

Improved Fast Recursive Algorithms for V-BLAST and G-STBC with Novel Efficient Matrix Inversion

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Abstract—In this paper, we propose a novel matrix inversion algorithm, which speeds up the corresponding step in the existing fast recursive algorithm for vertical Bell Laboratories layered space-time architecture (V-BLAST) by 1.67. Totally our improved recursive algorithm for V-BLAST speeds up the existing recursive algorithm for V-BLAST by 1.3. Furthermore, we develop an efficient recursive algorithm to implement the minimum mean-square error (MMSE) successive interference cancellation (SIC) detector with optimal ordering for Groupwise Space-Time Block Coded system (G-STBC), by exploiting the properties of the Alamouti structure in the equivalent channel matrix for G-STBC. Our recursive algorithm for G-STBC speeds up the low-complexity MMSE SIC algorithm with sub-optimal ordering for G-STBC by 2.57 approximately.

I. INTRODUCTION

Bell Laboratories Layered Space-Time architecture (BLAST), including the most practical version- vertical BLAST (V-BLAST) can achieve very high data rate and spectral efficiency in rich multi-path environments through exploiting the extra spatial dimension [1]. However, the required computational complexity is quite high. Recently several fast algorithms have been proposed for efficient implementation of V-BLAST [2]–[6]. The square-root algorithm in [2] can reduce the computational load of the conventional V-BLAST by $0.7M$, where M is the number of transmit antennas. The speedups of the fast recursive algorithm in [3] over the algorithm in [2] in the number of multiplications and additions are 1.54 and 1.89 respectively [4]. Improvements for different parts of the fast recursive algorithm [3] are proposed in [5] and [4], respectively, which are then incorporated in [6] to give the “fastest known algorithm”. The contributions of [6] also include a further improvement for the “fastest known algorithm” with the speedups claimed to be 1.3.

On the other hand, since space-time block code (STBC) provides diversity gain [7], the combination of STBC and BLAST can achieve both diversity gain and high data rates simultaneously [8]–[11], which is known as Groupwise Space-Time Block Coded system (G-STBC) or Multilayered STBC (MLSTBC). In G-STBC, the transmit antennas are divided into M subgroups, of which each has K transmit antennas and a corresponding STBC encoder. To detect one subgroup by suppressing the interference from the other $M - 1$ subgroups, the serial nulling and group interference cancellation detector in [8] for G-STBC requires $N \geq (M-1) \times K + 1$ antennas. The detectors in [9]–[11] only require $N \geq M$ antennas, which

exploit the temporal and spatial structure of STBC to generate an equivalent channel model similar to the one in the standard V-BLAST, and then extend the detectors for V-BLAST to the equivalent channel model. Minimum mean-square error (MMSE) successive interference cancellation (SIC) detectors with optimal ordering for G-STBC are proposed in [9] for $M = N = 2$ and in [10] for arbitrary M and N ($N \geq M$), respectively. To reduce the required computational complexity, the efficient MMSE-SIC detector for V-BLAST with sub-optimal ordering in [12] is extended to decode G-STBC in [11].

In this paper, firstly we further improve the recursive algorithm for V-BLAST in [6] by proposing a novel fast algorithm to compute the inverse matrix, i.e., the initial detection error covariance matrix \mathbf{Q} obtained by matrix inversion. Then we extend our improved recursive algorithm for V-BLAST to decode G-STBC. Specifically, we consider the simplest and most popular Alamouti STBC [7] in this paper. Our recursive algorithm for G-STBC exploits the properties of the Alamouti structure [13] in the equivalent channel model to further reduce the computational complexity.

System models for V-BLAST and G-STBC are overviewed in section II, followed by the description of the recursive algorithm for V-BLAST in [6] in Section III. A fast algorithm to compute the inverse matrix is proposed in Section IV to improve the recursive algorithm in [6]. Then the recursive algorithm for G-STBC is developed in Section V. The complexity of the presented algorithms for V-BLAST and G-STBC is evaluated in Section VI. Finally, we make conclusion in Section VII.

In the following sections, uppercase and lowercase bold face letters represent matrices and column vectors, respectively. $(\bullet)^T$, $(\bullet)^*$, and $(\bullet)^H$ denote matrix transposition, matrix conjugate, and matrix conjugate transposition, respectively. \mathbf{I}_M is the identity matrix with size M , and $\mathbf{0}_{M \times M}$ is the $M \times M$ zero matrix.

II. SYSTEM MODEL

The considered V-BLAST system consists of M transmit antennas and $N (\geq M)$ receive antennas in a rich-scattering and flat-fading wireless channel. At the transmitter, the data stream is de-multiplexed into M sub-streams, and each sub-stream is encoded and fed to its respective transmit antenna. Each receive antenna receives the signals from all M transmit

antennas. Let $\mathbf{s} = [s_1, s_2, \dots, s_M]^T$ denote the vector of transmit symbols from M antennas and assume $E(\mathbf{ss}^H) = \sigma_s^2 \mathbf{I}_M$. Then the received signal is given by

$$\mathbf{x} = \mathbf{H} \cdot \mathbf{s} + \mathbf{n} \quad (1)$$

where \mathbf{n} is the $N \times 1$ complex Gaussian noise vector with zero mean and covariance $\sigma_n^2 \mathbf{I}_N$, and $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_M] = [\underline{\mathbf{h}}_1, \underline{\mathbf{h}}_2, \dots, \underline{\mathbf{h}}_N]^H$ is the $N \times M$ complex channel matrix with statistically independent entries. Vectors \mathbf{h}_m and $\underline{\mathbf{h}}_n^H$ represent the m^{th} column and the n^{th} row of \mathbf{H} respectively.

The linear MMSE detection of \mathbf{s} is

$$\mathbf{y} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1} \mathbf{H}^H \mathbf{x} \quad (2)$$

where $\alpha = \sigma_n^2 / \sigma_s^2$. Let $\mathbf{R} = \mathbf{H}^H \cdot \mathbf{H} + \alpha \mathbf{I}_M$, and then

$$\mathbf{Q} = \mathbf{R}^{-1} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1} \quad (3)$$

is the covariance matrix for the detection error $\mathbf{e} = \mathbf{s} - \mathbf{y}$ [2], [3], i.e., $E\{\mathbf{ee}^H\} = \sigma_n^2 \mathbf{Q}$. So the best estimate, i.e., the component of \mathbf{y} with the highest SNR, is corresponding to the minimal diagonal element in \mathbf{Q} .

The conventional V-BLAST scheme detects M components of \mathbf{s} by M iterations with the optimal ordering. In each iteration, the component with the highest post detection SNR among all the undetected components is detected by an MMSE filter and then its effect is subtracted from the received signal vector [1]–[3].

On the other hand, the considered G-STBC system consists of $M \times K$ transmit antennas and N receive antennas where $M \leq N$. And it includes M parallel and independent STBC encoders, of which each has K transmit antennas. In this paper, we consider the Alamouti STBC [7] with two transmit antennas as an example and then $K = 2$. At the transmitter, a single data stream $s_{11}, s_{12}, \dots, s_{m1}, s_{m2}, \dots, s_{M1}, s_{M2}$ is de-multiplexed into M sub-streams. Each sub-stream s_{m1}, s_{m2} ($m = 1, 2, \dots, M$) is then encoded by an independent Alamouti STBC encoder, and the corresponding STBC encoded symbols are fed to its respective $K = 2$ transmit antennas and transmitted in two time slots. The received signals over two time slots can be represented as

$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1,2M} \\ h_{21} & h_{22} & \dots & h_{2,2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \dots & h_{N,2M} \end{pmatrix} \begin{pmatrix} s_{11} & -s_{12}^* \\ s_{12} & s_{11}^* \\ \vdots & \vdots \\ s_{M1} & -s_{M2}^* \\ s_{M2} & s_{M1}^* \end{pmatrix} + \mathbf{n}$$

which may be transformed into the equivalent channel model [9]–[11] as

$$\mathbf{x}' = \mathbf{H}' \mathbf{s}' + \mathbf{n}' \quad (4)$$

where

$$\mathbf{x}' = [x_{11} \ x_{12}^* \ \dots \ x_{N1} \ x_{N2}^*]^T, \quad (5)$$

the equivalent channel matrix

$$\mathbf{H}' = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1,2M-1} & h_{1,2M} \\ h_{12}^* & -h_{11}^* & \dots & h_{1,2M}^* & -h_{1,2M-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{N1} & h_{N2} & \dots & h_{N,2M-1} & h_{N,2M} \\ h_{N2}^* & -h_{N1}^* & \dots & h_{N,2M}^* & -h_{N,2M-1} \end{pmatrix}, \quad (6)$$

and

$$\mathbf{s}' = [s_{11} \ s_{12} \ \dots \ s_{M1} \ s_{M2}]^T. \quad (7)$$

III. THE EXISTING RECURSIVE ALGORITHM FOR V-BLAST IN [6]

In this section, we describe the recursive algorithm for V-BLAST proposed in [6]. In the subsequent derivations,

$$\mathbf{H}_m = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_m] \quad (8)$$

denotes the first m columns of \mathbf{H} , and $\mathbf{R}_m = \mathbf{H}_m^H \mathbf{H}_m + \alpha \mathbf{I}_m$.

In the *initialization* phase, with $\mathbf{R}_{[0]} = \alpha \mathbf{I}_M$, the initial $\mathbf{R} = \mathbf{R}_M$ is computed recursively by

$$\mathbf{R}_{[l]} = \sum_{n=1}^l \underline{\mathbf{h}}_n^H \underline{\mathbf{h}}_n + \alpha \mathbf{I}_M = \mathbf{R}_{[l-1]} + \underline{\mathbf{h}}_l^H \underline{\mathbf{h}}_l \quad (9)$$

for $l = 1, 2, \dots, N$, and then $\mathbf{R}_{[N]} = \mathbf{R}$.

To get the initial $\mathbf{Q} = \mathbf{Q}_M$ from $\mathbf{Q}_1 = \mathbf{R}_1^{-1}$, $\mathbf{Q}_m = \mathbf{R}_m^{-1}$ is computed from \mathbf{Q}_{m-1} recursively for $m = 2, 3, \dots, M$ by

$$\mathbf{T}_{m-1} = \mathbf{Q}_{m-1} + \frac{\mathbf{Q}_{m-1} \mathbf{v}_{m-1} \mathbf{v}_{m-1}^H \mathbf{Q}_{m-1}}{\gamma_m - \mathbf{v}_{m-1}^H \mathbf{Q}_{m-1} \mathbf{v}_{m-1}} \quad (10)$$

and

$$\mathbf{Q}_m = \begin{bmatrix} \mathbf{T}_{m-1} & -\gamma_m^{-1} \mathbf{T}_{m-1} \mathbf{v}_{m-1} \\ -\gamma_m^{-1} \mathbf{v}_{m-1}^H \mathbf{T}_{m-1} & \gamma_m^{-1} + \gamma_m^{-2} \mathbf{v}_{m-1}^H \mathbf{T}_{m-1} \mathbf{v}_{m-1} \end{bmatrix}, \quad (11)$$

where γ_m and the $(m-1) \times 1$ vector \mathbf{v}_{m-1} are in the m^{th} column of \mathbf{R}_m , i.e.,

$$\mathbf{R}_m = \begin{bmatrix} \mathbf{R}_{m-1} & \mathbf{v}_{m-1} \\ \mathbf{v}_{m-1}^H & \gamma_m \end{bmatrix}, \quad (12)$$

and \mathbf{R}_m is the sub-matrix of \mathbf{R} [3].

In the *recursion* phase, \mathbf{Q} is deflated recursively by

$$\mathbf{Q}_{|m-1} = \mathbf{T}_{|m-1} - \psi_{|m}^{-1} \mathbf{w}_{|m-1} \mathbf{w}_{|m-1}^H \quad (13)$$

for $m = M, M-1, \dots, 2$, where $\mathbf{T}_{|m-1}$, $\psi_{|m}$ and the $(m-1) \times 1$ vector $\mathbf{w}_{|m-1}$ are got from the matrix $\mathbf{Q}_{|m}$ by

$$\mathbf{Q}_m = \begin{bmatrix} \mathbf{T}_{m-1} & \mathbf{w}_{m-1} \\ \mathbf{w}_{m-1}^H & \psi_m \end{bmatrix}. \quad (14)$$

The recursive algorithm for V-BLAST in [6] is as follows:

Initialization: Set $\mathbf{x}_M = \mathbf{x}$ and $\mathbf{f} = [1, 2, \dots, M]^T$. Compute $\mathbf{z}_M = \mathbf{H}^H \mathbf{x}_M$. Compute the initial \mathbf{R}_M recursively using (9) and get \mathbf{B}_M as a byproduct where

$$\mathbf{B}_m = \mathbf{H}_m^H \mathbf{H}_m = \mathbf{R}_m - \alpha \mathbf{I}_m. \quad (15)$$

Compute the initial $\mathbf{Q}_M = \mathbf{Q}_{|M}$ recursively by (10) and (11).

Recursion: for $m = M, M-1, \dots, 2$.

- (a) Find $l_m = \arg \min_{i=1,2,\dots}^m q_{ii}$, where q_{ii} is the i^{th} diagonal element of $\mathbf{Q}_{|m}$. Permute rows and columns l_m and m in $\mathbf{Q}_{|m}$ and \mathbf{B}_m . Permute rows l_m and m in \mathbf{f} and \mathbf{z}_m .
- (b) $p_m = \mathbf{f}_m$, the estimation of the p_m -th signal is

$$y_{p_m} = \mathbf{q}_m^H \mathbf{z}_m \quad (16)$$

where \mathbf{q}_m denotes the m^{th} column of $\mathbf{Q}_{|m}$.

- (c) $\hat{s}_{p_m} = Q[y_{p_m}]$, where $Q[\cdot]$ indicates the slicing procedure according to the constellation in use.
- (d) Cancel the effect of s_{p_m} in \mathbf{z}_m by

$$\mathbf{z}_{m-1} = \mathbf{z}_m^{[-1]} - \hat{s}_{p_m} \mathbf{b}_m^{[-1]} \quad (17)$$

where $\mathbf{z}_m^{[-1]}$ and $\mathbf{b}_m^{[-1]}$ are got by removing the last element in \mathbf{z}_m and \mathbf{b}_m , respectively, and \mathbf{b}_m is the m^{th} column in \mathbf{B}_m . Get \mathbf{B}_{m-1} by removing the last column and row of \mathbf{B}_m .

- (e) Determine $\mathbf{T}_{|m-1}$, $\psi_{|m}$ and $\mathbf{w}_{|m-1}$ from $\mathbf{Q}_{|m}$ by (14). Then compute $\mathbf{Q}_{|m-1}$ by (13).

Solutions: When $m = 1$, only the above steps (b) and (c) is executed to get $\hat{s}_{p_1} = Q(y_{p_1})$.

IV. A NOVEL FAST ALGORITHM TO COMPUTE THE INVERSE MATRIX \mathbf{R}^{-1}

In this section, we propose a novel fast algorithm to compute the inverse matrix $\mathbf{Q} = \mathbf{R}^{-1}$, which improves the corresponding step of the recursive algorithm in the last section.

Following [5] and [6], we try to compute \mathbf{Q}_m from \mathbf{Q}_{m-1} and substitute (12) and (14) into $\mathbf{R}_m \mathbf{Q}_m = \mathbf{I}_m$ to get

$$\begin{bmatrix} \mathbf{R}_{m-1} & \mathbf{v}_{m-1} \\ \mathbf{v}_{m-1}^H & \gamma_m \end{bmatrix} \begin{bmatrix} \mathbf{T}_{m-1} & \mathbf{w}_{m-1} \\ \mathbf{w}_{m-1}^H & \psi_m \end{bmatrix} = \mathbf{I}_m. \quad (18)$$

From (18) we can deduce

$$\begin{cases} \mathbf{R}_{m-1} \mathbf{w}_{m-1} + \mathbf{v}_{m-1} \psi_m = 0 \end{cases} \quad (19a)$$

$$\begin{cases} \mathbf{v}_{m-1}^H \mathbf{w}_{m-1} + \gamma_m \psi_m = 1 \end{cases} \quad (19b)$$

$$\begin{cases} \mathbf{R}_{m-1} \mathbf{T}_{m-1} + \mathbf{v}_{m-1} \mathbf{w}_{m-1}^H = \mathbf{I}_{m-1}. \end{cases} \quad (19c)$$

Finally from (19) we can derive

$$\begin{cases} \psi_m = (\gamma_m - \mathbf{v}_{m-1}^H \mathbf{Q}_{m-1} \mathbf{v}_{m-1})^{-1} \end{cases} \quad (20a)$$

$$\begin{cases} \mathbf{w}_{m-1} = -\psi_m \mathbf{Q}_{m-1} \mathbf{v}_{m-1} \end{cases} \quad (20b)$$

$$\begin{cases} \mathbf{T}_{m-1} = \mathbf{Q}_{m-1} + \psi_m^{-1} \mathbf{w}_{m-1} \mathbf{w}_{m-1}^H, \end{cases} \quad (20c)$$

and the derivation is shown as follows. From (19a) and $\mathbf{Q}_m = \mathbf{R}_m^{-1}$ we deduce (20b), which is substituted into (19b) to obtain (20a). From (19c) and (20b) we deduce $\mathbf{T}_{m-1} = \mathbf{Q}_{m-1} - \mathbf{Q}_{m-1} \mathbf{v}_{m-1} \mathbf{w}_{m-1}^H$ and $\mathbf{Q}_{m-1} \mathbf{v}_{m-1} = -\psi_m^{-1} \mathbf{w}_{m-1}$ respectively, and then the latter is substituted into the former to derive (20c). Equation (20c) is equivalent to (13) which has been proved in [6] according to the block matrix inversion lemma [14].

To avoid unnecessary divisions, we can compute

$$\mathbf{u}_m = \mathbf{Q}_{m-1} \mathbf{v}_{m-1} \quad (21)$$

firstly, which is then substituted into (20) to get

$$\begin{cases} \psi_m = (\gamma_m - \mathbf{v}_{m-1}^H \mathbf{u}_m)^{-1} \end{cases} \quad (22a)$$

$$\begin{cases} \mathbf{w}_{m-1} = -\psi_m \mathbf{u}_m \end{cases} \quad (22b)$$

$$\begin{cases} \mathbf{T}_{m-1} = \mathbf{Q}_{m-1} + \psi_m \mathbf{u}_m \mathbf{u}_m^H. \end{cases} \quad (22c)$$

Notice that the \mathbf{Q}_m , \mathbf{T}_{m-1} , ψ_m and \mathbf{w}_{m-1} in the *initialization* phase are usually different from the $\mathbf{Q}_{|m}$, $\mathbf{T}_{|m-1}$, $\psi_{|m}$ and $\mathbf{w}_{|m-1}$ in the *recursion* phase, since usually they are corresponding to different sets of m transmit antennas.

In the recursive algorithm for V-BLAST in the last section, we can use (20) and (14) instead of (11) and (10) to compute the initial \mathbf{Q}_M recursively. Then we get our improved recursive algorithm for V-BLAST, in which (20) can be simplified to be (21) and (22) to avoid unnecessary divisions.

V. THE FAST RECURSIVE ALGORITHM FOR G-STBC

Mathematically (4) is similar to (1), and our improved recursive algorithm for V-BLAST can be directly extended to the equivalent channel model (4), as the algorithms for V-BLAST are extended to decode G-STBC in [10] and [11]. However, since the equivalent channel matrix \mathbf{H}' in (6) consists of 2×2 Alamouti sub-blocks [7] defined by $\begin{bmatrix} a_1 & -a_2^* \\ a_2 & a_1^* \end{bmatrix}$ and the Alamouti structure remains invariant under several matrix operations [13], we can develop a recursive algorithm for G-STBC with reduced complexity.

In (6), let $\mathbf{H}' = [\mathbf{h}'_1 \ \mathbf{h}'_2 \ \cdots \ \mathbf{h}'_{2M-1} \ \mathbf{h}'_{2M}] = [\bar{\mathbf{h}}'_1 \ \bar{\mathbf{h}}'_2 \ \cdots \ \bar{\mathbf{h}}'_m \ \cdots \ \bar{\mathbf{h}}'_M]$, where \mathbf{h}'_m is the m^{th} column of \mathbf{H}' and $\bar{\mathbf{h}}'_m = [\mathbf{h}'_{2m-1} \ \mathbf{h}'_{2m}]$. Define $\bar{\mathbf{H}}' = [\bar{\mathbf{h}}'_1 \ \bar{\mathbf{h}}'_2 \ \cdots \ \bar{\mathbf{h}}'_m]$, $\bar{\mathbf{R}}_m = \bar{\mathbf{H}}'^H \bar{\mathbf{H}}'_m + \alpha \mathbf{I}_{2m}$ and $\bar{\mathbf{Q}}_m = \bar{\mathbf{R}}_m^{-1}$. Then we can extend (12) to be

$$\bar{\mathbf{R}}_m = \begin{bmatrix} \bar{\mathbf{R}}_{m-1} & \bar{\mathbf{v}}'_{m-1} \\ \bar{\mathbf{v}}'^H_{m-1} & \bar{\gamma}'_m \end{bmatrix}, \quad (23)$$

and extend (14) to be

$$\bar{\mathbf{Q}}_m = \begin{bmatrix} \bar{\mathbf{T}}_{m-1} & \bar{\mathbf{w}}'_{m-1} \\ \bar{\mathbf{w}}'^H_{m-1} & \bar{\psi}'_m \end{bmatrix}. \quad (24)$$

In this paper, matrices, vectors and scalars with upper lines, e.g. $\bar{\mathbf{H}}'_m$, $\bar{\mathbf{R}}_m$, $\bar{\mathbf{h}}'_m$, $\bar{\mathbf{v}}'_{m-1}$, $\bar{\gamma}'_m$ and $\bar{\psi}'_m$, represent block matrices, block vectors and block scalars, respectively, which all consist of 2×2 sub-blocks as block entries.

Since \mathbf{H}' in (6) consists of 2×2 Alamouti sub-blocks, $\bar{\mathbf{H}}'^H \bar{\mathbf{H}}'$ must be Hermitian with diagonal 2×2 sub-blocks that are non-negative scaled multiples of the identity matrix and with off-diagonal sub-blocks that are 2×2 Alamouti matrices [13, Property 3 in the subsection “B. Block Matrices”], and the same is $\bar{\mathbf{R}}_m$ in (23). That is to say, in (23)

$$\bar{\gamma}'_m = \gamma'_m \mathbf{I}_2 \quad (25)$$

where γ'_m is a non-negative scalar, and the $(m-1) \times 1$ block vector $\bar{\mathbf{v}}'_{m-1}$ consists of 2×2 Alamouti sub-blocks. Let

$$\bar{\mathbf{v}}'_{m-1} = [\mathbf{v}'_{2(m-1)} \ \mathbf{v}''_{2(m-1)}], \quad (26)$$

where the $2(m-1) \times 1$ vectors $\mathbf{v}'_{2(m-1)}$ and $\mathbf{v}''_{2(m-1)}$ are in the $(2m-1)^{th}$ and $2m^{th}$ column of $\bar{\mathbf{R}}_m$ respectively.

With (23) and (24), we can extend (18) to be

$$\begin{bmatrix} \bar{\mathbf{R}}_{m-1} & \bar{\mathbf{v}}'_{m-1} \\ \bar{\mathbf{v}}'^H_{m-1} & \bar{\gamma}'_m \end{bmatrix} \begin{bmatrix} \bar{\mathbf{T}}_{m-1} & \bar{\mathbf{w}}'_{m-1} \\ \bar{\mathbf{w}}'^H_{m-1} & \bar{\psi}'_m \end{bmatrix} = \mathbf{I}_{2m} \quad (27)$$

Then from (27) we can deduce

$$\left\{ \begin{array}{l} \bar{\mathbf{R}}_{m-1} \bar{\mathbf{w}}'_{m-1} + \bar{\mathbf{v}}'_{m-1} \bar{\psi}'_m = \mathbf{0}_{2 \times 2} \\ \bar{\mathbf{v}}'^H_{m-1} \bar{\mathbf{w}}'_{m-1} + \bar{\gamma}'_m \bar{\psi}'_m = \mathbf{I}_2 \end{array} \right. \quad (28a)$$

$$\left\{ \begin{array}{l} \bar{\mathbf{v}}'^H_{m-1} \bar{\mathbf{w}}'_{m-1} + \bar{\gamma}'_m \bar{\psi}'_m = \mathbf{I}_2 \\ \bar{\mathbf{R}}_{m-1} \bar{\mathbf{T}}_{m-1} + \bar{\mathbf{v}}'_{m-1} \bar{\mathbf{w}}'^H_{m-1} = \mathbf{I}_{2(m-1)}. \end{array} \right. \quad (28b)$$

$$\left\{ \begin{array}{l} \bar{\mathbf{v}}'_{m-1} = \left(\bar{\gamma}'_m - \bar{\mathbf{v}}'^H_{2(m-1)} \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{2(m-1)} \right)^{-1} \\ \bar{\psi}'_m = \bar{\psi}'_m \mathbf{I}_2 \\ \bar{\mathbf{w}}'_{m-1} = -\bar{\psi}'_m \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} \\ \bar{\mathbf{T}}_{m-1} = \bar{\mathbf{Q}}_{m-1} + \bar{\psi}'_m^{-1} \bar{\mathbf{w}}'_{m-1} \bar{\mathbf{w}}'^H_{m-1}, \end{array} \right. \quad (28c)$$

Finally from (28) we can derive

$$\left\{ \begin{array}{l} \bar{\psi}'_m = \left(\bar{\gamma}'_m - \bar{\mathbf{v}}'^H_{2(m-1)} \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{2(m-1)} \right)^{-1} \\ \bar{\psi}'_m = \bar{\psi}'_m \mathbf{I}_2 \end{array} \right. \quad (29a)$$

$$\left\{ \begin{array}{l} \bar{\mathbf{w}}'_{m-1} = -\bar{\psi}'_m \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} \\ \bar{\mathbf{T}}_{m-1} = \bar{\mathbf{Q}}_{m-1} + \bar{\psi}'_m^{-1} \bar{\mathbf{w}}'_{m-1} \bar{\mathbf{w}}'^H_{m-1}, \end{array} \right. \quad (29b)$$

$$\left\{ \begin{array}{l} \bar{\mathbf{v}}'_{m-1} = \left(\bar{\gamma}'_m - \bar{\mathbf{v}}'^H_{2(m-1)} \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{2(m-1)} \right)^{-1} \\ \bar{\psi}'_m = \bar{\psi}'_m \mathbf{I}_2 \end{array} \right. \quad (29c)$$

$$\left\{ \begin{array}{l} \bar{\mathbf{w}}'_{m-1} = -\bar{\psi}'_m \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} \\ \bar{\mathbf{T}}_{m-1} = \bar{\mathbf{Q}}_{m-1} + \bar{\psi}'_m^{-1} \bar{\mathbf{w}}'_{m-1} \bar{\mathbf{w}}'^H_{m-1}, \end{array} \right. \quad (29d)$$

and the derivation is as follows.

From (28a) and $\bar{\mathbf{Q}}_m = \bar{\mathbf{R}}_m^{-1}$, we get the $(m-1) \times 1$ block vector $\bar{\mathbf{w}}'_{m-1} = -\bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} \bar{\psi}'_m$, which is substituted into (28b) to get

$$\left(\bar{\gamma}'_m - \bar{\mathbf{v}}'^H_{m-1} \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} \right) \bar{\psi}'_m = \mathbf{I}_2. \quad (30)$$

In appendix A, we will prove that

$$\bar{\mathbf{v}}'^H_m \bar{\mathbf{Q}}_m \bar{\mathbf{v}}'_m = \left(\bar{\mathbf{v}}'^H_{2m-1} \bar{\mathbf{Q}}_m \bar{\mathbf{v}}'_{2m} \right) \mathbf{I}_2 = \phi_m \mathbf{I}_2 \quad (31)$$

is non-negative scaled multiples of the identity matrix. (31) and (25) are substituted into (30) to deduce (29b) and (29a), and then (29b) is substituted into (28a) to get (29c), where

$$\bar{\mathbf{w}}'_{m-1} = \left[\begin{array}{cc} \mathbf{w}'_{2(m-1)} & \mathbf{w}''_{2(m-1)} \end{array} \right], \quad (32)$$

and the $2(m-1) \times 1$ vectors $\mathbf{w}'_{2(m-1)}$ and $\mathbf{w}''_{2(m-1)}$ are in the $(2m-1)^{th}$ column and $2m^{th}$ column of $\bar{\mathbf{Q}}_m$, respectively. Since $\bar{\mathbf{Q}}_m = \bar{\mathbf{R}}_m^{-1}$ consists of 2×2 Alamouti sub-blocks [13, Lemma “Invariance Under Inversion”], in (29c) only the first column of $\bar{\mathbf{w}}'_{m-1}$, i.e., $\mathbf{w}'_{2(m-1)} = -\bar{\psi}'_m \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{2(m-1)}$, is computed, from which $\mathbf{w}''_{2(m-1)}$ can be got directly. Finally from (28c) and (29c) we can deduce $\bar{\mathbf{T}}_{m-1} = \bar{\mathbf{Q}}_{m-1} - \bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} \bar{\mathbf{w}}'^H_{m-1}$ and $\bar{\mathbf{Q}}_{m-1} \bar{\mathbf{v}}'_{m-1} = -\bar{\psi}'_m^{-1} \bar{\mathbf{w}}'_{m-1}$ respectively, and then the latter is substituted into the former to get (29d).

In addition, from (29d) it is easy to derive

$$\bar{\mathbf{Q}}_{m-1} = \bar{\mathbf{T}}_{m-1} - \bar{\psi}'_m^{-1} \bar{\mathbf{w}}'_{m-1} \bar{\mathbf{w}}'^H_{m-1}, \quad (33)$$

which can be employed to deflate the matrix $\bar{\mathbf{Q}}_{|m}$ recursively.

Then our full recursive algorithm for G-STBC can be summarized as follows:

Initialization: Set $\mathbf{x}'_M = \mathbf{x}'$ where \mathbf{x}' is defined in (5), and set $\mathbf{f} = [1, 2, \dots, M]^T$. Compute $\mathbf{z}'_M = \bar{\mathbf{H}}'^H \mathbf{x}'_M$. Compute the initial $\bar{\mathbf{R}}_M = \bar{\mathbf{H}}'^H_M \bar{\mathbf{H}}' M + \alpha \mathbf{I}_{2M}$, and get

$$\bar{\mathbf{B}}_M = \bar{\mathbf{H}}'^H_M \bar{\mathbf{H}}' M = \bar{\mathbf{R}}_M - \alpha \mathbf{I}_{2M} \quad (34)$$

as a byproduct. Get $\bar{\mathbf{Q}}_1 = \bar{\mathbf{R}}_1^{-1} = (\mathbf{h}'_1 \mathbf{h}'_1 + \alpha)^{-1} \mathbf{I}_2$, and then compute $\bar{\mathbf{Q}}_m$ from $\bar{\mathbf{Q}}_{m-1}$ recursively by (29) and (24) till the initial $\bar{\mathbf{Q}}_M = \bar{\mathbf{Q}}_{|M}$.

TABLE I
COMPLEXITY COMPARISON BETWEEN OUR NEW RECURSIVE ALGORITHM FOR V-BLAST AND OUR RECURSIVE ALGORITHM FOR G-STBC

	For V-BLAST	For G-STBC
To compute \mathbf{R}_M or $\bar{\mathbf{R}}_M$	$\frac{1}{2} M^2 N$	$2M^2 N$
To compute \mathbf{Q}_M or $\bar{\mathbf{Q}}_M$	$\frac{1}{2} M^3$	$2M^3$
To deflate $\mathbf{Q}_{ m}$ or $\bar{\mathbf{Q}}_{ m}$	$\frac{1}{6} M^3$	$\frac{2}{3} M^3$
Total	$\frac{1}{2} M^2 N + \frac{2}{3} M^3$	$2M^2 N + \frac{8}{3} M^3$

Recursion: for $m = M, M-1, \dots, 2$.

- (a) Find $l_m = \arg \min_{i=1,3,5,\dots} q'_{ii}$, where q'_{ii} is the diagonal element of $\bar{\mathbf{Q}}_{|m}$. Exchange rows and columns (l_m, l_m+1) with rows and columns $(2m-1, 2m)$ in $\bar{\mathbf{Q}}_{|m}$ and $\bar{\mathbf{B}}_m$. Exchange rows (l_m, l_m+1) with rows $(2m-1, 2m)$ in \mathbf{z}'_{m-1} . Permute rows $\frac{1}{2}(l_m+1)$ and m in \mathbf{f} .
- (b) $p_m = \mathbf{f}_m$, the estimations of the p_m -th group of STBC encoded symbols is

$$[y_{p_m1} \ y_{p_m2}]^T = [\mathbf{q}'_{2m-1} \ \mathbf{q}'_{2m}]^H \mathbf{z}'_m, \quad (35)$$

where \mathbf{q}'_{2m-1} and \mathbf{q}'_{2m} denotes the $(2m-1)^{th}$ and $2m^{th}$ column of $\bar{\mathbf{Q}}_{|m}$, respectively.

$$(c) [\hat{s}_{p_m1} \ \hat{s}_{p_m2}]^T = [Q\{y_{p_m1}\} \ Q\{y_{p_m2}\}]^T.$$

(d) Cancel the effect of s_{p_m1} and s_{p_m2} in \mathbf{z}'_m by

$$\mathbf{z}'_{m-1} = \mathbf{z}'_{m-1}^{[-2]} - \left[\begin{array}{cc} \mathbf{b}'_{2m-1}^{[-2]} & \mathbf{b}'_{2m}^{[-2]} \end{array} \right] \left[\begin{array}{c} \hat{s}_{p_m1} \\ \hat{s}_{p_m2} \end{array} \right], \quad (36)$$

where $\mathbf{z}'_{m-1}^{[-2]}$, $\mathbf{b}'_{2m-1}^{[-2]}$ and $\mathbf{b}'_{2m}^{[-2]}$ are got by removing the last two elements in \mathbf{z}'_m , \mathbf{b}'_{2m-1} and \mathbf{b}'_{2m} , respectively, and \mathbf{b}'_k denotes the k^{th} column in $\bar{\mathbf{B}}_m$. Get $\bar{\mathbf{B}}_{m-1}$ by removing the last two columns and rows of $\bar{\mathbf{B}}_m$.

- (e) Determine $\bar{\psi}'_{|m}$, $\bar{\psi}'_{|m}$, $\bar{\mathbf{T}}_{|m-1}$, and $\bar{\mathbf{w}}'_{|m}$ from $\bar{\mathbf{Q}}_{|m}$ by (24) and (29b). Then compute $\bar{\mathbf{Q}}_{|m-1}$ by (33).

Solutions: When $m = 1$, only the above steps

- (b) and (c) is executed to get $[\hat{s}_{p_11} \ \hat{s}_{p_12}]^T = [Q\{y_{p_11}\} \ Q\{y_{p_12}\}]^T$.

VI. COMPLEXITY EVALUATION

The dominant computational complexities of our new recursive algorithm for V-BLAST and our recursive algorithm for G-STBC are compared in Table I where one item represents the number of multiplications and additions. Notice that in our recursive algorithm for G-STBC, only $\frac{1}{4}$ elements in $\bar{\mathbf{R}}_M$, $\bar{\mathbf{T}}_m$, $\bar{\mathbf{Q}}_m$ and $\bar{\mathbf{Q}}_{|m}$ need to be computed, since they are Hermitian and block matrices with 2×2 Alamouti sub-blocks.

The complexity to get the initial \mathbf{Q} by (10) and (11) is claimed to be $\frac{1}{2} M^3$ multiplications and additions [6, equation (19)], and in fact it should be $\frac{5}{6} M^3$ which mainly includes the computations of $\mathbf{g}_{m-1} = \mathbf{Q}_{m-1} \mathbf{v}_{m-1}$ and $\mathbf{Q}_{m-1} + \mathbf{g}_{m-1} \mathbf{g}_{m-1}^H / \beta_m$ in (10), and $\mathbf{T}_{m-1} \mathbf{v}_{m-1}$ in (11). Then actually the “fastest known algorithm” for V-BLAST from improvements before [6] and the corresponding improved recursive algorithm in [6] require $\frac{1}{2} M^2 N + \frac{4}{3} M^3$ and $\frac{1}{2} M^2 N + M^3$ of multiplications and additions, respectively. So

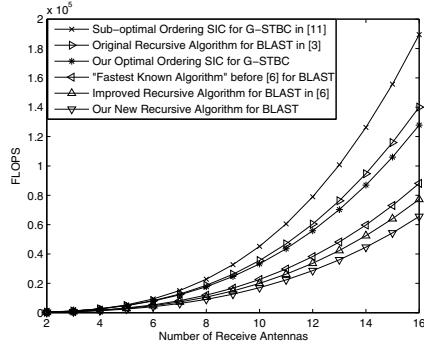


Fig. 1. Comparison of computational complexity among the MMSE-SIC algorithm with sub-optimal ordering for G-STBC in [11], the Original Recursive Algorithm for V-BLAST in [3], our MMSE-SIC algorithm with optimal ordering for G-STBC, the “fastest known algorithm” for V-BLAST from improvements before [6], the improved recursive algorithm for V-BLAST in [6], and our new recursive algorithm for V-BLAST.

the actual speedups of the algorithm in [6] over the previous “fastest known algorithm” should be $(\frac{11}{6})/(\frac{3}{2}) = 1.22$ instead of the 1.3 claimed in [6]. While the speedups of our new recursive algorithm for V-BLAST over the algorithm in [6] are $(\frac{3}{2})/(\frac{7}{6}) = 1.3$ when $M = N$. Moreover, our matrix inversion step to compute $\mathbf{Q} = \mathbf{R}^{-1}$ speeds up the corresponding step in [5], [6] by $(\frac{5}{6})/(\frac{1}{2}) = 1.67$.

In [11, equation (19)], it is claimed that the MMSE-SIC detector with sub-optimal ordering for G-STBC requires $8M^2N + 16M^3$ flops, where one flop can be one multiplication or one addition, and we simply assume the equal number of multiplications and additions. So when $M = N$, our MMSE-SIC algorithm with optimal ordering for G-STBC speeds up the MMSE-SIC algorithm with sub-optimal ordering for G-STBC in [11] by $(\frac{24}{14})/(\frac{14}{3}) = 2.57$ approximately.

Assuming $N = M$, we carried out numerical experiments to count the average flops per time slot for our new recursive algorithms for V-BLAST and G-STBC, the recursive algorithms for V-BLAST introduced in [3] and [6], and the algorithm for G-STBC in [11] for different N , the number of receive antennas. For V-BLAST the number of transmit antennas is M while for G-STBC the number of transmit antennas is $2M$. All results are shown in Fig. 1. It can be seen that they are consistent with the theoretical flops calculation.

VII. CONCLUSION

In this paper, firstly we improve the recursive algorithm for V-BLAST in [6] by proposing a novel fast algorithm to compute the inverse matrix, which speeds up the corresponding matrix inversion step in [5], [6] by 1.67. Correspondingly the speedups of our improved recursive algorithm for V-BLAST over the recursive algorithm in [6] in the number of multiplications and additions are 1.3. Then, by exploiting the properties of the Alamouti structure in the equivalent channel matrix, we develop a recursive algorithm to implement the MMSE-SIC detector with optimal ordering for G-STBC, which speeds up the MMSE-SIC detector with sub-optimal ordering for G-STBC in [11] by 2.57 approximately.

APPENDIX A THE PROOF OF (31)

By extending (17) of [12] to $\bar{\mathbf{H}}'$ for G-STBC, we get

$$\bar{\mathbf{Q}}_m = (\underline{\mathbf{H}}_m^H \underline{\mathbf{H}}_m)^{-1} = \mathbf{L}_m^{-1} \mathbf{L}_m^{-H}, \quad (37)$$

where $\underline{\mathbf{H}}_m = [\bar{\mathbf{H}}_m'^T \sqrt{\alpha} \mathbf{I}_{2m}]^T$ is QR decomposed into an orthogonal Θ_m and a triangular \mathbf{L}_m , i.e., $\underline{\mathbf{H}}_m = \Theta_m \mathbf{L}_m$. $\underline{\mathbf{H}}_m$ consists of 2×2 Alamouti sub-blocks, since the identity matrix can be seen as a special case of the Alamouti structure [13]. Then the QR factors of $\underline{\mathbf{H}}_m$, i.e., Θ_m and \mathbf{L}_m , also consist of 2×2 Alamouti sub-blocks [13, Lemma “Invariance Under QR Factorization”], and so does \mathbf{L}_m^{-1} [13, Lemma “Invariance Under Inversion”] and $\bar{\mathbf{v}}_m^H \mathbf{L}_m^{-1}$ [13, Property 1 in the subsection “B. Block Matrices”]. Thus the product $(\bar{\mathbf{v}}_m^H \mathbf{L}_m^{-1}) (\bar{\mathbf{v}}_m^H \mathbf{L}_m^{-1})^H$ which is a 2×2 matrix must be non-negative scaled multiples of the identity matrix [13, Property 3 in the subsection “B. Block Matrices”]. Moreover, it is easy to deduce $(\bar{\mathbf{v}}_m^H \mathbf{L}_m^{-1}) (\bar{\mathbf{v}}_m^H \mathbf{L}_m^{-1})^H = \bar{\mathbf{v}}_m^H \bar{\mathbf{Q}}_m \bar{\mathbf{v}}_m$ by (37) and verify that the first diagonal element in $\bar{\mathbf{v}}_m^H \bar{\mathbf{Q}}_m \bar{\mathbf{v}}_m$ must be $\bar{\mathbf{v}}_{2m}^H \bar{\mathbf{Q}}_m \bar{\mathbf{v}}_{2m}$. So (31) has been proved.

REFERENCES

- [1] P. W. Wolniansky, G. J. Foschini, G. D. Golden and R. A. Valenzuela, “V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel”, *Proc. ISSE*, 1998, pp. 295-300.
- [2] B. Hassibi, “An efficient square-root algorithm for BLAST”, *Proc. ICASSP*, June 2000, pp. 737-740.
- [3] J. Benesty, Y. Huang and J. Chen, “A fast recursive algorithm for optimum sequential signal detection in a BLAST system”, *IEEE Trans. on Signal Processing*, pp. 1722-1730, July 2003.
- [4] H. Zhu, Z. Lei, F.P.S. Chin, “An improved recursive algorithm for BLAST”, *Signal Process.*, vol. 87, no. 6, pp. 1408-1411, Jun. 2007.
- [5] L. Szczecinski and D. Massicotte, “Low complexity adaptation of MIMO MMSE receivers, Implementation aspects”, *Proc. IEEE Globecom*, Nov. 28 - Dec. 2, 2005.
- [6] Y. Shang and X. G. Xia, “An Improved Fast Recursive Algorithm for V-BLAST With Optimal Ordered Detections”, *IEEE International Conference on Communications 2008 (ICC 2008)*, Session SP11.
- [7] S.M. Alamouti, “A simple transmit diversity technique for wireless communications”, *IEEE J. Select. Areas in Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.
- [8] V. Tarokh, A. Naguib, N. Seshadri, and A.R. Calderbank, “Combined array processing and space-time coding”, *IEEE Trans. Inf. Theory*, vol. 45, No. 4, pp. 1121-1128, May 1999.
- [9] A.F. Naguib, N. Seshadri and A.R. Calderbank, “Applications of space-time block codes and interference suppression for high capacity and high data rate wireless systems”, *Signals, Systems & Computers, 1998, Conference Record of the Thirty-Second Asilomar Conference*, Nov. 1998.
- [10] X. Fan, H. Zhang, H. Luo and J. Huang, “Optimal MMSE successive interference cancellation in group-wise STBC MIMO systems”, *Journal of Systems Engineering and Electronics*, vol. 17, No. 1, 2006, pp. 85-90.
- [11] M. Gomaa and A. Ghayeb, “A Low Complexity MMSE Detector for Multiuser Layered Space-Time Coded MIMO Systems”, *CCECE* 2007.
- [12] R. Buhnke, D. Wubben, V. Kuhn and K.D. Kameyer, “MMSE extension of V-BLAST based on sorted QR decomposition”, *Proc. IEEE VTC*, Oct. 2003.
- [13] A.H. Sayed, W.M., Younis and A. Tarighat, “An invariant matrix structure in multiantenna communications”, *Signal Processing Letters, IEEE*, vol. 12, Nov. 2005, pp. 749-752.
- [14] H. Lutkepohl, *Handbook of Matrices*, New York: John Wiley & Sons, 1996.