# Low Complexity Multiuser Scheduling in MIMO Broadcast Channel with Channel Quantization

Feng She, Hanwen Luo, and Wen Chen Department of Electronic Engineering Shanghai Jiaotong University Shanghai 200030, P. R. China Email: {shefeng630;wenchen}@sjtu.edu.cn

Abstract—This paper deals with the design and analysis of low complexity user scheduling algorithm in multiantenna broadcast (downlink) systems under zero-forcing multiplexing. By using quantization technology, the channel matrix can be divided into serval unoverlapped channel regions. Based on the quantized channel regions, we can get semi-orthogonal region sets. The presented user scheduling algorithm in this paper is based on using the semi-orthogonal region sets. Simulation results shows that this algorithm can achieve a sum rate close to the full searching algorithms while with much lower complexity than those of the previous algorithms.

#### I. Introduction

Multiple-input multiple-output (MIMO) system is well motivated for wireless communications through fading channels due to the potential improvements in transmission rate or diversity gain [1]. It is well known that multiple antennas can be easily deployed at base station in cellular systems. However, mobile terminals usually have a small number of antennas due to the size and cost constraint. Thus, it may not be able to obtain significant capacity benefit from the multiple transmit antennas. This is true with the transmit strategy of time division multiple access (TDMA) [2]. To solve the problem, multiuser must be served simultaneously. One way to accomplish this is called dirty paper coding (DPC), which is a multiuser encoding strategy based on interference presubtraction [4]. Since DPC is with high complexity, [7] presents zero forcing dirty paper coding (ZF-DPC) as a suboptimal solution, which reduces the complexity, but ZF-DPC still has a very high implementation cost due to successive encoding. As a much simple transmit strategy, zero forcing beamforming (ZFBF) techniques have been proposed for space division multiple access (SDMA) to remove the cochannel interference in MIMO downlink systems [8], [10]. Compared with DPC and ZF-DPC, ZFBF can greatly reduce the complexity while keeping the throughput region close to optimal when the number of user K is large enough [3]. Then ZFBF greatly reduces the complexity while keeping the throughput region close to optimal.

In general, finding the optimal active user sets in ZFBF requires an exhaustive search over all users. We consider the problem of jointly multiplexing and scheduling multiple users in the wireless downlink systems. Multiuser scheduling is the problem of allocating resource (such as power and bandwidth) in order to perform desirably with respect to criteria such

as throughput or performance. This problem has attracted great interest in the recent years [11], [12]. In [12], the authors propose a semi-orthogonal user scheduling algorithm to reduce interference among different data streams. In [17], a similar idea is used to develop a greedy user sets selection, which is shown to achieve the optimal asymptotic sum rate. In [3], a better user sets selection scheme based on clique (full connected subgraph) graph is proposed. Because both greedy search algorithm and clique search algorithm are of high complexity, it is necessary to find an user scheduling algorithm with low complexity. In [2], a semi-orthogonal user scheduling (SUS) algorithm is proposed. Though SUS is of low complexity, it can not guarantee the transmitter can get the optimal user sets. This motivates us to find a low complexity user scheduling algorithm that can get better user sets than SUS in sum-rate sense.

In this paper, we propose a low complexity user scheduling algorithm which is based on channel quantization. The underlying idea is that the multiuser channel can be modeled as a weighted graph by quantized channel with single user channel gain as node weights.

The contributions of this paper are as follows.

- We present a new clique graph construction algorithm with low complexity for MIMO broadcast systems.
- We propose a low complexity user scheduling algorithm with quantized channel region sets.

The paper is organized as follows. We outline the system model in section II. In section III, we introduce the transmit strategies. The proposed scheduling algorithm is presented in section IV. We analyze the complexity in section V. The simulation results are presented in section VI. Finally we conclude this paper in section VII.

Notation used in this paper are as follows:  $(\cdot)^T$  denote matrix transposition,  $(\cdot)^H$  denotes matrix conjugate-transposition, and  $tr(\cdot)$  is trace of channel matrix,  $E[\cdot]$  denotes statistical expectation, and  $\|\cdot\|^2$  denotes the mean square norm of a vector.

#### II. MULTIUSER BROADCAST CHANNEL MODEL

We consider a single-cell MIMO BC system with a single base station supporting data traffic to K users. The base station is with  $N_t$  transmit antennas and each of the user terminal has single receive antenna. We assume  $K \geq N_t$ . For simplicity,

we assume that all the users experience independent fading. Thus, the signal received by user k is given by

$$\mathbf{y}_k = h_k x + n_k, \quad k \in \{1, \dots, K\},$$
 (1)

where  $x \in \mathbb{C}^{N_t \times 1}$  is the transmit signal vector with a power constraint  $tr(E[xx^H]) \leq P$ , and  $n_k$  is complex Gaussian noise with unit variance per vector component, i.e.,  $E[nn^H] = I$ , and  $h_k \in \mathbb{C}^{1 \times N_t}$  is the multiple-input single-output (MISO) channel gain matrix to the kth user.

At transmitter, we employ the ZFBF transmit strategy. In ZFBF, the scheduler first selects an active user set  $S \subset \{1,2,\ldots,K\}$ , where the set size  $|S| \leq N_t$ . Then, the transmitter assigns different orthogonal beamforming directions to each data stream in such a way that the interference at each receiver is completely suppressed.

Denote  $h_i$ ,  $i \in \{1, \dots, |S|\}$  as the channel to the *i*th active user, and define  $H(S) = [h_1^T, \dots, h_{|S|}^T]$ . Then the transmit signal is represented as

$$\mathbf{x} = \sum_{i=1}^{|S|} \sqrt{P_i} w_i s_i,\tag{2}$$

where  $s_i$ ,  $w_i$ , and  $P_i$  are data symbol, beamforming vector, and transmit power for the *i*th active user, respectively. Then the received signal at the *i*th active user is given by

$$\mathbf{y}_{i} = \sqrt{P_{i}} h_{i} w_{i} s_{i} + \sum_{j=1, j \neq i}^{|S|} \sqrt{P_{j}} h_{i} w_{j} s_{j} + n_{j}.$$
 (3)

From channel realization,  $H = \{h_1^T, h_2^T, \dots, h_K^T\}$ , Multiuser MIMO can be represented as a node weight graph [3]. In this paper, we assume that the base station has perfect channel state information (CSI) of all the downlink channels, while each of the users only has the CSI of its own downlink channel and does not know the CSI of downlink channel of other users.

#### III. MULTIUSER TRANSMIT STRATEGIES

In this section, we briefly introduce the exemplary MIMO-BC transmission schemes of DPC and ZFBF.

## A. Dirty paper coding (DPC)

It is now well known that DPC achieves the sum capacity of the multiple antenna broadcast channel as well as the full capacity region [15]. In precise, DPC is a precoding technique that precancel interference at the transmitter [4], [7], [14], [15], [16].

We now describe the transmit signal when DPC is utilized. Let  $s_k \in C^{N_t \times 1}$  be the  $N_t$  dimensional vector of data symbols intended for user k and  $v_k \in C^{N_t \times N_r}$  be its precoding matrix. Then the transmit signal vector x can (roughly) be represented as [7]

$$x = v_1 s_1 \oplus (v_2 s_2 \oplus \cdots \oplus (v_{K-2} s_{K-2} \oplus (v_{K-1} s_{K-1} \oplus v_K s_K)) \cdots), \quad (4)$$

where  $\oplus$  represents the nonlinear dirty paper sum. Without loss of generality, we have assumed that the encoding process is performed in descending numerical order. Dirty-paper decoding at the kth receiver will cancel the interference of  $v_{k+1}s_{k+1}, \cdots, v_Ks_K$ , and thus the effective received signal at user k is:

$$y_k = H_k v_k s_k + \sum_{j=1}^{k-1} H_k v_j s_j + \mathbf{n}_k,$$
 (5)

where the second term is the multi-user interference which is not canceled by DPC principle. If the  $s_k$  are chosen Gaussian, the rate of the kth user is expressed as:

$$R_k = \log_2 \frac{|I + H_k(\sum_{j=1}^k \sigma_j) H_k^H|}{|I + H_k(\sum_{j=1}^{k-1} \sigma_j) H_k^H|}, \tag{6}$$

where  $\sigma_j = v_j E[s_j s_j^H] v_j^H$  denotes the transmit covariance matrix of user j. Since DPC is optimal transmit scheme, the sum capacity of the MIMO BC can be expressed as:

$$C_{DPC}(H, P) = \max \sum_{k=1}^{K} \log_2 \frac{|I + H_k(\sum_{j=1}^k \sigma_j) H_k^H|}{|I + H_k(\sum_{j=1}^{k-1} \sigma_j) H_k^H|},$$
(7)

The duality of the MIMO BC and the MIMO MAC allows the sum capacity to be alternatively written as [13]

$$C_{DPC}(H, P) = \max \sum_{k=1}^{K} \log_2 |I + \sum_{k=1}^{K} \sigma_K H_k^H Q_k H_k|,$$
 (8)

where  $Q_k$  represents the  $N_t \times N_t$  transmit covariance matrices.

#### B. Multi-antenna ZFBF

In this subsection, ZFBF in multi-antenna broadcast channel (BC) is introduced. As mentioned above, multiple transmit antennas can potentially yield an  $N_t$ -fold increase in the sum capacity, where  $N_t$  is the number of transmit antennas. [2] showed that employing ZFBF to a set of  $N_t$  nearly orthogonal users with large channel norms is asymptotically optimal as the number of users grow large.

In multiuser MISO systems, we first select a user subset S to be served together and then build the corresponding channel matrix H(S). Then the beamforming matrix  $\mathcal{W}(S)$  is written as

$$W(S) = H(S)^{H} (H(S)H(S)^{H})^{-1}.$$
 (9)

As a result, the achievable throughput of ZFBF for a given user set S is given by

$$R_{ZFBF}(S) = \max \sum_{i \in S} \log_2(1 + P_i).$$
 (10)

We define  $b_i$  as the effective channel gain to the *i*th user. Then the power constraint is

$$\sum_{i} P_i / b_i \le P,\tag{11}$$

where  $b_i = \frac{1}{[(H(S)H(S)^H)^{-1}]_{i,i}}$ , and  $P_i$  in (10) can be easily obtained by waterfilling as

$$P_i = (\mu b_i - 1)^+, \tag{12}$$

where  $(x)^+$  denotes  $\max\{x,0\}$ , and  $\mu$  is waterlevel.

## IV. SCHEDULING UNDER ZERO-FORCING MULTIPLEXING

In this section, we provide scheduling algorithm of clique search based on channel quantization.

For finite user number of K, the probability of existence of an orthogonal set is zero. Thus, we consider the user sets which are "nearly" orthogonal in scheduling scheme. To be precise, we define two vectors  $v_1$  and  $v_2$  to be  $\alpha$ -orthogonal if

$$\frac{|v_1 v_2^H|}{\|v_1\| \|v_2\|} \le \alpha. \tag{13}$$

#### A. Channel quantization and codebook

The problem of channel quantization is the problem of vector (or matrix) quantization (VQ). In this paper, we attempt to divide the channel space into several unoverlapped channel regions. In other words, the space of channel matrix is divides into N non-overlapped regions. For each of these regions, there is a codeword to denote the channel vector in the region. The set of codewords is called codebook.

We consider codebooks construction from fast Fourier transform matrices [5], [6]. This class of codewords in the codebook can be thought of as subset of m columns of the  $N \times N$  FFT matrix [19]. More precisely, the codebook consists of m distinct columns chosen from an  $N \times N$  FFT matrix, with index set  $\mathbf{u} = [u_1 \ u_2 \dots u_m]$ , denoted  $\mathcal{C}_{FFT}(\mathbf{u}, N)$ , be the codebook of size N with codewords taken to be columns of

$$\frac{1}{\sqrt{m}} \begin{pmatrix}
1 & 1 & \cdots & 1 \\
e^{j\frac{\pi}{N}u_1} & e^{j\frac{\pi}{N}u_2} & \cdots & e^{j\frac{\pi}{N}u_m} \\
e^{j\frac{\pi^2}{N}u_1} & e^{j\frac{\pi^2}{N}u_2} & \cdots & e^{j\frac{\pi^2}{N}u_m} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j\frac{\pi(N-1)}{N}u_1} & e^{j\frac{\pi(N-1)}{N}u_2} & \cdots & e^{j\frac{\pi(N-1)}{N}u_m}
\end{pmatrix} (14)$$

These codebook is known to achieve the smallest  $\mu$  for a given N in very special cases.

Using this construction, the RMS inner product magnitude is the same for all codewords. That is,

$$\mu_{rms}(\mathcal{C}_{FFT}) = \max_{j} \sqrt{\sum_{i \neq j} |c_i^H c_j|^2}$$

$$= \sqrt{\sum_{i \neq k} |c_i^H c_k|^2}, \ \forall c_k \in \mathcal{C}_{FFT}.$$
(15)

Moreover, it is important note that under the assumption of uncorrelated Gaussian channel vectors each user channel vector is equally like to be quantized to any code index. Lastly, from (14), we can see that the correlation between very two users is only a function of the magnitude of the difference of the indices in user scheduling [9]. That is,

$$|c_i^H c_j| = g(|i-j|) \tag{16}$$

for some function g. Thus, these properties of FFT based codebook will be very valuable in scheduling of users with complexity constraint.

#### B. Selecting semi-orthogonal user sets by codebook

The main idea of our user scheduling algorithm with quantized channel is to use semi-orthogonal codewords to construct the semi-orthogonal user sets. The user scheduling algorithm is intended for systems with  $K\gg N_t$ , and for sum-rate maximization.

Before user scheduling, we firstly construct semi-orthogonal relationship among codewords in a codebook with a certain  $\alpha$ which is defined in (13). Because the each codeword denote a channel region, then we can thought the semi-orthogonal relationship among codewords is the same as the semi-orthogonal relationship as channel regions. Thus, from a codebook, we construct a channel-region semi-orthogonal relationship table  $T(\mathcal{R}, \alpha)$ , where  $\mathcal{R}$  denotes the region of channel vector, and  $\alpha$  captures pair-wise semi-orthogonal relation between the region when  $\frac{|c_i \cdot c_j^H|}{\|c_i\| \cdot \|c_j\|} \le \alpha$ , which means that the two regions  $R_i$  and  $R_j$  are  $\alpha$ -orthogonal region (note  $c_i$  and  $c_j$  are codewords in region  $R_i$  and  $R_j$  respectively). Fig. 1 is a semi-orthogonal region relation table for a codebook of the size 16. By using the codebook, the channel vector space is divided into 16 unoverlapped region. Take a codebook of size 16, Fig. 1 shows the semi-orthogonal relationship among the 16 quantized channel regions with  $\alpha = 0.1$ . Here, "o" denotes that the two region are not semi-orthogonal, and " $\sqrt{}$ " denotes that the two region are semi-orthogonal. Based on the semiorthogonal table, we can further search for semi-orthogonal region sets which include no more than  $N_t$  semi-orthogonal regions. If

$$\frac{|c_i c_j^H|}{\|c_i\| \cdot \|c_j\|} \le \alpha, \ i, j \in S, |S| \le N_t. \tag{17}$$

where  $c_i$  is the codeword denoting the ith quantized channel region, we can think the channel regions which are denoted by the codewords as a semi-orthogonal channel region set. Based on the semi-orthogonal region sets, the user set can be chosen. In other words, the semi-orthogonal channel region sets reflects the orthogonal relationship between users. Thus, in user scheduling, we only need to consider users in the  $\alpha$ -orthogonal channel regions sets. Then, by (17), the user sets (or clique graph) can be easily constructed.

Therefore, by (17), the semi-orthogonal channel region sets and semi-orthogonal user sets (or clique graph) can be easily constructed. With certain  $\alpha$  and certain codebook, we can calculate the channel region sets which meet with the semi-orthogonal condition. To decide which region that the channel vectors belong to, we define distance as  $d(c,h) = |c \cdot h^T|^2$ . Then we use the following nearest condition

if 
$$d(c_i, h) < d(c_i, h)$$
 then  $h \in R_i$  (18)

where  $c_i$  and  $c_j$  are the codewords in channel region  $R_i$  and (16)  $R_j$  respectively.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	О	О	√	√	0	√	√	0	√	О	√	√	О	√	√	o
2	0	0	o	√	√	0	√	√	0	√	О	√	√	О	√	√
3	~	0	o	0	√	√	0	√	7	o	√	o	√	√	o	√
4	~	√	o	0	0	√	√	0	~	√	О	√	o	√	√	О
5	0	√	√	0	0	0	√	√	0	√	√	o	√	О	√	√
6	~	0	√	√	0	0	0	√	7	О	√	√	o	√	o	√
7	~	√	o	√	√	0	0	0	<b>√</b>	√	o	√	√	o	√	o
8	0	√	√	0	√	√	0	0	0	√	√	o	√	√	О	√
9	~	0	√	√	0	√	√	0	0	О	√	√	o	√	√	О
10	0	√	o	√	√	0	√	√	0	О	О	√	√	О	√	√
11	<b>√</b>	0	√	0	√	√	0	√	<b>√</b>	О	О	o	√	√	o	√
12	~	√	o	√	0	√	√	0	<b>√</b>	√	О	o	О	√	√	О
13	0	√	√	0	√	0	√	√	0	√	√	o	О	О	√	√
14	√	О	√	√	О	√	o	√	√	О	√	√	О	О	o	√
15	√	√	О	√	√	o	√	О	√	√	О	√	√	О	О	О
16	0	√	√	o	√	√	0	√	О	√	√	o	√	√	0	О

Fig. 1. The semi-orthogonal relationship among codewords in codebook with size 16, and  $\alpha=0.1$ .

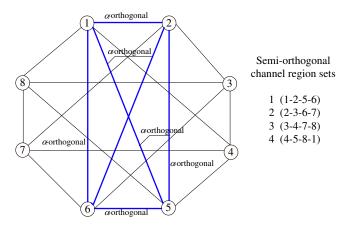


Fig. 2. The semi-orthogonal regions sets of codebook with  $\alpha=0.1$  and size 8.

Here, we express the construction of channel region sets by taking a codebook of size 8. As mentioned above, code book of size 8 means that the channel space matrix is divided into 8 un-overlapped regions. Then as shown in Fig. 2, we can get four groups of semi-orthogonal channel region sets by  $\alpha=0.1$ .

In user scheduling, we only need to compare the user sets which in the corresponding semi-orthogonal channel region sets. In this way, the scheduling algorithm complexity will decrease greatly. In the next section, we will investigate the scheduling algorithm by using the seni-orthogonal channel region sets.

## C. Low complexity scheduling algorithm with quantization

In this section, we provide user scheduling algorithm which is based on the  $\alpha$ -orthogonal channel region sets. The motivation of using those sets is that the optimal ZFBF user sets is a clique with high probability when the number of user K is very large.

As we assumed the transmitter knows perfect channel state information (CSI) of users, the transmitter know which channel vector belong to which channel region. We can assume that the transmitter can get the information by feedback from terminals.

Based on the knowledge, the transmitter carries out the following scheduling algorithm.

S1 In each time slot, the transmitter calculate the project term of each user by

$$p_{hi} = |h_i c_k^H|^2, (19)$$

where  $h_i$  is the channel vector of user i which is in channel region k, and  $c_k$  is the codeword denoting the channel region. Then in each channel region, the user with the largest project term  $p_{hi}$  will be selected.

S2 Calculating the capacity of each user set. The user set with the highest capacity will be the selected active user set.

The semi-orthogonal user sets can be constructed by the principle mentioned above. To each of those user sets, we first sort the channel gain  $|h_i h_i^H|$  in decreasing order, i.e.,  $||h_{\pi 1}||^2 \ge ||h_{\pi 2}||^2 \ge \ldots \ge ||h_{\pi \ell}||^N$ , where  $\ell \le N_t$  is the user number in the user set. Then, in each set, we calculate

$$g_t = h_{\pi t} - \sum_{i=1}^{t-1} \left( \frac{h_{\pi t} g_i^H}{\|g_i\|^2} \right) \cdot g_i, \quad t = 1, \dots, \ell.$$
 (20)

Let  $q_r = \sum_{i=1}^{\ell} \|g_i\|^2$ , where  $r = 1, \dots, R$ . Here, R is the number of semi-orthogonal channel or user set of a certain codebook with certain  $\alpha$  constraint. Then, the user selection criteria is as following

$$r = \arg\max q_r, \ r \in \{1, \dots, R\}.$$
 (21)

As a result, the users in channel user set r is the selected active user set in this time slot.

In the others time slots, the transmitter only need to repeat S1 and S2.

#### V. COMPLEXITY ANALYSIS

In this section, the complexity of the proposed user scheduling algorithm is analyzed. Due to DPC and greedy user selection algorithm are with high complexity, we only compare the complexity of proposed algorithm with that of SUS algorithm which is with low complexity. The SUS algorithm which is mentioned in [2] consists two stages: user selection using semi-orthogonal algorithm and a beamforming weight vector calculation. We note that the latter stage has a small fixed complexity, requiring only one  $N_t \times N_t$  matrix inversion  $W(S) = H(S)^{-1}$  to obtain beamforming weights. Henceforth, we concentrate on the complexity of SUS algorithm.

In step2) of SUS algorithm presented in [2], the complexity of computing  $\frac{\|g_{(i)}^Hg_{(i)}\|}{\|g_{(i)}\|^2}g_{(i)}$  is  $N_t^2+N_t+1$ , and each user need one  $(1\times N_t)\times (N_t\times N_t)$  vector-matrix multiplication with the complexity is  $N_t^2.$ 

In step 3) of SUS algorithm, each user need to calculate the channel vector norm, and the complexity is  $N_t$ .

In step 4), each user need to compute  $\frac{|h_k g_{(i)}^H|}{\|h_k\| \cdot \|g_{(i)}\|}$ , thus the complexity is  $N_t + 3$ .

Let T be the number of total times of user search in SUS algorithm, then the total computing complexity of SUS algorithm is

$$C_{SUS} = [N_t^2(T - K) + (N_t^2 + N_t + 1)(M - 1)] + N_t T + (N_t + 3)T$$

$$= (2N_t + 3)K + (N_t^2 + 2N_t + 3)(T - K)$$
(22)

Now, let us discuss the complexity of the proposed scheduling algorithm. As mentioned above, the transmitter knows which channel region the user channel vectors belong to. This can be realized by feedback from users. Therefore, we focus on calculating the complexity of user scheduling. As mentioned in section IV, there are 2 steps in user set selection.

In step 1, each user in each channel region needs to compute  $\|h_i c_k^H\|^2$ . Then the complexity is  $N_t + 1$ , and the total calculating complexity is  $(N_t + 1)K$ , where K is user number.

In step 2), we need to compute  $\|h_k\|^2$ ,  $\|g_i\|^2$  and  $\left(\frac{h_{\pi t}g_i^H}{\|g_i\|^2}\right)g_i$ . The complexity of computing  $\|h_k\|^2$  is  $N_t+1$ ; The complexity of computing  $\|g_i\|^2$  is  $N_t$ , which need to be computed in each channel regions (N times); The complexity of  $\left(\frac{h_{\pi t}g_i^H}{\|g_i\|^2}\right)g_i$  is  $(N_t+1)+N_t=2N_t+1$ , which need to be computed M(M-1)R/2 times per group. Thus the total computational complexity is

$$C_{FFT} = (M+1)K + MR + (2M+3)M(M-1)R/2.$$
 (23)

From (22), and (23), we can see that the computational complexity of SUS and our presented scheduling algorithm approximate to linear function of user number K by following result

$$\lim_{K \to \infty} \frac{C_{SUS}}{C_{FFT}} = \frac{2N_t + 3}{N_t + 1} \tag{24}$$

Thus, when K is large enough, the complexity of the proposed algorithm will be sure lower than that of SUS algorithm. For example, when  $N_t=4$ ,  $\alpha=0.3$ ,  $T\approx 1.3K$ , and codebook size of 24, and the semi-orthogonal sets is R=78. Then we get

$$C_{SUS} - C_{FFT} \approx (11K + 27 \times 0.3K + 63)$$
  
-  $(5K + 4.24 + 66.78) = 14.1K - 5181, (25)$ 

we can find that when K>368, the complexity of channel quantization based algorithm is lower than that of SUS algorithm.

#### VI. SIMULATION RESULT AND DISCUSSION

In this section, we provide some numerical examples to illustrate the performance of the proposed user scheduling algorithm. In the considered multiuser MIMO downlink systems, the number of transmit antenna is  $N_t=4$ , and each user has single receive antenna. We let the power in simulation be P=10dB. With spatial multiplexing, the number of the

active users in one time slot can not exceed the number of the transmit antennas.

We assume that the discrete-time channel impulse response is generated according to the Hiperlan2 Channel Model C in [18]. The channels between different transmit and receive antennas are assumed to be independently.

A. Experiment 1: The first experiment test the semi-orthogonal channel region sets for different  $\alpha$ . The codebook used in this experiment is designed by the principle in section III. Here, N=8,16,32,64 denote the size of the different codebooks. From Fig. 3, we can see that the number of semi-orthogonal channel region sets will increase with the increment of  $\alpha$  which is defined as (13). We can also find that codebooks with bigger size will have more channel region sets than that of codebooks with smaller size. This implies that big size codebook will result in higher complexity in user scheduling while get better performance. Thus, there exists a tradeoff between codebook size and complexity.

B. Experiment 2: In this experiment, we test the multiuser capacity of MIMO broadcast systems with different codebook sizes and channel region sets. Fig. 4 shows that the capacity increase with the increment of the number of users. This is because that the systems can get multi-user diversity with big number of users. In this experiment, the codebook size are 24 and 32, and  $\alpha$  used here are 0.01 and 0.4 respectively. As illustrated in experiment 1, with bigger  $\alpha$ , the number of semi-orthogonal channel region sets is bigger. Fig. 4 shows that with certain  $\alpha$ , the bigger size codebooks will achieve higher system capacity.

C. Experiment 3: The third experiment is about the capacity of the presented user scheduling algorithm, DPC, TDMA and semi-orthogonal user scheduling (SUS) algorithm presented in [2]. For SUS algorithm, we choose optimal  $\alpha$  range from 0.25 to 0.36. Fig. 5 shows that the presented algorithm, DPC and SUS can achieve higher capacity than that of TDMA. We can also find that the codebook of size 24 with 78 semi-orthogonal channel region sets and the codebook of size 32 with 144 semi-orthogonal channel region sets can achieve a moderate higher capacity than SUS. This implies that the presented scheduling algorithm will get higher capacity while has much lower complexity than SUS which will also be shown in the next experiment.

D. Experiment 4: The fourth experiment is about the complexity of the presented scheduling algorithm and that of SUS algorithm. By analyzing SUS algorithm proposed in [2], we can get that the computing complexity order  $C_{SUS} = (2N_t+3)K+(N_t^2+N_t+1)(N_t-1)+(N_t^2+2N_t+3)(T-K)$ . According to the scheme algorithm in section IV, we can get the computing complexity is  $C_{FFT} = (N_t+1)K+N_tR+(2N_t+3)N_t(N_t-1)R/2$ . In this experiment,  $N_t=4$ ,  $\alpha=0.3$ ,  $T\approx 1.3k$ , the codebook size is 24, and the number of semi-orthogonal channel region sets R=78. From Fig. 6, we can see that the proposed algorithm is with much lower complexity than that of SUS when user number is greater than 400. This implies that the complexity of our algorithm will be lower than that of SUS algorithm when the system is with large number.

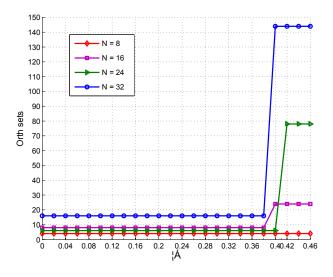


Fig. 3. The number of orthogonal sets with different codebooks.

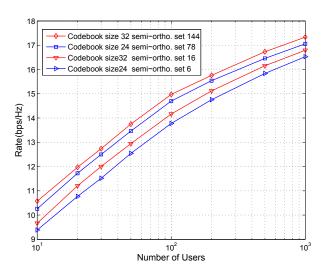


Fig. 4. The throughput of different codebooks with different  $\alpha$ .

# VII. CONCLUSION

In this paper, we present a low complexity user scheduling algorithm with channel quantization in MIMO broadcast systems . The objective of the user scheduling is to reduce the computing complexity of user selection and achieve sumrate optimization. The proposed user scheduling algorithm is with very low complexity in user scheduling than previous works. We also show that the sum-capacity will increase with the increment of the number of user. This is because that with large number of users, the transmitter can choose users with good channel condition including channel gain and orthogonality among user channel vectors.

In this paper, we only investigated users with single receive antenna. The case of users with multiple receive antennas would be an important extension of the paper. Moreover, this

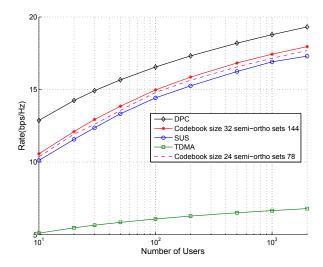


Fig. 5. The throughput of different transmit strategy

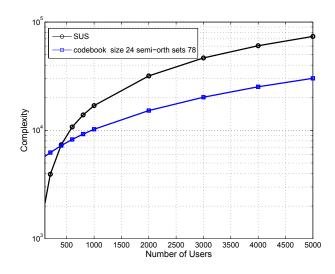


Fig. 6. The comparison of complexity between quantization and SUS

paper assumes that the base station has perfect channel state information. When the transmitter only has partial channel knowledge, it will be a new practical problem.

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