPSK constellation with normalized power, and the channel SNR is defined as $\text{SNR} = \sigma_h^2 / \sigma_n^2$. Without loss of generality, we set $N_s = N_r = 2$ such that all nodes in the system have two antennas. The channels are supposed to be static within a frame comprising 200 symbols, and all results given below are averaged over 100 000 different channel realizations.
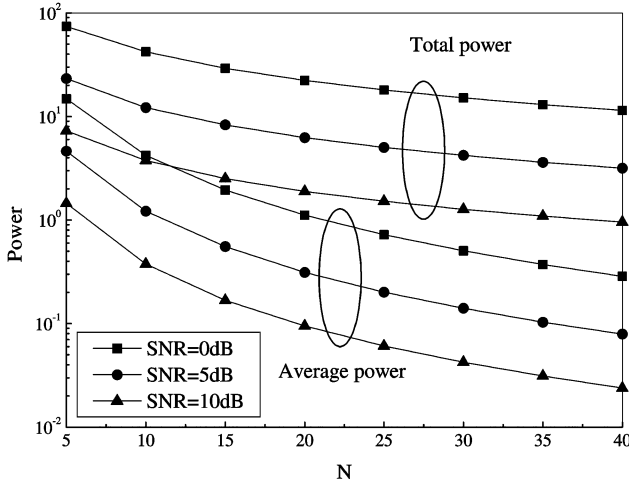
Fig. 2. Comparison of average power per relay node and total power of all relay nodes.
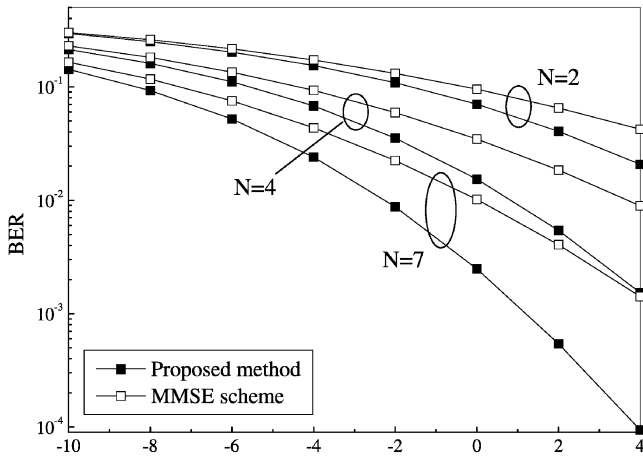

Fig. 3. BER performances of the proposed method and MMSE scheme.

Fig. 1 gives the comparison of MSE performances obtained from simulation and large N approximation (21). When N is sufficiently large (i.e., $N \geqslant 10$), theoretical results match well with simulations, and MSE drops quickly as N increases. For example, the MSE level is about 0.03 for SNR $= 10$ dB and $N = 10$, which is small enough to make the SNR requirements fulfilled. Fig. 2 shows the relay-power for fixed channel SNR when N varies. Proper relay selection is performed with $\gamma = 50$, and no constraint is imposed on $tr\{\mathbf{T}_{1,k}\}$. Clearly, both average power and total power decrease quickly as N increases, and lower power is required when channel SNR becomes higher. Another observation is that average power drops more quickly

than total power. This is consistent with the results in (25) and (26). Finally, we compare the proposed method with MMSE relaying scheme [6] in terms of bit-error rate (BER), as shown in Fig. 3. For each channel realization, we assume that the two schemes use the same total power. It is observed that the proposed method outperforms MMSE scheme in all cases. Such performance gain is due to adaptive power allocation among relay nodes, whereas local power constraints are set in MMSE scheme such that all relay nodes use the same power.

VI. CONCLUSION

In this letter, we have proposed one linear relaying scheme for MIMO relay system to attain a predetermined set of SNRs on different substreams. Power-efficient relaying strategy is derived in closed form after solving a two-step optimization problem. As the number of relays increases, theoretical analysis shows that the SNR requirements can be asymptotically fulfilled with lower relay-power consumption.

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