

Distributed Relay-Source Matching for Cooperative Wireless Networks Using Two-sided Market Games

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Abstract— In this paper, we address the incentive-based relay selection problem over multi-source and multi-relay wireless networks. A two-side market game approach is employed to jointly consider the benefits of all sources and relays. The equilibrium concept in such games is called *core*. The outcomes in the core of the game cannot be improved upon by any subset of players. These outcomes correspond exactly to the price-lists that competitively balance the benefits of all sources and relays. When the price assumes only discrete values, the core of the game is defined as *discrete core*. The Distributed Source-Relay Assignment (DSRA) algorithm is proposed for competitive price adjustment and converges to the discrete core of the game. With small enough measurement of price, the algorithm can achieve the optimal performance compared with centralized one in terms of total profit of the system.

I. INTRODUCTION

Extensive researches in recent years have shown that the users' cooperative relaying transmissions [1]- [3] play important roles in wireless networks. Efficient schemes for coordinating such transmissions and designing incentive mechanisms are fundamental problems, and have attracted a lot of interests. The idea of pricing as an incentive mechanism to encourage selfish users to provide cooperation has been studied in many peer to peer scenarios. For example, in the virtual currency systems [4] [5], each node was associated with a certain amount of virtual currency to construct protocols that rewarded packet forwarding in wireless ad hoc networks.

Wang *et al.* [6] proposed a distributed algorithm for relay selection and price updating based on a buyer/seller game for one source scenario. Shastry *et al.* proposed a pricing non-cooperative game framework to stimulate the cooperative diversity in WLAN for the situation with one user and one certain relay in [7]. A distribute relay selection without pricing scheme was proposed in [8] which just considered the scenario of single user with multiple relays. Based on coalitional and repeated game theory, in [9], Han and Poor proposed a scheme to form stable cooperative coalition for AF protocol. Mathur *et al.* [10] investigated the stability of the grand coalition, i.e., the coalition of all users in an interference channel to transmit and receive cooperatively. However, the above two works do not consider relay selection and are only suitable for the scenario where each node has traffic to send.

The focus of this paper is to study the networks with some transmit-receive links and some idle nodes (i.e., nodes without traffic) in frequency divisible channels (e.g., orthogonal

frequency-division multiple access (OFDMA) networks). The wireless devices communicate in half-duplex mode (either transmit or receive, but not both). The source in each link can request cooperation from the idle nodes. Due to the different channel conditions, the sources may evaluate the same relay differently. In other words, multiple sources will compete with each other to get help from multiple relays. And, the multiple relays complete with each other to ask price. There are two questions to be answered for such an environment: the first one is how to select the relays for each source; the second one is how to decide the competitive prices for the multiple sources and multiple relays. To address these problems, we propose a source-relay assignment game framework to jointly consider the benefits of all sources and relays. The assignment game [11] is a model for a two-sided market in which indivisible units are exchanged for money.

The remainder of the paper is organized as follows: Section II describes the system model and formulates the problems. In Section III, the source-relay assignment game approach is proposed to analysis the problems. In Section III, we design DSRA algorithm. Several simulation results are shown in Section IV. Finally, we conclude the paper in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The system model is shown in Figure 1. The sources provide payment to the relays for getting cooperation. The idle users serve as relays for the busy users by getting the reward. We shall refer to idle users and busy users simply as relays and sources respectively. The transmitted energy of the sources for each symbol is $P_s = E$.

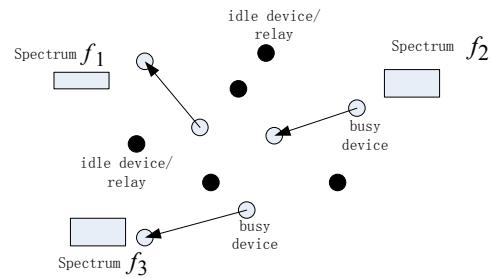


Fig. 1. System model.

There are N busy users forming the source set $S = \{s_1, \dots, s_i, \dots, s_N\}$. Denote the set $D = \{d_1, \dots, d_i, \dots, d_N\}$

as the destinations corresponding to the sources. The available M idle users form the relay set $R = \{r_1, \dots, r_j, \dots, r_M\}$. In this paper, the nonorthogonal amplify-and-forward (NAF) transmission pattern [3] is used. Other protocols can be considered in a similar way. The nonorthogonal protocols lead to higher spectral efficiency than orthogonal AF (amplify-and-forward) protocol. we assume that each source select at most one relay.

In NAF protocol, the data is transmitted by the cooperation frame where a cooperation frame is defined as two consecutive symbol intervals. The source transmits on every symbol interval in a cooperation frame. The relay, on the other hand, transmits only once per cooperation frame. It simply repeats the noise and signal it observed during the previous symbol interval. The channel between the source i and its destination is denoted as g_{s_i,d_i} . Source i has the bandwidth W_i . Thus the maximum data rate of direct transmission is:

$$I_{s_i,d_i} = W_i \cdot \log(1 + \frac{P_s * |g_{s_i,d_i}|^2}{\delta_v^2}) \quad (1)$$

where δ_v^2 is the noise power at the destination.

Let β_{s_i,r_j} denote the amplify gain of the relay j to source i . The relay maintains a constant energy of E at the output, thus β_{s_i,r_j} is [3]:

$$\beta_{s_i,r_j} = \sqrt{\frac{E}{E|h_{s_i,r_j}|^2 + \delta_w^2}} \quad (2)$$

where h_{s_i,r_j} is the channel gain between source i and relay j . δ_w^2 is the noise power at the relay.

Note that a source-relay pair occupies a certain spectrum. We assume that the channels do not change during the process of relay-source assignment. The signals received by the destination of s_i with the cooperation of r_j in two symbol intervals are:

$$y_1^i = g_{s_i,d_i}x_1^i + v_i \quad (3)$$

$$y_2^i = g_{s_i,d_i}x_2^i + g_{r,d}\beta_{s_i,r_j}(h_{s_i,r_j}x_1^i + w_j) + v_i \quad (4)$$

In above equations (3) and (4), x_1^i is the information symbol in first half of a cooperation frame, x_2^i is the information symbol in second half of a cooperation frame. v_i represents the noise observed by the destination. w_j is the noise sample observed by the relay j . In order to decode the message, the destination needs to know the relay repetition gain β_{s_i,r_j} , the source-relay channel h_{s_i,r_j} , and the relay-destination channel g_{r,j,d_i} .

The maximum data rate of source s_i by the two-slot NAF protocol with the cooperation of r_j , is given by [3]:

$$I_{s_i,r_j,d} = W_i \cdot \log \left(1 + \frac{|g_{s_i,d_i}|^2 E}{\delta_v^2} + \frac{(|g_{s_i,d_i}|^2 + |g_{r,j,d_i}|^2 |h_{s_i,r_j}|^2 \beta^2) E}{\delta_v^2 + |g_{r,j,d_i}|^2 \beta^2 \delta_w^2} + \frac{|g_{s_i,d_i}|^4 E^2}{\delta_v^2 (\delta_v^2 + |g_{r,j,d_i}|^2 \beta^2 \delta_w^2)} \right) \quad (5)$$

Then, the gross utility values (without payment) of data rate's increasing for the source i with relay j is:

$$\xi_{i,j} = a_i(I_{s_i,r_j,d} - I_{s_i,d_i}) \quad (6)$$

where a_i represents the gain per unit of rate increasing. In this paper, we set $a_i = 1$. The net utility (with payment) of source i with relay j is:

$$v_{i,j} = \xi_{i,j} - p_{i,j} \quad (7)$$

On the other hand, the relays are in different energy conditions and have different self-evaluations of the basic costs (c_j) to provide cooperation. For example, a idle device being in charging evaluates its basic energy cost to be a relay is $c = 0$. However, it also wants to earn as much reward as possible. The net revenue of relay j serving for source i can be defined as:

$$u_{i,j} = p_{i,j} - c_j \cdot P_j \quad (8)$$

where $p_{i,j}$ is the price of source i paying for relay j . P_j is the transmission energy on the relay r_j . In this paper, the amplify factor is set to its maximum value according to equation (2) , so $P_j = P_s = E$.

Assume that, the idle users who are willing to be relays broadcast flag packets to inform all sources. The sources with its destinations can obtain the channel conditions to the different relays. Thus, the sources have complete information of all relays. However, the relays have no information about each source's utility.

III. GAME THEORY ANALYSIS

In our model, each relay r_j can be seen as a seller and aims to earn the payment which not only covers its basic cost, but also gains as much extra revenue as possible. About the multi-source and multi-relay system, two fundamental problems need to be addressed: first, how to assign the source-relay pairs; second, what is the suitable competitive prices for sources to pay for the relays. A two-sided assignment game framework is proposed to address these problems.

A. The source-relay assignment game

Definition 1: The source-relay assignment game consists of:

- ◊ **Players:** The group of nodes (sources and relays).
- ◊ **Allocation:** A set of source-relay matching and the price scheduling.
- ◊ **Preference:** Each player prefers one allocation to another according to the matching partner it is assigned and the price between them.

The basic game problem is to decide how the inherent profitability of the total pairs, arising from the differences in subjective valuation, is to be shared among the sources-relay pairs. We have no way at present of ascertaining the price $p_{i,j}$, but we can postpone the question and focus on the outcome of the game. Firstly, Let us investigate the maximum profit of the system. The game consists of mixed source-relay pairs. Each pair contains the profit:

$$b_{i,j} = \max(0, u_{i,j} + v_{i,j}) = \max(0, \xi_{i,j} - c_j * P_j) \quad (9)$$

The assignment of relay j with source i is called as coalition \bar{ij} . It is impossible for the coalition \bar{ij} has minus benefit, because no player was enforced to match with another and no player can affect a profitable transaction independently.

Consider the maximum total profits of all coalition pairs. Obviously, there exist an optimal set of pairs maximizing the coalitions' total gain. There are N sources and M relays. Therefore, we introduce MN nonnegative real variables $x_{ij} = 0, 1, i \in S, j \in R$ and impose on them the $M + N$ constraints. Mathematically, the optimal sets of pairs (in term of total profits) are the solution of the following optimization problem:

$$\max_{x_{ij}} z_{\max} = \sum_{j \in R} \sum_{i \in S} b_{ij} x_{ij} \quad (10)$$

$$\text{s.t. } \sum_{j \in R} x_{ij} \leq 1, \sum_{i \in S} x_{ij} \leq 1 \quad (11)$$

where z_{\max} is the potential maximum profit in the multiple source and multiple relay systems. Equation (11) means one relay helps at most one source, and one source chooses at most one relay. In general, the maximum value is attained with $x_{ij} = 0$ or 1 [13]. The prices between all pairs form a price vector to allocate the total profit. For a core allocation, the price vector should be constructed to support it.

The game has a set of global outcomes named allocations. An allocation is defined to a set of source-relay matching and a price vector. The exact definition of the core is given in the following.

B. The "core" of the game

In the game, for each relay, due to the competition from other relays, it will reduce its price to attract the sources. To avoid the long duration of price-adjustment process, we restrict the price to discrete values. The distinguish between the core in discrete markets and in the continuous markets motive the definitions of "discrete core" and "core".

Definition 2: An rational allocation is an assignment of relays to sources together with a price vector p such that, if $a : \{1, \dots, N\} \rightarrow \{1, \dots, M\}$ is the function that represents the assignment (so that $a(i)$ is the relay to which the source i is assigned, and $a^{-1}(j) \equiv g(j)$ is the source matched with relay j), satisfy:

$$p_{i,a(i)} - c_{a(i)} > 0 \quad (12)$$

$$\xi_{g(j),j} - p_{g(j),j} > 0 \quad (13)$$

This definition means that any individual will not accept a price which makes its utility value be less than 0.

Definition 3: A (discrete) core allocation is an rational allocation $(a; p_{1f(1)}, \dots, p_{Nf(N)})$ such that there is no source-relay combination (i, j) and (discrete) price p that satisfy:

$$p - c_j > p_{i,a(i)} - c_{a(i)} \quad (14)$$

$$\xi_{i,j} - p > \xi_{g(j),j} - p_{g(j),j} \quad (15)$$

Definition 3 implies that: (a) The (discrete) core allocations are stable; (b) The core allocations are Pareto efficient in terms

of total profit of the system (bring together the net utility values of all players). Property (a) is obvious. Due to the page limit, the Property (b) is shown in simulations in this paper. Note that, in the discrete core, the allocation outcome is stable under the permitted prices. However, the achieved total profit will be affected.

IV. DISTRIBUTED SOURCES-RELAY ASSIGNMENT ALGORITHM

We have identified the (discrete) core allocations of the game. In this section, a distributed algorithm is proposed to make the outcome of the game converge to the discrete core allocation. After a user getting flag information from the relays, it can calculate its potential highest benefit by setting the lowest prices to the relays. Then, it sends a request to its favorite relay. Each relay which received the requests, rejects all but its favorite source according to the bidding price. The algorithm proceeds iteratively and terminates when a complete matching is obtained.

A. Algorithm design

There are two phases in each iteration, the requesting phasing (for each source to send request), and the selection phase (for each relay to select source) as described in Algorithm 1. We assume that the minimum unit of price is β . Hence, the prices for the transaction between sources and relays are n times more than β . The initial price is β . This is the permitted minimum initial price.

Algorithm 1 DSRA Algorithm

- 1: **Initialization:** For each source i
 - a). Set prices $p_{i,j}(0) = \beta$ for $j = 1, \dots, M$.
 - b). Calculates the utility values according to: $\xi_{i,j}(0) - p_{i,j}$. Make offer to its favorite relay.
- 2: **Selection phase:** For each relay j , who received requests,
 - a). Calculates the utility value according to: $p_{i,j} - c_{i,j}$.
 - b). Rejects all but its favorite. (The relays do not allow the minus utility value.)
- 3: **Requesting phase:** For each source i whose request is rejected by the relay,
 - a). If source i 's request is rejected by relay j in step $t - 1$, $p_{i,j}(t) = p_{i,j}(t - 1) - e$, if not $p_{i,j}(t) = p_{i,j}(t - 1)$. The price is not allowed to exceed $\xi_{i,j}$. {The total payment is not allowed to exceed ξ_{i,G_i} }.
 - b). Calculates the current utility values to each relay by : $v_{i,j}(t) = \xi_{i,j} - p_{i,j}(t)$ Find the best value made by the relay and send request to it. (Due to the random channel gain, the probability have the same evaluation to the relays is 0)
 - c). Send request with new bidding prices to the selected relays.

Algorithm ends: Repeat steps 2 and 3 until no rejects and requests are issued.

It is worth noting that DSRA is totally distributed. But, the traditional auction methods require a central auctioneer.

We do not consider the possible “manipulative” behaviors in which a source at some stages may choose a relay that is not its most preferred. Theorem 1 illustrates the convergence of DSRA. The whole proof is in appendix.

Theorem 1: DSRA algorithm will converge in a finite number of iterations to the discrete core allocation.

V. SIMULATION RESULTS AND ANALYSIS

To evaluate the performance of the proposed scheme, we perform simulations for the system. The channels between the nodes are modeled as flat Rayleigh fading. The noise powers are $\delta_v^2 = \delta_w^2 = 0.01$. W_i of each source is uniformly selected from $\{1, 2, 3\}$. We assume that the basic energy cost of each node is randomly set to 0 or 0.1. The first value is for the node being in charge. The second value is for the mobile node.

A. The case with two users and two relays

We first give a simple example to show the equilibrium outcome of the game. There are two users and two relays in the network. The bandwidths of the users 1 and 2 are set to 2 and 3 respectively. The basic energy costs for relays 1 and 2 are 0 and 0.3. And Let $E = 1, \beta = 1$. The following table shows the outcome of the DSRA algorithm.

TABLE I
THE OUTCOME OF THE GAME WITH 2 SOURCES AND 2 RELAYS

	Relay 1	Relay 2	v
Source 1	7.5667	6.6375	$v_1 = 6.6375 - 1.1 + 0.6$
Source 2	10.8214	7.5907	$v_2 = 10.8214 - 1.1$
u	$u_1 = 1.1$	$u_2 = 1.1 - 0.6$	

The unique optimal assignment is shown by overstriking. Each item in the Table I is calculated according to equation (9). We can verify that the DSRA converge to the utility vector u, v shown in Table I. This outcome is in the “core” of the game. No player can change the outcome unilaterally. In this example, the “discrete core allocation” with $\beta = 1$ is also in the “core” allocation. In general case, there are gaps between these two outcomes. In the following, we consider the case with multiple users and multiple relays.

B. The case with multiple users and multiple relays

We first plot the prices of some sources with $N = 10, M = 10, \beta = 1$ in Figure 2. We can see from the result that the prices changed with the iterations, then converge to the stable values. This is due to the fact that, initially, each user set its price as lower as possible. Then, the users being reject will increase their prices or find another relay. In fact, if there are more relays than the sources, the prices will shrink for the sources to get more utility value, and vice versa.

By theorem 1, DSRA will converge to the discrete core allocation. However, there is a gap in terms of total profit (bring together the net utility of all sources and relays) between the discrete core allocation and the core allocation. To compare the total profit of the proposed distributed algorithm with the optimal value, the simulation is set up in Figure 4. In

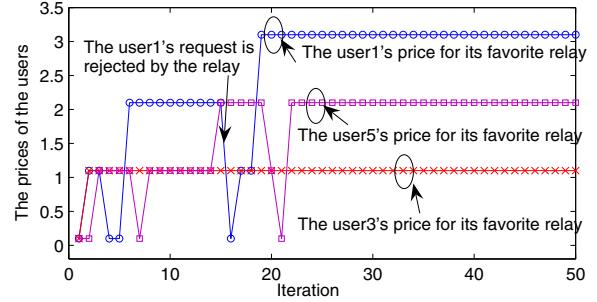


Fig. 2. Each curve represents the price set by a source. There are total 10 sources. We plot three of them for examples.

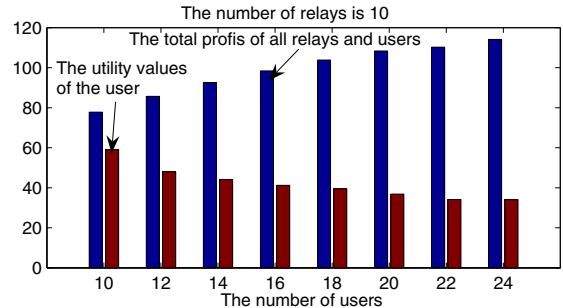


Fig. 3. The gross utility values of the users will shrink with the increasing number of users in the networks.

the centralized situation, it is no meaningful to consider the price adjust process. The total profit with the optimal value is given by equation 10. By varying the number of source-relay pairs, we can observe from Figure 4 that the DSRA algorithm can approximately achieve the same total utility value as the centralized optimal algorithm. The average iterations to the convergence of DSRA with different β are shown in Figure 5.

VI. CONCLUSION

In this paper, we proposed a game theoretic approach for the assignment of sources and relays in wireless cooperative networks. The relays are selfish, the sources should pay “virtual currency” for getting cooperation. Each relay consumes its energy as well as earns “currency”. The same relay has different values to different sources by different channel conditions. We

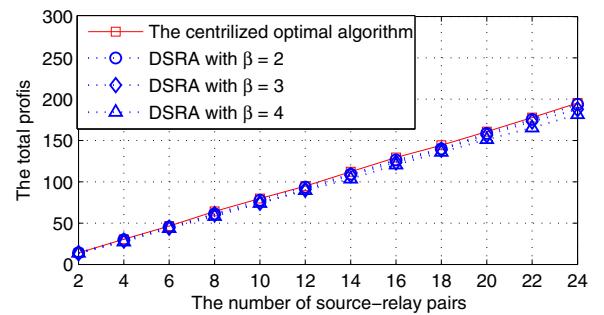


Fig. 4. The total profitability of all source-relay pairs.

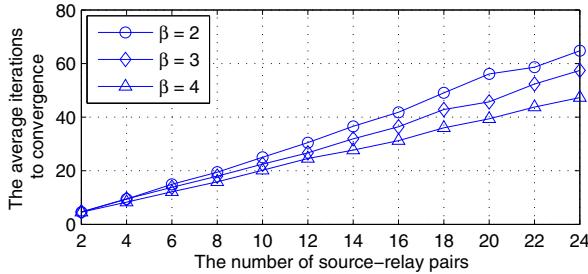


Fig. 5. The average number of iterations to convergency.

targeted to address two questions: which relay the source will select, and what is the prices. The source-relay assignment game model was employed to address the problems. The proposed distributed algorithm not only helps the sources to select the relay, but also forces the relays set reasonable prices. The algorithm is proven to converge to the “discrete core” allocation which is favorable to all sources and relays. It’s also shown that the distributed algorithm can achieve comparable performance with the centralized algorithm in terms of total profits.

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APPENDIX

In order to establish the theorem 1, we firstly proof serval Lemmas.

Lemma 1: After a finite number of iterations, no requests send by sources, each relay serves no more than one source, and the algorithm stops.

Proof: Firstly, we can immediately get that if a relay received at least one request at step t , it always has at least one source at any step $t' > t$. Suppose relay j , received more than two requests, it will reject all but its favorite. On the other hand, as long as the request was reject, the source will increase its price. Due to the price is a certain discrete value, so after a finite number iterations, the source even being rejected will not increase its price. Then no requests are issue, each relay serves no more than one source, the algorithm stops. ■

Lemma 2: The algorithm converges to an individually rational allocation (see definition 2).

Proof: Suppose t^* is the step at which the process stops. Then $p_{i,j}(t^*) - c(j) > 0$ is hold for all i, j at step t^* , since the price paid for the relay will nerve less than relay’s basic cost. Following from the initialization in Table I and the fact that

the user is not required to set the price larger than its basic utility, then we get $\xi_{i,j} - p_{i,j}(t^*) \geq 0$ immediately. ■

Theorem 1: DSRA algorithm will converge in a finite number of iterations to the discrete core allocation.

Proof: By Lemma 1, the algorithm stops in a infinite number of steps. Denote the result of allocation and the prices when the algorithm stops as $\{\phi; p_{1\phi(1)} \dots p_{N\phi(N)}\}$, where $\phi : \{1 \dots N\} \rightarrow \{1 \dots M\}$ is the matching function defined in definition 3. Suppose by way of contradiction that $\{\phi; p_{1\phi(1)} \dots p_{N\phi(N)}\}$ is not a discrete core allocation.

Then by Lemma 2, $\{\phi; p_{1\phi(1)} \dots p_{N\phi(N)}\}$ is individually rational allocation. So there must be exist source i , a relay j , and an discrete price p holding:

$$p - c_j > p_{i,a(i)} - c_{a(i)}, \quad \text{and} \quad (16)$$

$$\xi_{i,j} - p > \xi_{g(j),j} - p_{g(j),j} \quad (17)$$

For any discrete p satisfying the above equations, by Lemma 2, we can get $\xi_{i,j} - p > 0$. By the process of the algorithm, Source i must at one step have made an request to relay j at price p . On the other hand, due to the algorithm have been stopped, hence, $p_{i,a(i)} - c_{a(i)} > p - c_{i,j}$. This contradicts (16), completing the proof. ■

REFERENCES

- [1] J. N. Laneman and Gregory W. Wornell, “Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks,” IEEE Trans. Inf. Theory, vol.49, no.10, pp.2415-2425, Oct. 2003.
- [2] J. N. Laneman, D.N.C. Tse, and G. W. Wornell, “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior”, IEEE Trans. Inform. Theory, vol.50, no.12, pp.3062-3080, Dec. 2004.
- [3] Kambiz Azarian, Hesham El Gamal, Philip Schniter, “On the Achievable Diversity-Multiplexing Tradeoff in Half-Duplex Cooperative Channels,” IEEE Transaction on Information Theory, vol.51, no.12, pp.4152-4172, Dec. 2005.
- [4] O. Ileri, S. C. Mau, and N. B. Mandayam, “Pricing for Enabling Forwarding in Self-Configuring Ad Hoc Networks,” IEEE Journal on Selected Areas in Communications, vol.23, pp.151-162, Jan, 2005.
- [5] Zhu Ji, Wei Yu and K. J. Ray Liu, “A Game Theoretical Framework for Dynamic Pricing-Based Routing in Self-Organized MANETs,” IEEE Journal on Selected Areas In Communications, vol.26, no.7, pp.1204-1217, Sep. 2008.
- [6] N.Shastry and R.S.Adve, “Stimulating Cooperative Diversity in Wireless Ad Hoc Networks through Pricing”, Proc. IEEE ICC, 2006.
- [7] B. Wang, Z. Han, and K. J. Ray Liu, “Distributed Relay Selection and Power Control for Multiuser Cooperative Communication Networks Using Stackelberg Game,” in IEEE Transaction on Mobile Computing, vol. 8, no. 7, Jul. 2009, pp. 975-990.
- [8] Aggelos Bletsas, Ashish Khisti, David PReed, and Andrew Lippman, “A Simple Cooperative Diversity Method Based on Network Path Selection,” IEEE Journal on Selected Areas in Communications, vol.24, no.3, Mar.2006.
- [9] Z. Han and H. V. Poor, “Coalition games with cooperative transmission: A cure for the curse of boundary nodes in selfish packet-forwarding wireless networks,” IEEE Transactions on Communications, vol. 57, no. 1, Jan. 2009, pp. 203–213.
- [10] S. Mathur, L. Sankar, and N. B. Mandayam, “Coalitions in cooperative wireless networks,” IEEE Journal on Selected areas in Communications, vol. 26, no. 7, Sept. 2008, pp. 1104–1115.
- [11] Lloyd Shapley and Martin Shubik, “The assignment game I: the core,” International Journal of Game Theory, vol.1, pp.111-130, 1972.
- [12] Dierker, Egbert:“Equilibrium Analysis of Exchange Economies with Indivisible Commodities,” Econometrica, vol.39, pp.997-1008, 1971.
- [13] George B. Dantzig, Thapa, N. Mukund, Linear Programming: Introduction, Springer Press, 1997.