Minimum MSE-Based MIMO-OFDM Precoded Spatial Multiplexing Systems with Limited Feedback

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***Abstract*— This paper deals with design and performance analysis of transmit precoder optimization for MIMO-OFDM systems with limited feedback of channel state information (CSI). We assume that the receiver has perfect channel knowledge while the transmitter has only partial channel knowledge from limited feedback. We propose MSE-based optimal codebook design algorithm for MIMO-OFDM precoded spatial multiplexing systems under a specific average power constraint. The optimal precoder has the structure of a precoding and power allocation for each mode obtained by water-filling process. We derived the closed form solution for power allocation in the MSE-sense. Simulation results show that the MSE-based codebook construction algorithm with hybrid design of power allocation and precoding can achieve better performance than that of equal power allocattion based codebook in the previous works.**

I. Introduction

Multiple-input multiple-output (MIMO) system is well motivated for wireless communication through fading channels because it has the potential to improve transmission rate or diversity gain. The performance of a multi-antenna system depends on the degree of channel state information (CSI) available at the transmitter and the receiver [1]. However, in practical wireless systems, the transmitter could not obtain perfect CSI because of various factors, e.g., estimation error, feedback delay, feedback error, etc. For multi-antenna systems, even partial channel knowledge can be valuable in enhance system performance [2]. Thus, exploiting channel partial CSI at the transmitter in a MIMO wireless systems has attracted great attention recently [3], [4].

Transmit CSI can enhance MIMO system performance by using precoder. CSI at transmitter can be obtained by feedback from receiver in frequency division duplexing systems or by measuring the reverse channel in time division duplexing systems. Generally, it is assumed that perfect CSI is available at the receiver while the transmitter only get partial CSI or imperfect CSI. There are two ways to exploit partial CSI [4], [7]. One is to use the statistical characteristic of CSI [2], [7]; The other is to use limited feedback bits which indexing CSI. The latter way is to first quantize the channel at the receiver, and then send back the limited bits indexing the quantized CSI which is called codebook. One can design off-line a codebook containing transmission candidates [7], [8]. The previous works partial CSI based transmit precoding are based on various criteria such as the average signal to noise ratio (SNR) [9], [10], the outage probability [11], [12], and the symbol error rate [13], [14] etc. Optimal transmit algorithms for maximal capacity based limited feedback CSI have been investigated in [15]. However, those are all based on equally power allocation. On the other hand, the transmitter structure in the previous work do not coincide with the optimal transmitter structure in [16]- [20].

By dividing frequency-selective channels into an equivalent set of frequency-flat subchannels, orthogonal frequency division multiplexing (OFDM) has emerged as an attractive modulation scheme to handle frequency selective fading resulting from delay spreading by expanding the symbol duration [5], [6]. It is also an effective technique to combat inter-symbol interference (ISI) caused by wireless multi-path fading channel [6]. Our focus in this paper is to take the critical work of determining which CSI and by which way to be sent to the transmitter for MIMO-OFDM systems.

In this paper, we consider power allocation across different subcarriers. We propose hybrid precoding and power adaptive allocation algorithm based on limited feedback with spatial multiplexing. Meanwhile, the minimal MSE based codebook design algorithm is also investigated.

The paper is organized as follows. We outline MIMO-OFDM model and the problem statement in section II. In section III, the MSE design criterion is described. The minimum MSE based coodbook construction algorithm is presented in section IV. Section V proposes the optimal precoder design algorithm for MIMO-OFDM system with limited feedback. The simulation result are presented in section VI followed by the conclusion in section VII.

II. System Model

We consider N_t transmitter-antennas and N_r receiver antennas MIMO-OFDM wireless communication system with spatial multiplexing, where N_c subcarriers is employed in an OFDM symbol. By MIMO-OFDM system, frequency-selective channel can be decoupled into N_c parallel MIMO frequency-flat channels.

Assume that there are N_s symbols to be transmitted per subcarrier, which is then bounded by $\min(N_t, N_r)$. Collect the N_cN_s symbols transmitted over N_cN_t subcarriers in a

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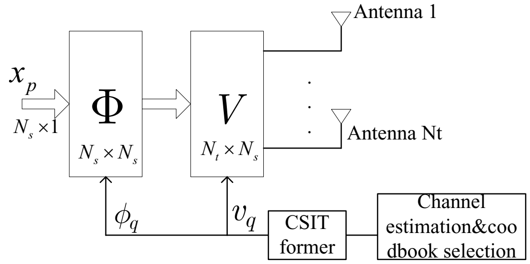

Fig. 1. The optimal transmission structure on each subcarrier

$N_c N_s \times 1$ vector x. The precoder is applied to each subcarrier to allocate power across the N_c subcarriers and N_t antennas. Therefore the precoder $\boldsymbol{F}$ is an $N_c N_t \times N_c N_s$ block diagonal matrix with N_c blocks of $N_t \times N_s$ matrices $\boldsymbol{F}_p,\ p = 1, \cdots, N_c$.

In this context, we consider a MIMO block-fading channel model where the channel state remains quasi-statistic within a fading block on each subcarrier, but behaves independently across a different fading block. The frequency responses of the N_c MIMO channels can be described by a $N_c N_r \times N_c N_t$ block diagonal channel matrix. $\boldsymbol{H}$ with N_c blocks of $N_r \times N_t$ matrices $\boldsymbol{H}_p,\ p = 1, \cdots, N_c$. If the channel keeps quasi-static, the $N_c N_s \times 1$ received symbol vector $\mathbf{r}$ is

$$\boldsymbol{r} = \boldsymbol{G}\boldsymbol{H}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{n}, \tag{1}$$

where the receiver $\boldsymbol{G}$ is an $N_c N_s \times N_c N_r$ block diagonal matrix with N_c blocks of $N_s \times N_r$ matrices $\boldsymbol{G}_p,\ p = 1, \cdots, N_c$, and $\boldsymbol{n}$ is the $N_c N_s \times 1$ noise vector, which has the Gaussian statistics.

In a MIMO-OFDM system, each subcarrier can be considered as a MIMO channel. Suppose $\boldsymbol{x}^T = [\boldsymbol{x}_1^T\ \cdots\ \boldsymbol{x}_{N_c}^T]$, $\boldsymbol{r}^T = [\boldsymbol{r}_1^T\ \cdots\ \boldsymbol{r}_{N_c}^T]$ and $\boldsymbol{n}^T = [\boldsymbol{n}_1^T\ \cdots\ \boldsymbol{n}_{N_c}^T]$, where $\boldsymbol{x}_k$, $\boldsymbol{r}_k$ and $\boldsymbol{n}_k$ are respectively the transmitted symbol vector, the received symbol vector and the noise vector over the kth MIMO channel for $k = 1, \cdots, N_c$. From (1), the output-input relation for each subcarrier can be individually written out. For $p = 1, \ldots, N_c$, denote

$$\boldsymbol{y}_p = \boldsymbol{H}_p \boldsymbol{F}_p \boldsymbol{x}_p + \boldsymbol{n}_p. \tag{2}$$

Then for $p = 1, \cdots, N_c$, we have

$$\boldsymbol{r}_p = \boldsymbol{G}_p \boldsymbol{y}_p = \boldsymbol{G}_p \boldsymbol{H}_p \boldsymbol{F}_p \boldsymbol{x}_p + \boldsymbol{G}_p \boldsymbol{n}_p, \tag{3}$$

III. Minimum MSE Based Codebook Construction

A. Transmitter Optimization

Given an independent identical distribution ($i.i.d.$) block-fading MIMO channel with the transmit CSI of B bit. The average minimum mean square error (MSE) $P_e(V, \Phi)$ of the MIMO-OFDM system can be computed from the B bit index codeword. This optimal performance problem can be formulated as

$$\begin{cases} \min\ P_e(V, \Phi), \\ s.\ t.\ E\left(\sum_{p=1}^{N_c} tr\left(\tilde{x}_p \tilde{x}_p^H\right)\right) \le P_0 \end{cases} \tag{4}$$

where $\tilde{\boldsymbol{x}}_p = V_p \sqrt{\Phi_p} \boldsymbol{x}_p$ and P_0 is the average transmit power.

B. Optimal Transmitter Structure

The proposed feedback transmission scheme in a MIMO-OFDM system for one subcarrier is illustrated in Fig. 1. This is the optimal transmitter structure which hybrids adaptive power control and precoding model. Our objective is to jointly design the feedback scheme and the transmission parameters with limited feedback. Without loss of generality, in the following, we will not deliberately decern the N_c MIMO systems and simply denote $\mathbf{H}_p$, Φ_p and V_p for $p = 1, \cdots N_c$ by H, ϕ and v respectively.

For B bits feedback, let $N_B = 2^B$. There are N_B candidates v and ϕ in the codebook, denoted by $v(q)$ and $\phi(q)$ for $q = 1, \cdots N_B$. The power loading matrice $\phi(q)$, and precoding matrices $v(q)$, for $q \in [1, N_B]$, are designed offline and stored at both transmitter and receiver. Each feedback index $q \in [1, N_B]$ is corresponding to a stored transmission scheme in the codebook. The power loading matrices are diagonal matrices with nonnegative entries. The column vectors of the precoding matrix are unitary. Without loss of generality, let $E[\boldsymbol{x}_p^H \boldsymbol{x}_p] = 1$ for normalization.

Although the previous research has proposed many codebook construction criteria [9]–[11], there is no MSE based codebook construction criterion with optimal transmitter structure. [18] deduced the optimal MSE expression of linear transmitter and receiver, but it is based on perfect CSI on both sides. In this section we manage to directly use the exact MSE as the codebook design criterion. Let $\mathbf{W}$ be codebook designed off-line and known to the transmitter and the receiver, the proposed MSE based selection rule is

$$\boldsymbol{F}_{opt} = \arg\min_{\boldsymbol{F} \in \mathbf{W}} MSE(\boldsymbol{H}, \boldsymbol{F}). \tag{5}$$

The MSE expression can be computed by a simple expression in [20]. Let $P_e(\Phi)$ denote the MSE expression by allocated power Φ, we can write it as

$$P_e(\Phi) = \sum_{\ell=1}^{N_s} \frac{1}{1 + \phi_\ell \lambda_\ell}, \tag{6}$$

where the constants λ_ℓ in (6) is singular value of channel SVD decomposition and ϕ_ℓ is the power allocated on the ℓth subchannel. In (6), we omitted the transmission scheme candidate index q.

C. NMSE Based Codebook Construction Algorithm

The codebook construction for limited-bit feedback beamforming and precoding can be linked to a vector quantization problem [8]. We use Lloyd algorithm to search for good precoder codebooks based on MSE criterion. The Lloyd algorithm based codebook construction also provides an alterative

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systematic approach for the subspace packing problem in [21], [22].

In Lloyd algorithm, the quantizer quantizes the input space $\mathbb{C}^N$ into N_B separated regions $\{R_q\}_{q=1}^{N_B}$, where R_q denote the region with codeword $\{v(q), \phi(q)\}$. Design a quantizer means to find a codebook and a partition rule that jointly minimize the overall average distortion measure [23], where the centroid condition and the nearest neighbor rule condition teratively play crucial roles.

To design the MSE based codebook, we have to construct "the nearest neighbor rule" and "the cenroid condition" in this scenario [4]. Thus, joint optimization of the channel vector region and the transmission modes is required. The two key works are

1) Given the channel vector region, find an optimal design of the transmission modes.
2) Given a set of transmission modes, find an optimal design of channel vector region.

Notice that the optimization of finding the regions $\{R_1, \cdots, R_{N_B}\}$, and transmission scheme $\{v, \phi\}_{q=1}^{N_B}$ is equivalent to designing a vector quantizer with a modified distortion measure.

Let A_q denote the probability that the channel matrix H lies in the region R_q, i.e, $P(H \in R_q) = A_q$. P_0 stands for the average transmit power. By jointly designing $\{R_q\}_{q=1}^{N_B}$ and transmitter strategy $\{v_q, \phi_q\}_{q=1}^{N_B}$, our ultimate goal is to

$$\begin{cases} \min J = \sum_{q=1}^{N_B} (A_q \cdot p_e(q)) \\ s.\ t.\ \sum_{q=1}^{N_B} \left(A_q \sum_{\ell=1}^{N_s} (\phi_\ell(q)) \right) \leq P_0 \end{cases} \tag{7}$$

Using Lagrange multiplier μ, the distortion measure for optimizing the MSE performance is given by

$$D(H, v_q, \phi_q) = \sum_{q=1}^{N_B} A_q \frac{1}{N_s} \sum_{\ell=1}^{N_s} \frac{1}{1 + \phi_\ell(q)\lambda_\ell(q)} \\ \mu \left(\sum_{q=1}^{N_B} A_q \sum_{\ell=1}^{N_s} \phi_\ell(q) - P_0 \right). \tag{8}$$

The distortion measure is a function of H and ϕ_q. The partition index q is sent to the transmitter, and the transmitter selects the optimal schemes by the index q. Hence, the optimization problem could be solved by general Lloyd's algorithm, which can be outlined in the following two steps.

S1: Given a certain channel condition regions of $\{R_1, \cdots, R_{N_B}\}$, find the optimal transmission scheme $\{T_1, \cdots, T_{N_B}\}$, where the transmitter scheme $T_q \triangleq \{v_q, \phi_q\}$ denotes the multimode beamfoming matrix and power allocation along different mode. The optimal transmission scheme T_k, is given by the generalized region centroid condition.

$$T_q = \arg\min_{T} E_{H \in R_q}[D(H, T_q)]A_q. \tag{9}$$

S2: Given a transmission scheme $\{T_q\}$, find the optimal channel regions R_q, $1 \leq q \leq N_B$. The optimal region is given by the nearest neighbor rule.

$$R_q = \{H : D(H, T_q) \leq D(H, T_j),\ \forall j = 1, \cdots, N_B\} \tag{10}$$

By iteratively using centroid condition and the nearest neighbor rule, the overall distortion will decrease monotonically. We will use this principle to form a new codebook construction method. Here, we give the closed form solution for the power loading.

1) Solution of S1: For a given certain region R_q, $1 \leq q \leq N_q$, find the optimal transmission scheme T_q.

Given a certain region R_q, the probability A_q and the channel covariance matrix can be calculated for every qth region. Let the eigen decomposition of $H^H H$ in region R_q be $H^H H = V_q \Lambda_q V_q{}^H$, where $\Lambda_q = diag(\lambda_1(q), \cdots, \lambda_{N_t}(q))$ is an $N_t \times N_t$ diagonal matrix with $\lambda_1(q) \geq \lambda_2(q) \geq \cdots \geq \lambda_{N_t}(q)$, and V_q is $N_t \times N_t$ unitary matrix formed by corresponding eigen-vectors $v_i(q)$, i.e., $V_q = (v_1(q), \cdots, v_{N_t}(q))$. The optimal unquantized precoder for each channel condition should be $\boldsymbol{F}_{opt} = \overline{\boldsymbol{V}}_q$, where $\overline{\boldsymbol{V}}_q$ is the matrix constructed from the first N_s columns of $\boldsymbol{V}_q$ [4], [24]. Based on those decomposition matrices, we can derive the optimal power loading matrix belonging to the region R_q consequently, which is summarized in the following theorem.

Theorem 1: With optimal eigen-beamforming combined with power allocation, we get the optimal transmitter power allocation schemes by water-filling principle:

$$\phi_\ell(q) = \left[\frac{P_{tot} + \sum_{q=1}^{N_B} \sum_{\ell=1}^{\bar{N}_s} \lambda_\ell^{-1}(q)}{\sum_{q=1}^{N_B} \sum_{\ell=1}^{\bar{N}_s} \lambda_\ell^{-1/2}(q)} \lambda_\ell^{-1/2}(q) - \lambda_\ell^{-1}(q) \right]^+ \tag{11}$$

where $(x)^+ = \max(0, x)$, $q = 1, \cdots, N_B$, and $\phi_\ell(q)$ denotes the ℓth entry of the diagonal power allocation matrix of the qth region.

Since the worst eigen mode would be drop off in some cases, we have $\bar{N}_s \leq N_s$. Due to limited space, the proof is omitted. Notice that, the weaker subchannels may have a dominate MSE than the stronger subchannels. Thus, those eigenmodes whose channel gain are less than a certain threshold will be dropped, and the power is then distribute among the remaining eigenmodes. As a result, more power is allocated to the weaker eigenmodes to guarantee MSE performance.

2) Solution of S2: For a given transmission scheme, determine the optimal region $\{R_q\}_{q=1}^{N_B}$. The optimal partition region is given by the nearest neighbor rule. In our case, we will use the distortion measure as (8). Thus, we can get the new regions as following

$$R_q = \{H :\ D(H, T_q) \leq D(H, T_j); \\ \forall j \neq q \in [1, \cdots, N_B]\} \tag{12}$$

The above two necessary optimization conditions are essential for the codebook construction [31]. Firstly, for each region, the optimal codeword can be chosen to minimize the distortion. In our case, precoding together with power allocation are employed in centroid condition. Secondly, the nearest neighbor rule is used to find the optimal region for each codeword.

3) Solution of Codewords Selection: Here, we present the optimal codeword selection algorithm for MIMO system. It can also be employed for MIMO-OFDM systems. By (2), our codeword selection algorithm can be employed on each sub-carrier. Let $T_q = \{w_q, \phi_q\}$ denote the qth codeword in codebook $\{T_q\}_{q=1}^{N_B}$, then the description of the codewords selection is

St1 : Get CSI at receiver by channel estimation.

St2 : Perform eigen decomposition: $v^H H^H H v = V \Lambda V^H$, where $\Lambda = \mathrm{diag}(\lambda_1, \cdots, \lambda_{N_s})$ and matrix V denotes the eigen matrix of $v^H H^H H v$.

St3 : Let $D(H,T) = \sum_{\ell=1}^{N_s} \frac{1}{1+\lambda_\ell \phi_\ell}$, where N_s denotes the data stream in each subcarrier. The selected optimal codeword is

$$T^{opt} = \arg\min_T D(H,T)$$

IV. Simulation Result and Discussion

In this section, we provide some numerical examples to illustrate the performance of the optimal limited feedback design in section IV. We present Monte Carlo simulation for the MIMO-OFDM system with $N_t = 4,\ N_r = 2$. The number of sub-carriers is 64, the cyclic prefix length is 16 and the used constellation is QPSK. We assume that the discrete-time channel impulse response is generated according to the Hiperlan2 Channel Model C in [28]. The channels between different transmit and receiver antenna is assumed independent. The channel is fixed for a frame and randomly varies between frames. $n(k)$ is $i.i.d$ complex Gaussian with zero mean. The transmitter power was allocated across subcarriers by waterfilling principle. The receiver uses linear decoder with perfect channel knowledge. Assume that the feedback CSI has no delay and no transmission error.

Experiment 1: The first experiment compares the performance of system with ideal CSI and partial CSI at the transmitter. When the transmitter has perfect CSI, best performance can be achieved. In partial CSI scenario, we compare the performance of MMSE based codebook and the existing codebooks in [7], [27]. We use actual Q-function in codebook selection criterion for the existing the codebook. Fig. 2 shows that MMSE-based codebook can get moderate performance gain over the existing codebook, and the codebook in [7] is a little bit better than that in [27]. We can also observe that Chernoff- bound approximation selection is not as good as actual Q-function selection.

Experiment 2: The codebook construction algorithm couples two steps: to find the optimal transmission $T_1, \cdots, T_{N_B}$ for a certain channel condition region and find the optimal channel regions $\{R_k\}$ for a transmission scheme $\{T_k\}$, for $k \in [1, N_B]$. The two steps works iteratively to enable the final design of the transmission modes and the fading regions. Fig. 3 shows that after three to four times of iterations, the distortion

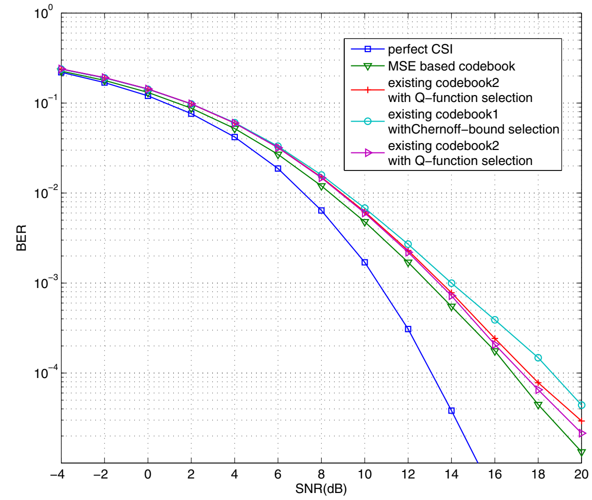

Fig. 2. BER performance comparison of optimal power allocation and codebook based limited feedback

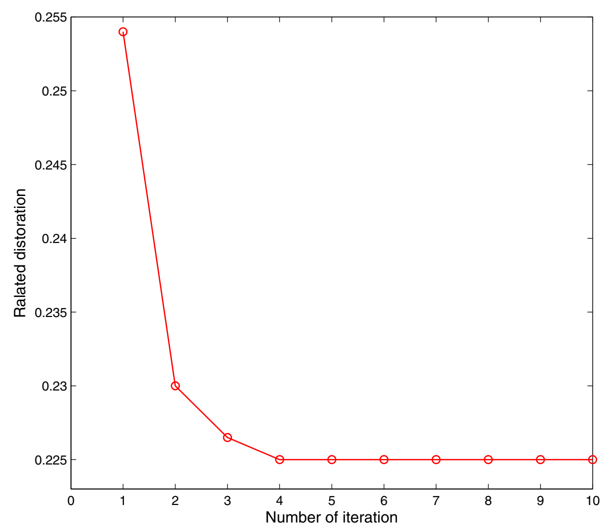

Fig. 3. Convergence of codebook construction algorithm

will converges. The Lloyd algorithm guarantees convergence in a few steps of iterations.

Experiment 3: The influence of size of the codebook is addressed in Fig. 4. The cases $B = 4, 6, 7, 8$ corresponds to the codebook sizes of 16, 64, 128, 256 respectively. Perfect CSI means $B = inf$. From Fig. 4, we observe:

- Feedback link can improve the system performance;
- When the feedback bits increases, the system has performance gain with $B = inf$ as the performance benchmark;
- the performance of $B = 6$ is pretty good. The gap between $B = 6$, $B = 7$ and $B = 8$ is small. So the size of codebook in a practical system is not necessarily

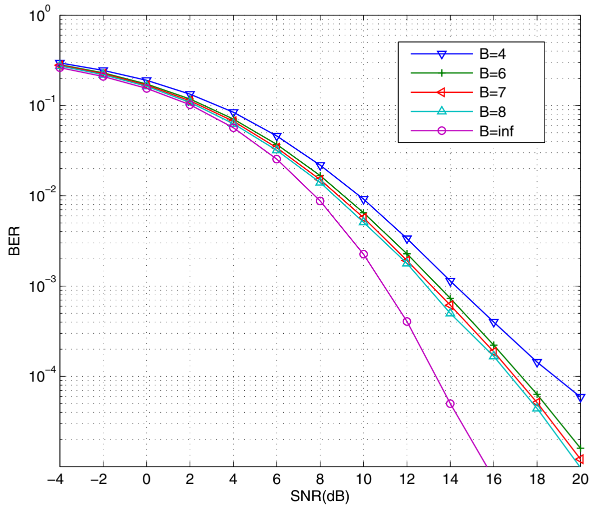

Fig. 4. The effect of codebook size

very large.

V. Conclusion

In this paper, we consider the precoded spatial multiplexing MIMO-OFDM system with limited feedback of CSI. We propose a new MMSE based codebook construction algorithm. The performance of the MMSE based codebook outperforms those of the existing codebooks. Then we develop the optimal algorithm for MIMO-OFDM system with limited feedback. The essential component of this paper is the codebook design using optimal structure of the transmitter and receiver, which employs the optimal transmission structure by combining precoding and power allocation. Since the codebook is designed offline, we do not need to care about the complexity of the presented codebook design algorithm.

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