

Transmission Capacity of Two Co-existing Wireless Ad hoc Networks with Multiple Antennas

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Abstract—This paper addresses bounds on the transmission capacities of two coexisting wireless networks (a primary and a secondary network), where each with multiple antennas shares the same spectrum and operates in the same geographic region. In the two coexisting network, the secondary (SR) network limits its interference to the primary (PR) network by carefully controlling its active transmitter density. Each transmitter uses a subset of its antennas to transmit multiple data streams, while each receiver uses partial zero forcing (PZF) to cancel interference using some of its spatial receive degrees of freedom (SRDOF). Considering general power-law wireless channels with path-loss exponent $\alpha > 2$ and small-scale Rayleigh fading, based on stochastic geometry tools and Markov's inequality, we first derive bound on the transmission capacity for the PR networks. Then the transmission capacity of the SR networks is derived when the transmission density of PR network maintains unchanged. Using the obtained bounds, we derive their optimal number of data streams to transmit and the optimal SRDOF to use for interference cancellation.

I. INTRODUCTION

Multiple antenna communication is currently of great interest in many wireless communication systems, and has become a key component of virtually every contemporary high-rate wireless standard (LTE, 802.11n, WiMAX) [1]. Prior works on finding the transmission capacity with multiple antennas has been discussed in [2]–[4]. In an ad-hoc network with multiple antennas, multiple antennas can either be used to increase the data rate by transmitting multiple data streams, or mitigate the interference by employing interference cancellation at the receiver. Transmitting multiple streams, however, will increase the interference power at other receivers, while using interference cancellation at the receiver limits the available spatial receive degrees of freedom (SRDOF) for decoding the signal of interest.

Recently, cognitive radio (CR) technology has been proposed as promising solutions to implement efficient reuse of the licensed spectrum by unlicensed devices [5]. In cognitive radio applications, secondary (SR) networks coexist with the licensed primary (PR) networks and share the same spectrum, while the PR users have a higher priority to access the spectrum and the SR users need to operate conservatively to limit their interference to the PR users. Along the line, Vu *et al.* considered the throughput scaling law for a single-hop cognitive radio network in [6], where a linear scaling law is obtained for the SR network under an outage constraint for the PR network. In [7], Jeon *et al.* considered a multi-hop cognitive network overlaid with a PR network and assumed

that the SR nodes know the location of each PR nodes. They showed that each network can achieve the same throughput scaling law as that when the other network is absent. In [8], based on the stochastic geometry, Yin *et al.* derived the transmission capacities for both the PR and SR network with deterministic power-law channel models and path-loss exponent $\alpha = 4$.

In this paper, we study a two coexisting network with per node equipped with multiple antennas in which transmitters are randomly located on an infinite plane according to a 2-D homogeneous Poisson point process (PPP), and each transmitter attempts communication with a receiver a fixed distance away from it. The PR network has a higher priority to access the spectrum without particular consideration for the SR network. The SR network, however, needs to limit its interference to the PR network by carefully controlling its active node density. Different from the approaches in [6], [7] and [8], we resort to stochastic geometry tools to evaluate the transmission capacity bounds for both the PR and SR networks without defining any preservation regions. Different from the approaches in [11], the transmitter (TX) uses a subset of its antennas to transmit multiple data streams, while the receiver uses partial zero forcing (PZF) to cancel interference. In particular, we are interested in finding the optimal number of transmission data streams and the optimal SRDOF for the two coexisting networks such that a target success probability is maintained and wish to quantify the rate at which this density can be increased as the number of antennas at each receiving node is increased.

II. SYSTEM MODEL

We consider a planar in which a network of PR nodes and a network of SR nodes coexist in the same geographic region, each node is equipped with N antennas. The distribution of PR active TXs follows a homogeneous PPP Π_1 of density λ_1 , and the distribution of SR active TXs follows another independent homogeneous PPP Π_2 of density λ_2 . Our aim is to evaluate the outage probability of the PR network, P_{out} , and that of the SR network, P'_{out} , which are functions of the TX node densities λ_1 and λ_2 . Time is slotted and in each time-slot potential TX follow a simple slotted ALOHA-like random access protocol. Similar to that in [9], in order to evaluate the outage probabilities, we condition on a typical PR (or SR) receiver at the origin, which yields the Palm distributions which also follows homogeneous PPP on a 2-D

plane with the density denoted by λ_1 and λ_2 , respectively. An attempted transmission is successful if the received signal-to-interference-plus-noise ratio (SINR) at the reference receiver (RX) is above a threshold, β_0 ; otherwise, the transmission fails, i.e., an outage occurs.

At the physical layer, we consider that each TX uses $k, k = 1, 2, \dots, N$, of its N antennas to transmit k independent data to its receiver with equally distributed power over all k antennas. we consider that each PR receiver uses m spatial receive SRDOF for canceling the $c(k, m_1) = \lfloor \frac{m_1}{k} \rfloor$ other nearest PR TX and the $c(k, m_2) = \lfloor \frac{m_2}{k} \rfloor$ nearest SR interferers in terms of distance from the receiver, where $m_1 + m_2 = m$, and use the rest $N - m$ SRDOF to decode the k streams transmitted by its transmitter, where $\lfloor \bullet \rfloor$ is floor function. Each SR receiver adopts the same strategy as PR receiver. Let $\mathbf{x}_i = [\mathbf{x}_i(1) \ \mathbf{x}_i(2), \dots, \mathbf{x}_i(k)]^T$, $\mathbf{x}'_i = [\mathbf{x}'_i(1) \ \mathbf{x}'_i(2), \dots, \mathbf{x}'_i(k)]^T$ be the $k \times 1$ signal transmitted from the PR TX T_i and SR TX T'_i , respectively, where each element $\mathbf{x}_i(n)$ and $\mathbf{x}'_i(n)$, $n = 1, 2, \dots, k$ are independent and $\mathcal{CN}(0, 1)$ distributed. For simplifying analysis, we limit our discussion to single-hop transmission, and assume that all PR TXs use the same transmission power P_1 , which supports the same transmission distance d . Similarly, all SR TXs use the same transmission power P_2 for the same transmission distance d . We consider that the channels suffer both the large-scale path-loss and the small-scale Rayleigh fading. Then the received signal at the typical PR RX R_0 is

$$\begin{aligned} \mathbf{y}_0 = & \sqrt{\frac{P_1}{k}} d^{-\frac{\alpha}{2}} \mathbf{H}_{00} \mathbf{x}_0 + \sum_{T_n \in \Pi_1 \setminus T_0} \sqrt{\frac{P_1}{k}} d_n^{-\frac{\alpha}{2}} \mathbf{H}_{0n} \mathbf{x}_n \\ & + \sum_{T'_n \in \Pi_2} \sqrt{\frac{P_2}{k}} d'_n^{-\frac{\alpha}{2}} \mathbf{H}'_{0n} \mathbf{x}'_n + \mathbf{z}_0, \end{aligned} \quad (1)$$

where $\mathbf{H}_{00} \in \mathcal{C}^{N \times k}$ is the channel between the PR TX T_0 and its intended RX R_0 , $\mathbf{H}_{0n}, \mathbf{H}'_{0n} \in \mathcal{C}^{N \times k}$ are the channels from T_n, T'_n to R_0 , d_n, d'_n is the distance from the n^{th} PR TX, n^{th} SR TX to the R_0 , respectively. α ($\alpha > 2$) is the path loss exponent, and $\mathbf{z}_0 \in \mathcal{C}^{k \times 1}$ is the additive white Gaussian noise with covariance $\eta \mathbf{I}$. We assume that each entries of \mathbf{H}_{00} , \mathbf{H}_{0n} , and \mathbf{H}'_{0n} are independent and identically distributed $\mathcal{CN}(0, 1)$.

We consider a PZF receiver where each stream is decoded by projecting the received signal to the null space of the other streams [4]. For the t^{th} stream, the received signal at the typical PR RX can be written as

$$\begin{aligned} \mathbf{y}_0(t) = & \sqrt{\frac{P_1}{k}} d^{-\alpha/2} \mathbf{H}_{00}(t) \mathbf{x}_0(t) + \sqrt{\frac{P_1}{k}} \sum_{j=1, j \neq t}^k d^{-\alpha/2} \\ & \mathbf{H}_{00}(j) \mathbf{x}_0(j) + \sum_{T_n \in \Pi_1 \setminus T_0(t)} \sqrt{\frac{P_1}{k}} d_n^{-\alpha/2} \sum_{j=1}^k \mathbf{H}_{0n}(j) \mathbf{x}_n(j) \\ & + \sum_{T'_n \in \Pi_2} \sqrt{\frac{P_2}{k}} d'_n^{-\alpha/2} \sum_{j=1}^k \mathbf{H}'_{0n}(j) \mathbf{x}'_n(j) + \mathbf{z}_0, \end{aligned} \quad (2)$$

for $t = 1, 2, \dots, k$, where $\mathbf{H}_{00}(j), \mathbf{H}_{0n}(j)$ and $\mathbf{H}'_{0n}(j)$ denote the j^{th} column of $\mathbf{H}_{00}, \mathbf{H}_{0n}$ and \mathbf{H}'_{0n} , respectively. To decode data stream $\mathbf{x}_0(t)$ while canceling the $c(k, m_1)$ PR TX interferers, and $c(k, m_2)$ SR TX interferers, the receiver multiplies a vector \mathbf{q}_t to the received signal, where \mathbf{q}_t lies in the null space of

$$\mathcal{H} = [\mathbf{H}_{00}(1) \cdots \mathbf{H}_{00}(t-1) \mathbf{H}_{00}(t+1) \cdots \mathbf{H}_{00}(k) \mathbf{H}_{01} \mathbf{H}_{02} \cdots \mathbf{H}_{0c(k, m_1)} \mathbf{H}'_{01} \mathbf{H}'_{02} \cdots \mathbf{H}'_{0c(k, m_2)}].$$

Given an outage constraint ϵ , transmission capacity C^ϵ is defined as the product of the maximum density λ^ϵ of transmissions, the common transmission data rate R , and $(1 - \epsilon)$ [10], i.e., $C^\epsilon = R \lambda^\epsilon (1 - \epsilon)$.

III. ANALYSIS OF THE TRANSMISSION CAPACITY: TWO COEXISTING NETWORK

A. Lower Bound on Transmission Capacity of the PR Network

In this subsection, we first derive the lower bound on the transmission capacity of the PR network. To cancel the nearest interferers, let the indices of the interferers be sorted in an increasing order in terms of their distance from R_0 , i.e., $d_1 \leq d_2 \leq \cdots \leq d_{c(k, m_1)} \leq \cdots$ and $d'_1 \leq d'_2 \leq \cdots \leq d'_{c(k, m_2)} \leq \cdots$. In order to decode the $\mathbf{x}_0(t)$ data stream, the R_0 uses PZF to remove the contribution from all the other data streams transmitted by T_0 , and all the data streams transmitted by the first $c(k, m_1)$ PR and the first $c(k, m_2)$ SR interferers. Since each channel coefficient is *i.i.d.* Rayleigh distributed, the rank of matrix \mathcal{H} is $m + k - 1$ with probability 1. Let \mathbf{S} be the orthogonal basis of the null space $\mathcal{N}(\mathcal{H})$ of the matrix \mathcal{H} . Then \mathbf{S} has dimension $N - (m + k - 1)$. To decode stream $\mathbf{x}_0(t)$, the receiver R_0 multiplies $\mathbf{q}_t^T \in \mathcal{N}(\mathcal{H})$ to the received signal. We assume that noise is neglected and $P_1 = P_2 = P$ for simplicity. After multiplying by \mathbf{q}_t^T , the received signal is

$$\begin{aligned} \mathbf{q}_t^T \sqrt{\frac{k}{P}} \mathbf{y}_0(t) = & d^{-\frac{\alpha}{2}} \mathbf{q}_t^T \mathbf{H}_{00}(t) \mathbf{x}_0(t) \\ & + \sum_{n=c(k, m_1)+1}^{\infty} d_n^{-\frac{\alpha}{2}} \sum_{j=1}^k \mathbf{q}_t^T \mathbf{H}_{0n}(t) \mathbf{x}_n(j) \\ & + \sum_{n=c(k, m_2)+1}^{\infty} d'_n^{-\frac{\alpha}{2}} \sum_{j=1}^k \mathbf{q}_t^T \mathbf{H}'_{0n}(t) \mathbf{x}'_n(j), \end{aligned} \quad (3)$$

where $t = 1, 2, \dots, k$. As shown in [4], the optimal \mathbf{q}_t^T is given by $\mathbf{q}_t^T = [(\mathbf{S} \mathbf{H}_{00}(t))^* \underbrace{0 \cdots 0}_{m+k-1}]$, and the signal power

$|\mathbf{q}_t^T \mathbf{H}_{00}(t)|^2$ is Chi-square distributed with $2(N - m - k + 1)$ degrees of freedom. Since \mathbf{q}_t^T is independent of \mathbf{H}_{0n} for $n \geq c(k, m_1) + 1$, and independent of \mathbf{H}'_{0n} for $n \geq c(k, m_2) + 1$, $\sum_{j=1}^k |\mathbf{q}_t^T \mathbf{H}_{0n}(j)|^2$ and $\sum_{j=1}^k |\mathbf{q}_t^T \mathbf{H}'_{0n}(j)|^2$ are Chi-square distributed with $2k$ degrees of freedom, respectively. Define $s = |\mathbf{q}_t^T \mathbf{H}_{00}(t)|^2$, $\rho_n = \sum_{j=1}^k |\mathbf{q}_t^T \mathbf{H}_{0n}(j)|^2$, and $\rho'_n = \sum_{j=1}^k |\mathbf{q}_t^T \mathbf{H}'_{0n}(j)|^2$. With PZF, the signal to interference ratio (SIR) at the typical PR RX is given by

$$\text{SIR} = \frac{d^{-\alpha}s}{\sum_{n=c(k, m_1)+1}^{\infty} d_n^{-\alpha} \rho_n + \sum_{n=c(k, m_2)+1}^{\infty} d'_n^{-\alpha} \rho'_n}. \quad (4)$$

First, we derive the transmission capacity of the PR network when the SR network is absent, which outage probability is given in the following lemma

Lemma 1 ([10]): For $c(k, m) > \lceil \frac{\alpha}{2} \rceil$, the outage probability when the PR TX sends k independent data streams with equal power allocation, and the PR RX cancels the $c(k, m)$ nearest interferers using PZF, is upper bounded by

$$\gamma_{out} \leq \frac{\beta_0 k}{N - m - k + 1} (\pi d^2 \lambda_1)^{\frac{\alpha}{2}} A;$$

If we set $\gamma_{out} = \epsilon_1$, the lower bound value on $\lambda_1^{\epsilon_1}$ is

$$\lambda_1^{\epsilon_1} = \frac{\epsilon_1^{\frac{2}{\alpha}}}{\pi d^2} \left(\frac{N - m - k + 1}{\beta_0 k A} \right)^{\frac{2}{\alpha}}, \quad (5)$$

where $A = (\frac{\alpha}{2} - 1)^{-1} (c(k, m) - \lceil \frac{\alpha}{2} \rceil)^{1-\frac{\alpha}{2}}$, $\lceil \bullet \rceil$ denotes ceil function.

When the SR network is present, it introduces interference to the PR network and the outage probability of the PR network will increase as $\Delta\epsilon$. Then the upper bound of the outage probability of PR network is given in the following theorem.

Theorem 1: For $c(k, m_1) > \lceil \frac{\alpha}{2} \rceil$ and $c(k, m_2) > \lceil \frac{\alpha}{2} \rceil$, the outage probability of the PR network with the presence of the SR network using PZF is upper bounded by

$$P_{out} \leq \frac{\beta_0 k}{N - m - k + 1} \left[(\pi d^2 \lambda_1)^{\frac{\alpha}{2}} E + (\pi d^2 \lambda_2)^{\frac{\alpha}{2}} F \right], \quad (6)$$

where $E = (\frac{\alpha}{2} - 1)^{-1} (c(k, m_1) - \lceil \frac{\alpha}{2} \rceil)^{1-\frac{\alpha}{2}}$, and $F = (\frac{\alpha}{2} - 1)^{-1} (c(k, m_2) - \lceil \frac{\alpha}{2} \rceil)^{1-\frac{\alpha}{2}}$.

Proof: To upper bound the outage probability while canceling the $c(k, m_1)$ nearest PR TX interferers and the $c(k, m_2)$ nearest SR TX interferers, let dominator of (4)

$$I_{sum} = \sum_{n=c(k,m_1)+1}^{\infty} d_n^{-\alpha} \rho_n + \sum_{n=c(k,m_2)+1}^{\infty} d'_n^{-\alpha} \rho'_n.$$

The outage upper bound is derived by the tail probability of the random variable $1/\text{SIR}$. Apply Markov's inequality as follows.

$$P_{out} = \mathbb{P}[\text{SIR} \leq \beta_0] = \mathbb{P}\left[\frac{1}{\text{SIR}} \geq \frac{1}{\beta_0}\right] \quad (7)$$

$$\leq \beta_0 \mathbb{E}\left[\frac{1}{\text{SIR}}\right] = \beta_0 d^{\alpha} \mathbb{E}[I_{sum}] \mathbb{E}\left[\frac{1}{s}\right]. \quad (8)$$

From [4], we can obtain for large N

$$\mathbb{E}\left\{\sum_{i=c(k,m_1)+1}^{\infty} d_i^{-\alpha}\right\} \leq (\pi \lambda_1)^{\frac{\alpha}{2}} \left(\frac{\alpha}{2} - 1\right)^{-1} \times (c(k, m_1) - \lceil \frac{\alpha}{2} \rceil)^{1-\frac{\alpha}{2}}, \quad (9)$$

and

$$\mathbb{E}\left\{\sum_{i=c(k,m_2)+\lceil \frac{\alpha}{2} \rceil+2}^{\infty} d'_i^{-\alpha}\right\} \leq (\pi \lambda_2)^{\frac{\alpha}{2}} \left(\frac{\alpha}{2} - 1\right)^{-1} \times (c(k, m_2) - \lceil \frac{\alpha}{2} \rceil)^{1-\frac{\alpha}{2}}. \quad (10)$$

Since the independence of d_i and ρ , as well as d'_i and ρ' , $\mathbb{E}\{I_{sum}\} = \sum_{i=c(k,m_1)+1}^{\infty} \mathbb{E}\{d_i^{-\alpha}\} \mathbb{E}\{\rho_i\} + \sum_{i=c(k,m_2)+1}^{\infty} \mathbb{E}\{d'_i^{-\alpha}\} \mathbb{E}\{\rho'_i\}$. Due to $\mathbb{E}\{\rho_i\} = k, \forall i \geq c(k, m_1)$ and $\mathbb{E}\{\rho'_j\} = k, \forall j \geq c(k, m_2)$, we can get

$$\begin{aligned} \mathbb{E}\{I_{sum}\} &\leq (\pi \lambda_1)^{\frac{\alpha}{2}} k \left(\frac{\alpha}{2} - 1\right)^{-1} \left(c(k, m_1) - \lceil \frac{\alpha}{2} \rceil\right)^{1-\frac{\alpha}{2}} \\ &\quad + (\pi \lambda_2)^{\frac{\alpha}{2}} k \left(\frac{\alpha}{2} - 1\right)^{-1} \left(c(k, m_2) - \lceil \frac{\alpha}{2} \rceil\right)^{1-\frac{\alpha}{2}}. \end{aligned} \quad (11)$$

Because $s = \chi_{2(N-m-k+1)}^2$ and $\mathbb{E}[1/\chi_{2h}^2] = 1/(h-1)$ for $h > 1$, the upper bound of the PR outage probability when the SR network is present is

$$P_{out} \leq \frac{\beta_0 k}{N - m - k + 1} \left[(\pi d^2 \lambda_1)^{\frac{\alpha}{2}} E + (\pi d^2 \lambda_2)^{\frac{\alpha}{2}} F \right]. \quad (12)$$

■

From Theorem 1, we can see that outage probability of the PR network will increase with the presence of the SR network, because $E \geq A$. If we set the target outage probability increment of the PR network as $\Delta\epsilon$, we have $P_{out} = \epsilon_1 + \Delta\epsilon$. In CR network, the transmission density of PR network maintains unchanged with the presence of SR network. Therefore, we can choose $\lambda_1 = \lambda_1^{\epsilon_1}$ as in (5). Then the transmission capacity of the PR network with the presence of SR network is given by

$$C_1^{\epsilon_1} \geq k R_1 \lambda_1^{\epsilon_1} (1 - \epsilon_1 - \Delta\epsilon), \quad (13)$$

where R_1 is the data rate adopted by successful PR links. From (12), the lower bound value of λ_2 corresponding to a target outage probability increment $\Delta\epsilon$ is given by

$$\lambda_2^{\Delta\epsilon} = \max\left(0, \frac{(\Delta\epsilon - (E/A - 1)\epsilon_1)^{\frac{2}{\alpha}}}{\pi d^2} \left(\frac{N - m - k + 1}{\beta_0 k F}\right)^{\frac{2}{\alpha}}\right). \quad (14)$$

Using (13), the optimal number of transmit antennas k and the fraction of total SRDOF that maximizes the lower bound on the PR transmission capacity are given by the next lemma.

Lemma 2: Use a single transmit antenna ($k = 1$), and a fraction of total SRDOF for interference cancellation ($c(k, m) = \theta N, \theta = \theta_1 + \theta_2, \theta = 1 - \frac{2}{\alpha}$) at the PR RX. Maximizing the scaling of transmission lower capacity with the number of antennas, the transmission capacity of PR network with the presence of the SR network scales as

$$C_1^{\epsilon_1} = O(N). \quad (15)$$

Proof: From (6), we know that $\frac{\beta_0}{N-m-k+1}$, $c(k, m_1)$, and $c(k, m_2)$ will enlarge as k increases. Therefore, the optimal value of k is $k = 1$. Use a single transmit antenna ($k = 1$) and a fraction of total SRDOF for interference cancellation $m = \theta N (0 < \theta \leq 1)$, that is $c(k, m) = m$. For simple analysis, we assume $c(k, m) > \lceil \frac{\alpha}{2} \rceil$. Then the expectation of this interference power $A = (\frac{\alpha}{2} - 1)^{-1} (m - \lceil \frac{\alpha}{2} \rceil)^{1-\frac{\alpha}{2}}$. From (6) and (14), we can obtain in the case of large N ,

$$C_1^{\epsilon_1} \geq \frac{NR_1\epsilon_1^{\frac{2}{\alpha}}(1-\epsilon_1-\Delta\epsilon)}{\pi d^2(\frac{\alpha}{2}-1)^{-\frac{2}{\alpha}}} (1-\theta)^{\frac{2}{\alpha}} \theta^{1-\frac{2}{\alpha}} \beta_0^{-\frac{2}{\alpha}}. \quad (16)$$

To find the optimal value of θ , i.e., $\theta_1 + \theta_2$, that maximizes the lower bound, we need to maximize the function $(1 - \theta)^{\frac{2}{\alpha}} \theta^{1 - \frac{2}{\alpha}}$. Solving by setting the derivative to zero, the optimal value of $\theta^* = 1 - \frac{2}{\alpha}$. From (16) the transmission capacity $C_1^{\epsilon_1}$ of the PR network scales as $O(N)$. ■

B. Lower Bound on Transmission Capacity of the SR Network

The outage probability of the SR network using PZF follows the same approach as section III-A, which is shown below

$$P'_{out} \leq \frac{\beta_0 k}{N - m - k + 1} \left[(\pi d^2 \lambda_2)^{\frac{2}{\alpha}} E + (\pi d^2 \lambda_1)^{\frac{2}{\alpha}} F \right]. \quad (17)$$

In turn, we suppose $P'_{out} = \epsilon_2$, and choose $\lambda_1 = \lambda_1^{\epsilon_1}$ as (5). The lower bound value on $\lambda_2^{\epsilon_2}$ is

$$\lambda_2^{\epsilon_2} = \max \left(0, \frac{(\epsilon_2 - \frac{F}{A} \epsilon_1)^{\frac{2}{\alpha}}}{\pi d^2} \left(\frac{N - m - k + 1}{\beta_0 k E} \right)^{\frac{2}{\alpha}} \right). \quad (18)$$

On the other hand, if we set the target outage probability of the PR network to be $\epsilon_1 + \Delta\epsilon$ as above, and set the target outage probability of the SR network to be ϵ_2 , simultaneously, we can choose the value of λ_2 as follows

$$\lambda_2 = \min(\lambda_2^{\Delta\epsilon}, \lambda_2^{\epsilon_2}). \quad (19)$$

Therefore, the lower bound on transmission capacity of the SR network is given by

$$C_2^{\epsilon_2} \geq k R_2 \lambda_2 (1 - \epsilon_2), \quad (20)$$

where R_2 is the data rate adopted by successful SR links. Using Lemma 1, the optimal number of antennas k and the fraction of total SRDOF that maximize the transmission capacity are given by the next theorem.

Theorem 2: Using a single transmit antenna ($k = 1$), and a fraction of total SRDOF for interference cancelation ($c(k, m_1) = \theta_1 m, c(k, m_2) = \theta_2 m, \theta_1, \theta_2 \in (0, 1)$) at the SR RX to maximize the scaling of transmission lower capacity with the number of antennas, the transmission capacity of SR network scales as

$$C_2^{\epsilon_2} = O(N). \quad (21)$$

Proof: The SR uses a fraction of total SRDOF for interference cancelation $m_1 = \theta_1 m, m_2 = \theta_2 m$ ($0 < \theta_1, \theta_2 \leq 1, \theta_1 + \theta_2 = 1, m = \theta N$). For simple analysis, we assume $c(k, m_1), c(k, m_2) > \lceil \frac{\alpha}{2} \rceil$. From (14), we can obtain the following in the case of large N ,

$$\lambda_2^{\Delta\epsilon} = \frac{(\Delta\epsilon + (1 - \theta_1^{1 - \frac{2}{\alpha}}) \epsilon_1)^{\frac{2}{\alpha}}}{\pi d^2 (\frac{\alpha}{2} - 1)^{-\frac{2}{\alpha}}} \frac{((1 - \theta) N - k + 1)^{\frac{2}{\alpha}}}{\beta_0^{\frac{2}{\alpha}} k (\theta \theta_2 N)^{\frac{2}{\alpha} - 1}}.$$

It is easy to see that the optimal the number of data transmission is $k = 1$. When $k = 1$, we can get the following

$$\lambda_2^{\Delta\epsilon} = \frac{\left(\Delta\epsilon + \left(1 - \theta_1^{1 - \frac{2}{\alpha}} \right) \epsilon_1 \right)^{\frac{2}{\alpha}}}{\pi d^2 (\frac{\alpha}{2} - 1)^{-\frac{2}{\alpha}}} \frac{(1 - \theta)^{\frac{2}{\alpha}} (\theta \theta_2)^{1 - \frac{2}{\alpha}} N}{\beta_0^{\frac{2}{\alpha}}}. \quad (22)$$

In order to maximize the SR transmission capacity, we first ensure $\lambda_2^{\Delta\epsilon} \geq 0$, i.e. $\theta_1 \geq \left(\frac{\epsilon_1}{\epsilon_1 + \Delta\epsilon} \right)^{\frac{2}{\alpha-2}}$. Take the same approach as Lemma 2 to find the optimal value of θ_1 , which results in the optimal value of $\theta_1^* = \left(\frac{\epsilon_1}{\epsilon_1 + \Delta\epsilon} \right)^{\frac{2}{\alpha}}$. From (18), we have the following in the case of large N ,

$$\lambda_2^{\epsilon_2} = \frac{\left(\epsilon_2 - \theta_2^{1 - \frac{2}{\alpha}} \epsilon_1 \right)^{\frac{2}{\alpha}}}{\pi d^2 (\frac{\alpha}{2} - 1)^{-\frac{2}{\alpha}}} \frac{(1 - \theta)^{\frac{2}{\alpha}} (\theta \theta_1)^{1 - \frac{2}{\alpha}} N}{\beta_0^{\frac{2}{\alpha}}}. \quad (23)$$

In order to ensure $\lambda_2^{\epsilon_2} \geq 0$, we can obtain $\theta_2 \geq \left(\frac{\epsilon_1}{\epsilon_2} \right)^{\frac{2}{\alpha-2}}$. The optimal value is derived by the same approach when $\theta_2^* = \left(\frac{\epsilon_1}{\epsilon_2} \right)^{\frac{2}{\alpha}}$. From (22) and (23), we can see that the transmission capacity of SR network scales as $O(N)$, and the lower bound on the SR network transmission capacity is maximized at the case of $k = 1$ and $\theta_1^* = \left(\frac{\epsilon_1}{\epsilon_1 + \Delta\epsilon} \right)^{\frac{2}{\alpha}}$ or $\theta_2^* = \left(\frac{\epsilon_1}{\epsilon_2} \right)^{\frac{2}{\alpha}}$. ■

C. Upper Bound on Transmission Capacity of the PR Network

As section III-A, the maximum transmission density of PR network maintains unchanged with the presence of SR network. To upper bound the outage probability, we consider the interference contribution from only the nearest l non-canceled interferers. From [4], the transmission capacity upper bound of the PR network can be derived when the SR network is absent.

$$P_{out} \geq 1 - \frac{N - m - k + 1}{\beta_0 d^\alpha (lk - 1)} (\pi \lambda_1)^{-\frac{\alpha}{2}} \left(c(k, m) - 1 + l + \frac{\alpha}{2} \right)^{\frac{\alpha}{2}}. \quad (24)$$

Setting this bound equal to ϵ_1 and solving for λ_1 yields the associated upper bound value to $\lambda_1^{\epsilon_1}$ as

$$\lambda_1^{\epsilon_1} = \frac{(N - m - k + 1)^{\frac{2}{\alpha}} (c(k, m) - 1 + l + \frac{\alpha}{2})}{\beta_0^{\frac{2}{\alpha}} (1 - \epsilon_1)^{\frac{2}{\alpha}} (lk - 1)^{\frac{2}{\alpha}} \pi d^2}. \quad (25)$$

When the SR network is present, it introduces interference to the PR network and the outage probability of the PR network will increase as $\Delta\epsilon$. The upper bound on transmission capacity of the PR network is $C_1^{\epsilon_1} \leq k R_1 \lambda_1^{\epsilon_1} (1 - \epsilon_1 - \Delta\epsilon)$, where R_1 is the data rate adopted by successful PR links. It is easy to see that the optimal transmit antenna is $k = 1$. Assume $l \ll N$ and $m = \theta N$. From (25), we can find that the optimal SRDOF is $\theta^* = \frac{\alpha}{\alpha+2}$ and the upper bound on transmission capacity of PR network scales as $\frac{2\alpha}{(\alpha+2)^2} O(N^{\frac{\alpha+2}{\alpha}})$.

D. Upper Bound on Transmission Capacity of the SR Network

The SR RX uses the $c(k, m_1)$ and $c(k, m_2)$ SRDOF to cancel the interference from the other SR users and the PR users. Then the outage probability of the SR is shown below

Theorem 3: The outage probability of the SR network with PZF is lower bounded by

$$P'_{out} \geq 1 - \frac{N - m - k + 1}{\beta_0 d^\alpha (lk - 1)} \left((\pi \lambda_1)^{-\frac{\alpha}{2}} + (\pi \lambda_2)^{-\frac{\alpha}{2}} \right) G^{\frac{\alpha}{2}}.$$

In turn, the upper bound value of SR transmission density is

$$\lambda_2^{\Delta\epsilon} = \frac{(N-m-k+1)^{\frac{2}{\alpha}}}{\beta_0^{\frac{2}{\alpha}}(lk-1)^{\frac{2}{\alpha}}\pi d^2} \times \max \left(0, \left(\frac{1-\epsilon_1-\Delta\epsilon}{G^{\frac{\alpha}{2}}} - \frac{1-\epsilon_1}{(c(k,m)-1+l+\frac{\alpha}{2})^{\frac{\alpha}{2}}} \right)^{-\frac{2}{\alpha}} \right),$$

where $G = (\max(c(k,m_1), c(k,m_2)) - 1 + l + \frac{\alpha}{2})$. \blacksquare

Proof: To lower bound the outage probability, we consider the interference contribution from only the nearest l non-canceled SR and PR interferers. Recall that

$$1 - P'_{out} = \mathbb{P}(s > d^\alpha \beta_0 I_{sum}) \quad (26)$$

$$\leq \mathbb{P}(s > d^\alpha \beta_0 (I_{c(k,m_1)} + I_{c(k,m_2)})) \quad (27)$$

$$\leq \mathbb{P}\left[\frac{d^{-\alpha}s}{I_{c(k,m_1)} + I_{c(k,m_2)}} \geq \beta_0\right]. \quad (28)$$

Using the Markov's inequality to the success probability, we can get $1 - P'_{out} \leq \frac{1}{\beta_0} \mathbb{E}\left[\frac{d^{-\alpha}s}{I_{c(k,m_1)} + I_{c(k,m_2)}}\right]$. Note that $I_{c(k,m_1)} = \sum_{i=1}^l d_{c(k,m_1)+i}^{-\alpha} \rho_{c(k,m_1)+i}$, $I_{c(k,m_2)} = \sum_{i=1}^l d_{c(k,m_2)+i}^{-\alpha} \rho'_{c(k,m_2)+i}$, where $\rho_{c(k,m_1)+i}$ and $\rho'_{c(k,m_2)+i}$ are χ_{2k}^2 distributed. Because d_i and d'_i are increasing with i and the function $(\cdot)^{-\alpha}$ is decreasing, we can get

$$1 - P'_{out} \leq \frac{1}{\beta_0} \mathbb{E}\left[\frac{d^{-\alpha}s}{\left(d_{c(k,m_1)+l}^{-\alpha} + d_{c(k,m_2)+l}^{-\alpha}\right) \sum_{i=1}^l \rho_{c(k,m_1)+i}}\right].$$

From [4], $d_{c(k,m_1)+l}^2$ and $d_{c(k,m_2)+l}^2$ are 1-D PPP with density $\pi\lambda_1$ and $\pi\lambda_2$, respectively, random variables $\pi\lambda_1 d_{c(k,m_1)+l}^2$ and $\pi\lambda_2 d_{c(k,m_2)+l}^2$ are Chi-square distributed with $2(c(k,m_1)+l)$, $2(c(k,m_2)+l)$ degrees of freedom, $\mathbb{E}[d_{c(k,m_1)+l}^{-\alpha}] = (\pi\lambda_1)^{\frac{\alpha}{2}} \frac{\Gamma(c(k,m_1)+l-\alpha/2)}{\Gamma(c(k,m_1)+l)}$, and $\mathbb{E}[d_{c(k,m_2)+l}^{-\alpha}] = (\pi\lambda_2)^{\frac{\alpha}{2}} \frac{\Gamma(c(k,m_2)+l-\alpha/2)}{\Gamma(c(k,m_2)+l)}$, respectively, where $\Gamma(\bullet)$ denotes Gamma function. It follows from the definition of the chi-square distribution that the sum of independent chi-square variables is also chi-square distributed. Therefore, $\mathbb{E}\left[\sum_{i=1}^l \rho_{c(k,m_1)+i}\right] = lk - 1$. Since s is independent of $d_{c(k,m_1)+1}$, $d'_{c(k,m_2)+1}$, and $\frac{\Gamma(i-\frac{\alpha}{2})}{\Gamma(i)}$ is decreasing function as the value of i increase, we can obtain

$$1 - P'_{out} \leq \frac{N-m-k+1}{d^\alpha \beta_0 (lk-1)} \left((\pi\lambda_1)^{-\frac{\alpha}{2}} + (\pi\lambda_2)^{-\frac{\alpha}{2}} \right) \times \left(\max(c(k,m_1), c(k,m_2)) - 1 + l + \frac{\alpha}{2} \right)^{\frac{\alpha}{2}}. \quad (29)$$

We define $G = (\max(c(k,m_1), c(k,m_2)) - 1 + l + \frac{\alpha}{2})$ for simple notation. If we choose $\lambda_1 = \lambda_1^{\epsilon_1}$ as in (25), from (29), and the upper bound value of maximum allowable value of λ_2

corresponding to a target outage probability increment $\Delta\epsilon$ is

$$\lambda_2^{\Delta\epsilon} = \frac{(N-m-k+1)^{\frac{2}{\alpha}}}{\beta_0^{\frac{2}{\alpha}}(lk-1)^{\frac{2}{\alpha}}\pi d^2} \times \max \left(0, \left(\frac{1-\epsilon_1-\Delta\epsilon}{G^{\frac{\alpha}{2}}} - \frac{1-\epsilon_1}{(c(k,m)-1+l+\frac{\alpha}{2})^{\frac{\alpha}{2}}} \right)^{-\frac{2}{\alpha}} \right). \quad (30)$$

In order to guarantee $\lambda_2^{\Delta\epsilon} \geq 0$, $G \leq \left(\frac{1-\epsilon_1-\Delta\epsilon}{1-\epsilon_1}\right)^{\frac{\alpha}{2}} (c(k,m)-1+l+\frac{\alpha}{2})$. Similar to section III-B, we set the target outage probability of the SR network to be ϵ_2 , the lower bound of maximum allowable value λ_2 corresponding to a target outage probability ϵ_2 is $\lambda_2^{\epsilon_2} \leq \frac{(N-m-k+1)^{\frac{2}{\alpha}}}{\beta_0^{\frac{2}{\alpha}}(lk-1)^{\frac{2}{\alpha}}\pi d^2} \max \left(0, \left(\frac{1-\epsilon_2}{G^{\frac{\alpha}{2}}} - \frac{1-\epsilon_1}{(c(k,m)-1+l+\frac{\alpha}{2})^{\frac{\alpha}{2}}} \right)^{-\frac{2}{\alpha}} \right)$.

Therefore, the value of $\lambda_2 = \min(\lambda_2^{\Delta\epsilon}, \lambda_2^{\epsilon_2})$. From G , it can be seen that the upper bound of the SR network approaches to the smallest value when $c(k,m_1) = c(k,m_2)$. Then the upper bound on transmission capacity of the SR network is $C_2 \leq kR_2\lambda_2(1-\epsilon_2)$.

IV. NUMERICAL RESULTS

We first evaluate lower bounds on the transmission capacity of PR (SR) network when the transmitter sends k independent information with equal power allocation, and the PR (SR) RX uses $N-m$ SRDOF to receive the k streams transmitted by the intended transmitter, and the rest m_1 , m_2 SRDOF for canceling the nearest other PR (SR) interferers and the nearest SR (PR) interferers using PZF. We set $\epsilon_1 = 0.1$, $\Delta\epsilon = 0.1$, $\epsilon_2 = 0.15$ and $\alpha = 3$. Fig. 1 shows that $k = 1$ and $m = (1 - 2/\alpha)N$ maximizes the lower bound on the transmission capacity of PR network. Fig. 2 shows that $k = 1$ and $\theta_2 = (\frac{\epsilon_1}{\epsilon_2})^{\frac{2}{\alpha}}$ maximizes the lower bound on the transmission capacity of SR network. From the simulation results, we can see that the lower bounds scale linearly with N .

We also evaluate upper bounds on the transmission capacity of PR network and SR network. Fig. 3 and Fig. 4 show that $k = 1$ maximizes the upper bound on the transmission capacity of PR network and SR network, and the upper bounds on the PR transmission capacity scales linearly with $\frac{2\alpha}{(\alpha+2)^2}O(N^{\frac{\alpha+2}{\alpha}})$. From the simulation results, we can see that the optimal SRODF of the PR network is $\frac{\alpha}{\alpha+2}N$. Meanwhile, we find that the bounds on transmission capacity with $k = 2$ is less than that of transmission capacity with $k = 1$. This means that this strategy with MRC on the subset of receive antennas to increase the power of the signal of the interest is more significant than canceling a few interferers, $(1 - \frac{2}{\alpha})N$ and $\frac{2}{\alpha}\theta N$ ($\theta = 1 - \frac{2}{\alpha}$), respectively.

V. CONCLUSION

We analyzed the bounds on the transmission capacities of two coexisting wireless networks with multiple antennas that

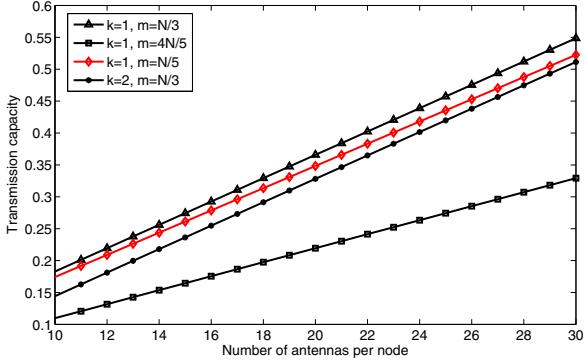


Fig. 1. The lower bound on transmission capacity for the PR network versus N with canceling the nearest interferers for $d = 1$, $R_1 = 1\text{bit/s}$, $\alpha = 3$

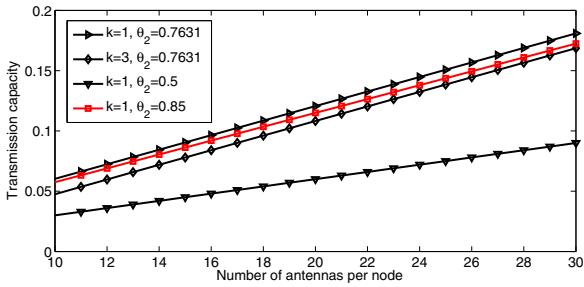


Fig. 2. The lower bound on transmission capacity for the SR network versus N with canceling the nearest interferers for $d = 1$, $R_2 = 1\text{bit/s}$, $\alpha = 3$

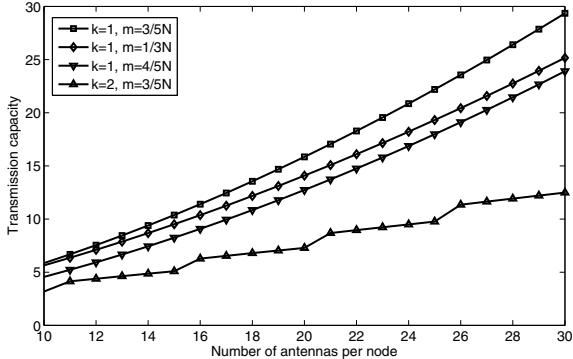


Fig. 3. The upper bound on transmission capacity for the PR network versus N with canceling the nearest interferers for $d = 1$, $R_1 = 1\text{bit/s}$, $\alpha = 3$, $l = 2$

operate in the same geographic region and share the same spectrum. Each transmitter uses a subset of its antennas to transmit multiple data streams, while the receiver uses PZF to cancel interference using some of its spatial receive SRDOF. Based on Markov's inequality, we derive bounds on the transmission capacity for the two networks. Using the obtained bounds, the optimal number of data streams to transmit, and the optimal SRODF to use for interference cancellation are derived, which maximizes the transmission capacity.

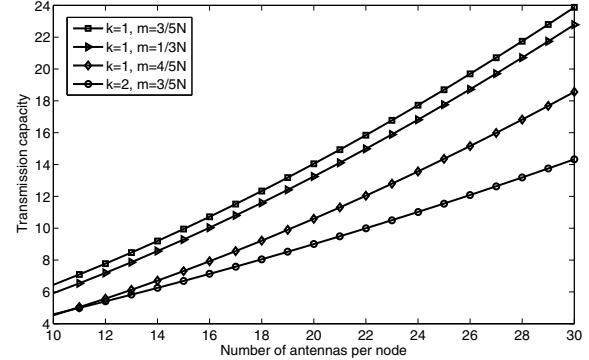


Fig. 4. The upper bound on transmission capacity for the SR network versus N with canceling the nearest interferers for $d = 1$, $R_2 = 1\text{bit/s}$, $\alpha = 3$, $l = 2$, $\theta_1 = \theta_2 = 1/2$

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