A Simple Construction of 16-QAM Codewords with Low PMEPR for OFDM Signals

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Abstract—Golay sequences have been introduced to construct 16-QAM (quaternary amplitude modulation) code for the orthogonal frequency division multiplexing (OFDM), reducing the peak-to-mean envelope power ratio (PMEPR). As an alternative way to construct Golay sequences, the construction of 16-QAM using Rudin-Shapiro polynomials (RSP) was also reported recently in several literatures. In this paper, we develop a simple efficient construction of 16-QAM using generalized RSP, which provides high code rate and large Hamming distance while tightly controlling the PMEPR.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) eliminates the need for complex equalizers in wide-band fading channels, while efficient hardware implementations can be realized using the fast Fourier transform (FFT). However, a major drawback of OFDM signals is the high peak-to-mean envelope power ratio (PMEPR) of the uncoded OFDM signal. Some popular PMEPR reduction techniques include signal distortion techniques [1], [2], coding [3], [4], [5], [6], multiple signal representation [7], [8], [9], [10], modified signal constellation [11], pilot tone methods [12] and others.

An idea introduced in [13] and developed in [14] is to use the Golay sequences [15] to encode OFDM signals with a PMEPR of at most 2. These sequences have been employed as pilot sequences by European Telecommunications Standards Institute (ETSI) Broad Radio Access Networks (BRAN). Recently Davis and Jedwab [3] made an attractive theoretical advance on this work and observed that the 2^h -ary Golay sequences of length 2^m can be obtained from certain second order cosets of the classical first order Reed-Muller code. As a consequence of this intrinsic observation, Davis and Jedwab [4] were able to obtain $(m!/2)2^{h(m+1)}$ codewords (DJ-code) for the phase shift keying (PSK) OFDM signals of 2^m carriers with good error-correcting capabilities, efficient encoding and decoding, and a PMEPR of at most 2. Further investigation on DJ-code has been done in [6], which employs Golay set [16] to increase the code rate by relaxing the PMEPR.

Since quadrature amplitude modulation (QAM) sequences are widely used in OFDM, R6Bing and Tarokh[17] has studied the Golay sequences for 16-QAM OFDM signals. By decomposing a 16-QAM symbol uniquely into a pair of quaternary

PSK (QPSK) symbols, they construct $[(m!/2)4^{(m+1)}]^2$ 16-QAM OFDM signals of length 2^m starting from Golay QPSK sequences and provide bound 3.6 on their PMEPR. More detailed construction of 16-QAM OFDM signals using Golay sequences has been recently reported in [18].

As a simple way to construct Golay sequences, the Rudin-Shapiro polynomials (RSP) [19] have been studied in [20], [21], [22]. They obtain $2^{h(m+1)}$ codewords for 2^h -ary OFDM signals of 2^m carriers. Although the code rate of RSP based construction is lower than that of DJ-code, this simple construction matches with some efficient decoding scheme [23]. Further study to increase the code rate has been reported in [20]. Recently, RSP has also been used to construct 16-QAM code in [24], which obtains $m4^{m+3} - 256(m-1)$ codewords of length 2^m . In this paper, we develop a simple way to design the 16-QAM OFDM signals using the generalized RSP [20]. We can obtain $4^{3(2k)+2}$ codewords of length 2^{2k} and provide bound 7.2 for PMEPR. Let $m = 2k$, we actually construct 4^{3m+2} codewords of length 2^m , which is more than those presented in [18], [24] for moderately large carriers.

II. PRELIMINARIES

Let j be the imaginary unit, i.e., $j^2 = -1$. For an M -ary phase modulation OFDM, let $\xi^{\mathbb{Z}_M} = \{\xi^k : k \in \mathbb{Z}_M\}$, where $\xi = \exp(2\pi j/M)$ and $\mathbb{Z}_M = \{0, \dots, M-1\}$.

A. OFDM signals, instantaneous power and PMEPR

For a codeword $c = (c_0, \dots, c_{n-1})$ with $c_\ell \in \xi^{\mathbb{Z}_M}$, the n subcarrier complex baseband OFDM signal can be mathematically simplified as

$$s_c(z) := \sum_{\ell=0}^{n-1} c_\ell z^\ell, \quad (1)$$

where $z = e^{j2\pi t}$. The instantaneous power of the complex envelope $s_c(z)$ is defined by

$$P_c(z) := |s_c(z)|^2. \quad (2)$$

The peak-to-mean envelope power ratio (PMEPR) of the codeword c is defined as

$$\text{PMEPR}(c) := \frac{1}{n} \sup_{|z|=1} P_c(z). \quad (3)$$

B. Aperiodic auto-correlation and Golay sequences

For a sequence $a \in \mathbb{C}^n$, the aperiodic auto-correlation function $R_a(\cdot)$ is defined by

$$R_a(\ell) = \begin{cases} \sum_{k=0}^{n-\ell-1} a_{k+\ell} \bar{a}_k, & \ell = 0, 1, \dots, n-1, \\ 0, & \text{otherwise,} \end{cases}$$

where $\bar{a}_k$ is the complex conjugate of a_k .

Golay sequences were originally introduced to deal with the optical problem of multislit spectrometry. Golay also predicted that it will have possible application in communication engineering [15], which was recently fulfilled by the works done in [3], [4], [5], [6]. The original Golay sequences are defined only for the binary sequences [15]. However, it can be easily extended to the M -ary sequences as recently done in [4], [5], [6].

A pair of sequences a and b of length n are said to form a Golay pair if

$$P_a(z) + P_b(z) = 2n. \quad (4)$$

The sequences a and b are called Golay sequences.

Obviously, $\text{PMEPR}(a) \leq 2$ if a is a Golay sequence, which tightly controls the PMEPR of the underlying OFDM signal $s_a(z)$ by 2. This is a big advantage of using Golay sequences to reduce PMEPR for OFDM signals. Using the aperiodic auto-correlation function, the instantaneous power $P_a(z)$ of the sequence a can be represented as

$$P_a(z) = R_a(0) + \sum_{\ell=1}^{n-1} [R_a(\ell)z^\ell + \bar{R}_a(\ell)z^{-\ell}].$$

Therefore, equation (4) is equivalent to

$$R_a(\ell) + R_b(\ell) = 2n\delta(\ell),$$

where $\delta(\ell)$ is Dirac sequence, which takes the value 1 at 0, and takes the value 0 elsewhere.

C. Rudin-Shapiro polynomials

Besides those introduced in the introduction, the early application of Rudin-Shapiro polynomials (RSP) [19] to constructing encoding and decoding schemes for OFDM can be found in [25].

For a $k \geq 0$, an RSP pair $(A(z), B(z))$ is recursively defined as

$$\begin{cases} A_{k+1}(z) = A_k(z) + \xi_k z^{2^k} B_k(z), \\ B_{k+1}(z) = A_k(z) - \xi_k z^{2^k} B_k(z), \end{cases} \quad (5)$$

where $A_0(z) = B_0(z) = 1$ and ξ_k is any element in $\xi^{\mathbb{Z}_M}$.

Formula (5) recursively produces the polynomials $A_k(z)$ and $B_k(z)$ of degree $2^k - 1$ for any $k > 0$. In general, for $n = 2^m$, let the sequences a and b be, respectively, the coefficients of the polynomials $A_m(z)$ and $B_m(z)$. The 2^m -subcarrier OFDM signals are $s_a(z) = A_m(z)$ and $s_b(z) = B_m(z)$. For example, for $m = 3$, we have $n = 8$ and the codewords

$$\begin{cases} a = (1 \ \xi_0 \ \xi_1 \ -\xi_1\xi_0 \ \xi_2 \ \xi_2\xi_0 \ -\xi_2\xi_1 \ \xi_2\xi_1\xi_0), \\ b = (1 \ \xi_0 \ \xi_1 \ -\xi_1\xi_0 \ \xi_2 \ -\xi_2\xi_0 \ \xi_2\xi_1 \ -\xi_2\xi_1\xi_0). \end{cases}$$

From (5), it is clear that

$$P_a(z) + P_b(z) = |s_a(z)|^2 + |s_b(z)|^2 = |A_m(z)|^2 + |B_m(z)|^2$$

Noting $|A_m(z)|^2 + |B_m(z)|^2 = 2[|A_{m-1}(z)|^2 + |B_{m-1}(z)|^2]$ and repeating the process, we have

$$P_a(z) + P_b(z) = 2^m [|A_0(z)|^2 + |B_0(z)|^2] = 2n.$$

This shows that a and b form a Golay pair. Therefore, RSP pair constitute a subset of Golay pair. Hence the PMEPR of an RSP is at most 2.

By the recursive formula (5), one can construct M^m Golay sequences. Since for any $\eta \in \xi^{\mathbb{Z}_M}$, ηa is also a Golay sequence if a is a Golay sequence, one can construct M^{m+1} Golay sequences by RSP.

III. NEW CONSTRUCTION OF 16-QAM CODEWORDS

Our construction of 16-QAM is based on the generalized RSP and the decomposition of 16-QAM symbols. In the following, we will firstly introduce the generalized RSP, then construct the 16-QAM codewords using generalized RSP.

A. Generalized RSP

We now introduce the generalized RSP, for which we write the formula (5) in the matrix form. Let

$$\mathbf{A}_k^2(z) = \begin{pmatrix} A_k(z) \\ B_k(z) \end{pmatrix}, \quad \mathbf{B}_k^2(z) = \begin{pmatrix} A_k(z) \\ z^{2^k} B_k(z) \end{pmatrix},$$

and

$$\mathbf{T}_k^2 = \begin{pmatrix} 1 & \xi_k \\ 1 & -\xi_k \end{pmatrix}.$$

Then one can rewrite formula (5) in the matrix form

$$\mathbf{A}_{k+1}^2(z) = \mathbf{T}_k^2 \mathbf{B}_k^2(z).$$

This immediately suggests an extension of RSP. Let $\theta = \exp(j2\pi/N)$. Extend $\mathbf{A}_k^2(z)$, $\mathbf{B}_k^2(z)$ and $\mathbf{T}_k^2$ respectively to $\mathbf{A}_k^N(z)$, $\mathbf{B}_k^N(z)$ and $\mathbf{T}_k^N$ as

$$\mathbf{A}_k^N(z) = \begin{pmatrix} A_{k+1}^0(z) \\ A_{k+1}^1(z) \\ \vdots \\ A_{k+1}^{N-1}(z) \end{pmatrix}, \quad \mathbf{B}_k^N(z) = \begin{pmatrix} A_k^0(z) \\ z^{N^k} A_k^1(z) \\ \vdots \\ z^{(N-1)N^k} A_k^{N-1}(z) \end{pmatrix},$$

$$\mathbf{T}_k^N = \begin{pmatrix} 1 & \xi_k^1 & \xi_k^2 & \dots & \xi_k^{N-1} \\ 1 & \theta \xi_k^1 & \theta^2 \xi_k^2 & \dots & \theta^{N-1} \xi_k^{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \theta^{N-1} \xi_k^1 & \theta^{2(N-1)} \xi_k^2 & \dots & \theta^{(N-1)(N-1)} \xi_k^{N-1} \end{pmatrix},$$

where $A_0^0 = \dots = A_0^{N-1} = 1$ and $\xi_k^1, \dots, \xi_k^{N-1}$ are any symbols taken from the constellation $\xi^{\mathbb{Z}_M}$. Naturally, the generalized RSP vector is iteratively defined by the formula

$$\mathbf{A}_k^N(z) = \mathbf{T}_k^N \mathbf{B}_k^N(z). \quad (6)$$

Each polynomial entry in $\mathbf{A}_k^N(z)$ is called a *generalized RSP*. Obviously, the generalized RSP degenerates to the ordinary RSP if $N = 2$. In this paper, we are interested in the case $N > 2$.

Fig. 1. Construction of 16-QAM from two QPSK.

B. PMEPR of 16-QAM sequences by generalized RSP

For a QPSK sequence Q constructed in the last subsection, we can write it as

$$Q = \alpha e^{j\pi/4}U + \beta e^{j\pi/4}V,$$

where $U = (\eta_0, \dots, \eta_{n-1})$ and $V = (\zeta_0, \dots, \zeta_{n-1})$ are the two QPSK sequences from generalized RSP (see Fig. 1).

Using the PMEPR of generalized RSP, one can obtain the PMEPR of 16-QAM sequences, which is summarized in the following theorem.

Theorem 1: For the 16-QAM sequence Q of length 4^k , constructed in the last subsection, the PMEPR is at most 7.2. Proof: Let $Z = (1, z, \dots, z^{n-1})^T$. Then the 16-QAM OFDM signal is $s_Q = QZ$. Since $Q = \alpha e^{j\pi/4}U + \beta e^{j\pi/4}V$, we have

$$s_Q(z) = \alpha e^{j\pi/4}UZ + \beta e^{j\pi/4}VZ.$$

If we can show $\text{PMEPR}(U) \leq B$ and $\text{PMEPR}(V) \leq B$, then it is easy to see

$$|s_Q(z)|^2 \leq (\alpha + \beta)^2 Bn = \frac{9Bn}{5}.$$

In order to show $\text{PMEPR}(Q) \leq 7.2$, we only need to show $B = 4$. In the following, we will verify this result.

Since U and V are two QPSK sequences constructed by generalized RSP. Without loss of generality, we can just investigate U . The derived result can be directly applied to V . Assume U^1Z, U^2Z, U^3Z together with $U^0Z (= UZ)$ form the generalized RSP vector. From (6), we have

$$\begin{aligned} \sum_{\ell=0}^3 |s_{U^\ell}(z)|^2 &= \sum_{\ell=0}^3 |A_m^\ell(z)|^2 \\ &= (\mathbf{A}_m^4)^\top \cdot \overline{\mathbf{A}_m^4}, \end{aligned}$$

where $(\mathbf{A}_m^4)^\top$ is the transpose of the matrix $\mathbf{A}_m^4$. Since $\mathbf{T}_{m-1}^N$

is an orthogonal matrix, we have $(\mathbf{T}_{m-1}^4)^\top \overline{\mathbf{T}_{m-1}^4} = 4\mathbf{I}_4$ and

$$\begin{aligned} \sum_{\ell=0}^3 |s_{U^\ell}(z)|^2 &= (\mathbf{B}_{m-1}^4(z))^\top (\mathbf{T}_{m-1}^4)^\top \overline{\mathbf{T}_{m-1}^4} \mathbf{B}_{m-1}^4(z) \\ &= 4 (\mathbf{B}_{m-1}^4(z))^\top \mathbf{I}_4 \overline{\mathbf{B}_{m-1}^4(z)} \\ &= 4 \sum_{\ell=0}^3 (\mathbf{B}_{m-1}^\ell(z))^\top \overline{\mathbf{B}_{m-1}^\ell(z)} \\ &= 4 \sum_{\ell=0}^3 (\mathbf{A}_{m-1}^\ell(z))^\top \overline{\mathbf{A}_{m-1}^\ell(z)} \\ &= \dots \\ &= 4^m \sum_{\ell=0}^3 (\mathbf{A}_0^\ell(z))^\top \overline{\mathbf{A}_0^\ell(z)} \\ &= 4^{m+1} = 4n, \end{aligned}$$

where $\mathbf{I}_4$ is the identity 4×4 matrix. This clearly shows that $\text{PMEPR}(U) = 4$. By the same reason, $\text{PMEPR}(V) = 4$. Therefore $B = 4$, which completes the proof.

C. Code rate of 16-QAM by generalized RSP

For the generalized RSP of degree $4^k - 1$, there are $3k$ variables involved in $A_k^0(z)$, and each variable has 4 choices. Hence, one can construct 4^{3k} QPSK sequences by generalized RSP. Since for any $\xi \in \xi^{\mathbb{Z}_4}$, ξa is a generalized RSP if a is a generalized RSP, one can totally construct 4^{3k+1} distinct QPSK sequences by generalized RSP. Since we choose different QPSK sequences to construct the 16-QAM OFDM signals, we have totally $[4^{3k+1}]^2$ different choices. This gives the code rate of the 16-QAM by generalized RSP.

Theorem 2: For the 16-QAM codes of length 4^k by generalized RSP, the code rate is

$$\frac{6k+2}{4^k}.$$

Let $m = 2k$, the code rate is $(3m+2)/2^m$, which is higher than the constructions presented in [17], [18], [24]. Fig. 2 shows the code rate versus the code length for different constructions. One will see our performance is better than [18], [24] for the moderately large carriers. Since generalized RSP is a subset of Golay set, our curve is below that in [17]. But the generalised RSP based method in this paper is simple, efficient and practical..

D. Hamming distance of 16-QAM by generalized RSP

For two 16-QAM symbols $q_1 = q(\eta_1, \zeta_1)$ and $q_2 = q(\eta_2, \zeta_2)$, we can see $q_1 \neq q_2$ if either $\eta_1 \neq \eta_2$ or $\zeta_1 \neq \zeta_2$. Take a 16-QAM sequence $Q = \alpha e^{j\pi/4}U + \beta e^{j\pi/4}V$, where $U = (\eta_0, \dots, \eta_{n-1})$ and $V = (\zeta_0, \dots, \zeta_{n-1})$ are two QPSK sequences. Then the Hamming distance (HD) of Q is the smaller one between the Hamming distances of U and V . In the following theorem, the Hamming distance of 16-QAM sequences by generalized RSP is given.

Theorem 3: For the 16-QAM sequences of length 4^k by generalized RSP, the minimum Hamming distance is 4^{k-1} .

Proof: Take the 16-QAM sequence $Q = \alpha e^{j\pi/4}U + \beta e^{j\pi/4}V$ of length 4^k . Since the hamming distance of Q is

Fig. 2. The code rates of the constructions by [17], [18], [24] and this paper versus the length of the code. Our code rate is higher than those in [18], [24]. Our code rate is slightly below that in [17], but our method is simple, efficient and practical.

determined by those of U and V , we just need to find out the Hamming distances of U and V . In the following, we will use induction to show that the Hamming distances of U and V are 4^{k-1} .

For $k = 1$ we have $A_1^0(z) = 1 + \xi_0^1 z + \xi_0^2 z^2 + \xi_0^3 z^3$. For different choices of $\xi_0^1, \xi_0^2, \xi_0^3$, we obtain different codes. Then the Hamming distance of U is $1 = 4^{k-1}$.

For the case $k = m + 1$, we have

$$A_{m+1}^0 = A_m^0 + \xi_m^1 z^{4^m} A_m^1 + \xi_m^2 z^{2 \cdot 4^m} A_m^2 + \xi_m^3 z^{3 \cdot 4^m} A_m^3$$

Since the degrees of $A_m^0, \dots, A_m^3$ are $4^m - 1$, the coefficients of $z^0, \dots, z^{3 \cdot 4^m}$ will not add to each other. For the different choices of ξ_m^ℓ , the derived codes are different at least at 4^m places, the length of A_m^ℓ . Therefore the Hamming distance of the code is $4^{(m+1)-1}$. For the case of fixed choice of ξ_m^ℓ , suppose the Hamming distance of the code by A_m^ℓ are 4^{m-1} . Since each A_m^ℓ includes all ξ_i^ℓ for $\ell = 0, 1, 2, 3$ and $0 \leq i < m$, there are must one ξ_i^ℓ changes. This implies that all A_m^ℓ are not fixed choices for two different A_{m+1}^0 . Therefore the code rate of A_{m+1}^0 is $4 \times 4^{m-1} = 4^{(m+1)-1}$.

By induction, this proves the Hamming distance of U is 4^{k-1} . For the same reason, the Hamming distance of V is also 4^{k-1} . Therefore the Hamming distance of Q is 4^{k-1} , which completes the proof.

Let $2k = m$, the Hamming distance is actually 2^{m-2} for this kind of codes of length 2^m .

IV. CONCLUSIONS

In this paper, we introduce the generalized RSP, which can recursively produce a large number of QPSK sequences. Then we use these QPSK sequences to construct 16-QAM codewords for OFDM signals. By this construction, we can obtain 4^{3m+2} 16-QAM codewords of length 2^m , while controlling the PMEPR by 7.2. Moreover, we find that the Hamming distance is 2^{m-2} for this code of length 2^m . Since this construction is very simple and efficient, it is very practical.

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