

Efficient Square-root Algorithms for the Extended V-BLAST with Selective Per-Antenna Rate Control

Hufei Zhu
 Dept. of Corporate Research,
 Huawei Technologies Co., Ltd.

Wen Chen
 Dept. of Electronic Engineering
 Shanghai Jiao Tong Univ.

Bin Li
 Dept. of Corporate Research
 Huawei Technologies

Abstract—We propose efficient algorithms for the extended Vertical Bell Labs Layered Space-Time architecture (V-BLAST) with selective per-antenna rate control (S-PARC). Instead of computing SNIRs from nulling vectors, we compute SNIRs from diagonal entries in the estimation error covariance matrix P , which are obtained from entries in F , i.e. the triangular square-root of P . Then the square-root V-BLAST algorithm is applied, to compute F for an antenna subset from F for another subset. Moreover, when computing F , we can reuse intermediate results to further reduce the computational complexity dramatically. Assume M transmit/receive antennas. The proposed algorithm for the S-PARC scheme minimizing total transmit power has the speedup of $0.14M + 1.82$, with respect to the corresponding S-PARC algorithm computing SNIRs from nulling vectors, while the proposed algorithm for the S-PARC scheme maximizing total information rate has the speedup of $(M + 58)/24$.

I. INTRODUCTION

V-BLAST (Vertical Bell Labs Layered Space-Time architecture) [1] is a multiple-input-multiple-output (MIMO) wireless communication system that achieves very high spectral efficiency in rich multi-path environments. The extended V-BLAST with per-antenna rate control (PARC) can achieve the open-loop capacity of the flat fading MIMO channel [2], [3], when a minimum mean square error (MMSE) receiver with successive decoding and cancellation (SDC) [3] is employed. Compared to conventional (non-selective) PARC, selective-PARC (S-PARC) [4], [5] achieves significant system-level performance gains [6], by adaptively selecting both the number and the best subset of transmit antennas. S-PARC is a promising technique that has been adopted in 3GPP [6].

Compared to the conventional V-BLAST, S-PARC requires much higher computational complexity [4], though usually it also achieves higher spectral efficiency [6]. It considers several possible sets of active transmit antennas [4] to select the best one. For each considered antenna set, S-PARC computes the corresponding set of nulling vectors to estimate the post-detection Signal-to-Noise-and-Interference Ratios (SNIRs) [4], [5], which requires a complexity comparable to that of a conventional V-BLAST detector. When S-PARC considers k antenna sets, its complexity is nearly k times that of a conventional V-BLAST detector. Thus low-complexity S-PARC schemes are proposed in [4], [5]. However, the schemes in [4], [5] only reduce the number of considered antenna sets, to save the computational load with some performance loss.

In this paper, we propose efficient algorithms to implement existing low-complexity S-PARC schemes. Firstly, we develop

efficient algorithms for SNIR calculation, which can avoid computing nulling vectors. Then we apply the square-root V-BLAST algorithm proposed by us in [7] to implement S-PARC efficiently, where we reuse intermediate results to further reduce the computational complexity dramatically.

This paper is organized as follows. System models and the existing efficient algorithms for V-BLAST and S-PARC are presented in Sections II and III, respectively. In Section IV, we propose efficient implementations of S-PARC. The complexities of the presented S-PARC algorithms are evaluated in Section V. Finally we conclude this paper in Section VI.

In the following sections, $(\bullet)^T$, $(\bullet)^*$ and $(\bullet)^H$ denote matrix transposition, matrix conjugate, and matrix conjugate transposition, respectively. $\mathbf{0}_M$ is the $M \times 1$ zero column vector, while \mathbf{I}_M is the identity matrix of size M .

II. SYSTEM MODEL AND THE EFFICIENT SQUARE-ROOT ALGORITHM FOR V-BLAST

The considered V-BLAST system consists of M transmit antennas and $N(\geq M)$ receive antennas in a rich-scattering and flat-fading wireless channel. At the transmitter, the data stream is de-multiplexed into M sub-streams. Then each sub-stream is encoded and fed to its respective transmit antenna. Each receive antenna receives the signals from all M transmit antennas. Let $\mathbf{a} = [a_1, a_2, \dots, a_M]^T$ denote the vector of transmit symbols from M antennas and assume $E(\mathbf{a}\mathbf{a}^H) = \sigma_a^2 \mathbf{I}_M$. Then the received signal vector is given by

$$\mathbf{r} = \mathbf{H} \cdot \mathbf{a} + \mathbf{w}, \quad (1)$$

where \mathbf{w} is the complex zero-mean Gaussian noise vector with covariance $\sigma_w^2 \mathbf{I}_N$, and $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_M]$ is the $N \times M$ complex channel matrix with statistically independent entries. The vector \mathbf{h}_m denotes the m^{th} column of \mathbf{H} .

The linear MMSE estimation of \mathbf{a} is

$$\hat{\mathbf{a}} = \mathbf{G} \cdot \mathbf{r} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1} \mathbf{H}^H \mathbf{r}, \quad (2)$$

where $\alpha = \sigma_w^2 / \sigma_a^2$. Correspondingly let

$$\left\{ \begin{array}{l} \mathbf{R} = \mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M, \\ \mathbf{P} = \mathbf{R}^{-1} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1}, \end{array} \right. \quad (3a)$$

$$\left\{ \begin{array}{l} \mathbf{R} = \mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M, \\ \mathbf{P} = \mathbf{R}^{-1} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1}, \end{array} \right. \quad (3b)$$

where \mathbf{P} is the estimation error covariance matrix [7]. Set $\alpha = 0$ in (2) and (3). Then we obtain the linear zero-forcing (ZF) estimation [1] and the corresponding \mathbf{R} and \mathbf{P} , while \mathbf{P} is also the estimation error covariance matrix.

The conventional V-BLAST scheme detects M entries of \mathbf{a} iteratively with ordered successive interference cancellation (SIC). Suppose that the M^{th} entry in \mathbf{a} , i.e. \hat{a}_M , is detected firstly. Then the interference of a_M is cancelled by

$$\mathbf{r}^{(M-1)} = \mathbf{r} - \mathbf{h}_M \hat{a}_M = \mathbf{r} - \mathbf{h}_M a_M + \mathbf{h}_M (a_M - \hat{a}_M), \quad (4)$$

where $a_M - \hat{a}_M$ is the remaining interference. Assume $a_M - \hat{a}_M = 0$. Then we obtain the reduced order problem

$$\mathbf{r}^{(M-1)} = \mathbf{H}_{M-1} \mathbf{a}_{M-1} + \mathbf{w}, \quad (5)$$

where

$$\mathbf{H}_{M-1} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_{M-1}] \quad (6)$$

is the deflated channel matrix. Define \mathbf{H}_m by (6) which includes the first m columns of \mathbf{H} . Define \mathbf{P}_m and \mathbf{R}_m from \mathbf{H}_m by (3). Then we have [7]

$$\begin{bmatrix} \mathbf{R}_{m-1} & \mathbf{v}_{m-1} \\ \mathbf{v}_{m-1}^H & r_m \end{bmatrix} = \mathbf{R}_m, \quad (7)$$

where \mathbf{v}_{m-1} is a column vector of length $m-1$, and r_m is the m^{th} diagonal entry.

The square-root V-BLAST algorithm in [7] uses the matrix \mathbf{F} instead of \mathbf{P} to compute the linear MMSE estimation, where \mathbf{F} is the upper triangular square-root of \mathbf{P} that satisfies

$$\mathbf{F} \cdot \mathbf{F}^H = \mathbf{P}. \quad (8)$$

In the *initialization* phase, \mathbf{F}_m , i.e. the square-root of \mathbf{P}_m , is computed from \mathbf{F}_{m-1} iteratively by [7]

$$\mathbf{F}_m = \begin{bmatrix} \mathbf{F}_{m-1} & \mathbf{u}_{m-1} \\ \mathbf{0}_{m-1}^T & \lambda_m \end{bmatrix}. \quad (9)$$

In (9), λ_m and \mathbf{u}_{m-1} are computed by

$$\left\{ \begin{array}{l} \lambda_m = \sqrt{1/(r_m - \mathbf{y}_{m-1}^H \mathbf{y}_{m-1})}, \\ \mathbf{u}_{m-1} = -\lambda_m \mathbf{F}_{m-1} \mathbf{y}_{m-1}, \end{array} \right. \quad (10a)$$

$$(10b)$$

where

$$\mathbf{y}_{m-1} = \mathbf{F}_{m-1}^H \mathbf{v}_{m-1}. \quad (11)$$

The iterations start from the initial \mathbf{F}_1 computed by [7]

$$\mathbf{F}_1 = \sqrt{\mathbf{P}_1} = \sqrt{\mathbf{R}_1^{-1}}. \quad (12)$$

Then in the *iterative detection* phase, \mathbf{F}_m is upper-triangularized by a unitary transformation Σ to obtain the upper-triangular \mathbf{F}_{m-1} [7], i.e.,

$$\mathbf{F}_m \Sigma = \begin{bmatrix} \mathbf{F}_{m-1} & \mathbf{u}_{m-1} \\ \mathbf{0}_{m-1}^T & \lambda_m \end{bmatrix}. \quad (13)$$

III. SYSTEM MODEL AND EXISTING LOW-COMPLEXITY SCHEMES FOR S-PARC

In the extended V-BLAST with PARC, the source data is demultiplexed into several independent sub-streams. Each sub-stream is coded and modulated by an independent channel coder and modulator, and then it is fed to its respective transmit antenna. Usually the modulation and channel coding scheme (MCS) for each sub-stream is adjusted independently according to the feedback, which carries the instantaneous post-detection SNIR of that sub-stream. The SDC receiver for PARC is a special kind of SIC receiver, which uses re-encoded versions of reliable post-decoding symbols to achieve perfect interference cancellation [3]. Thus the remaining interference is always zero in (4).

S-PARC considers several possible sets of active transmit antennas to select the best one, which achieves the greatest total information rate [5] or the minimal total transmit power [4]. For each considered antenna set, usually the corresponding set of nulling vectors are computed to estimate the post-detection SNIRs [4]. The SNIR of the i^{th} entry \hat{a}_i in $\hat{\mathbf{a}}$ can be computed from the MMSE nulling vector by [8]

$$SNIR_i = \frac{|\mathbf{g}_i^H \mathbf{h}_i|^2 \sigma_a^2}{\sum_{j \neq i, j=1}^m \left(|\mathbf{g}_i^H \mathbf{h}_j|^2 \sigma_a^2 \right) + |\mathbf{g}_i^H|^2 \sigma_w^2}, \quad (14)$$

or it can be computed from the ZF nulling vector by [4]

$$SNIR_i = \sigma_a^2 / (|\mathbf{g}_i^H|^2 \sigma_w^2). \quad (15)$$

In (14) and (15), \mathbf{g}_i^H denotes the MMSE or ZF nulling vector, which is the i^{th} row of $\mathbf{G} = \mathbf{P} \mathbf{H}^H$.

For SIC detection, the active antenna set is an ordered set (e.g., $\{1, 2\} \neq \{2, 1\}$), whose order corresponds to the detection order [4]. Then theoretically we should consider all $\sum_{m=1}^M P_m^M = \sum_{m=1}^M \frac{M!}{(M-m)!}$ ordered sets to find the best one. Fortunately, when ideal rate adaptation is adopted, the detection ordering brings rather small benefit [2], [4]. So at the cost of slight performance loss, S-PARC can employ any fixed detection order, and then only consider all $\sum_{m=1}^M C_m^M = 2^M - 1$ antenna combinations [4], [5]. Correspondingly the required complexity grows exponentially with M [4], which is too high for real-time implementations. We call this scheme as the *all-combination* S-PARC, e.g., the all-combination scheme in [4] and the so-called “PARC” in [5].

In [4], $2^M - 1$ is further reduced to a number not more than M with some performance loss. Use the term “mode” to refer to the number of selected(/active) transmit antennas [5]. The scheme in [4] computes the total transmit power of mode- M firstly. Then the transmit antenna with the lowest SNIR for the linear receiver is removed to obtain the mode-($M-1$) selection, whose total transmit power is then calculated. This recursion is repeated till the total transmit power of mode-($m-1$) is more than that of mode- m , or till mode-1 is reached.

On the other hand, the so-called “S-PARC” in [5] considers only those mode- m ($m = 1, 2, \dots, M$) selections that obey the “subset property”, to decide the best mode- m selection and the corresponding fixed detection order. The considered

mode- m selections must include all the selected antennas in the best mode- $(m-1)$ selection as a subset, which are detected lastly with the order identical to that fixed for mode- $(m-1)$. Thus “S-PARC” in [5] only considers $\sum_{m=1}^M (M-m+1) = \frac{M(M+1)}{2}$ antenna selections [5].

IV. EFFICIENT IMPLEMENTATIONS OF S-PARC

In this section we propose low-complexity algorithms for the S-PARC scheme in [4], “S-PARC” in [5], and the *all-combination* S-PARC, where SDC receivers [3] are assumed.

A. Improvement I: To Compute SNIRs Efficiently from Diagonal Entries in \mathbf{P} instead of Nulling Vectors

S-PARC schemes usually require much complexity to obtain or compare many SNIRs, which are usually computed from nulling vectors by (14) or (15) [4], [8]. We can propose simple algorithms to compute SNIRs from p_i , the i^{th} diagonal entry in \mathbf{P} . The SNIR can be computed by

$$SNIR_i = 1 / (\alpha p_i) - 1 \quad (16)$$

when MMSE filters are employed, or it can be computed by

$$SNIR_i = 1 / (\alpha p_i) \quad (17)$$

when ZF filters are employed. In appendix A we derive (16) and (17), which are much simpler than (14) and (15).

B. Improvement II: Efficient S-PARC Algorithms Based on \mathbf{F} , the Triangular Square-root of \mathbf{P}

As mentioned above, a mode- m selection can adopt any fixed detection order, which can be assumed to be t_m, t_{m-1}, \dots, t_1 (where t_m is detected firstly). Correspondingly this selection can be represented as an ordered active antenna set $\Pi_m = \{t_1, t_2, \dots, t_m\}$. Then represent the channel matrix for Π_m as

$$\mathbf{H}_m^{\{t_m\}} = [\mathbf{h}_{t_1} \quad \mathbf{h}_{t_2} \quad \dots \quad \mathbf{h}_{t_m}], \quad (18)$$

where the superscript $\{t_m\}$ denotes the antenna set $\{t_1, t_2, \dots, t_m\}$. From $\mathbf{H}_m^{\{t_m\}}$, we define the corresponding $\mathbf{R}_m^{\{t_m\}}$, $\mathbf{P}_m^{\{t_m\}}$ and $\mathbf{F}_m^{\{t_m\}}$ by (3a), (3b) and (8), respectively. $\mathbf{v}_{m-1,|t_m}^{\{t_{m-1}\}}$ and r_{t_m} are in $\mathbf{R}_m^{\{t_m\}}$, as shown in (7), while $\lambda_{m,|t_m}^{\{t_{m-1}\}}$ and $\mathbf{u}_{m-1,|t_m}^{\{t_{m-1}\}}$ are in $\mathbf{F}_m^{\{t_m\}}$, as shown in (9).

We can use (16) or (17) to compute the SNIR of antenna t_m from p_m in $\mathbf{P}_m^{\{t_m\}}$. Since the interference of antenna t_m is cancelled, the SNIR of antenna t_{m-1} is computed from p_{m-1} in $\mathbf{P}_{m-1}^{\{t_{m-1}\}}$. p_m can be computed by

$$p_m = |\lambda_m|^2 = 1 / (r_m - \mathbf{y}_{m-1}^H \mathbf{y}_{m-1}), \quad (19)$$

which is deduced from (8), (9) and (10a). Then p_i in $\mathbf{P}_i^{\{t_i\}}$ can be computed from λ_i in $\mathbf{F}_i^{\{t_i\}}$, by (19) for $i = m, m-1, \dots, 1$. Moreover, $\mathbf{F}_{m-1}^{\{t_{m-1}\}}$ is the submatrix in $\mathbf{F}_m^{\{t_m\}}$ [7], as shown in (9). Then λ_i in $\mathbf{F}_i^{\{t_i\}}$ is the i^{th} diagonal entry of $\mathbf{F}_m^{\{t_m\}}$.

In Table I we propose an efficient algorithm for the S-PARC scheme in [4]. As in [4], ZF filters are employed.

“S-PARC” in [5] obtains the SNIRs for Π_m from those for Π_{m-1} . It only computes the SNIR of antenna t_m , since after

TABLE I
THE PROPOSED ALGORITHM FOR THE S-PARC SCHEME IN [4]

A1	Compute (11), (10) and (9) iteratively to obtain the triangular \mathbf{F}_M for mode- M . Let $m = M$.
A2	Use all diagonal entries of the triangular \mathbf{F}_m to compute the SNIRs (for the SDC receiver) by (19) and (17), from which we get the total transmit power of mode- m [4].
A3	Find the maximal length row of \mathbf{F}_m , and permute it to be the last row. It corresponds to the transmit antenna with the lowest SNIR for the linear receiver [7], which is removed to obtain the mode- $(m-1)$ selection [4].
A4	Obtain the triangular \mathbf{F}_{m-1} from \mathbf{F}_m by (13), where Σ is the sequence of Givens rotations introduced in [7]. If the total transmit power of mode- m is less than that for mode- $(m+1)$, go back to step A2 with \mathbf{F}_{m-1} . Also reduce m by 1.

TABLE II
THE PROPOSED ALGORITHM FOR “S-PARC” [5]

S1	Compute \mathbf{R}_M , which includes all the required $\mathbf{R}_{m-1, s}, \mathbf{v}_{m-1, s}$ and r_{ms} ($m = 1, 2, \dots, M$) [7].
S2	Consider M mode-1 selections, i.e. $\{x_1\}$ where $x_1 = 1, 2, \dots, M$. Compute all M $\mathbf{P}_1^{\{x_1\}}$ s from $\mathbf{R}_1^{\{x_1\}}$ s by (3b). Find $t_1 = \arg \min_{x_1} \mathbf{P}_1^{\{x_1\}}$. Then antenna t_1 is the best selection for mode-1. From $\mathbf{P}_1^{\{t_1\}}$, compute $\mathbf{F}_1^{\{t_1\}}$ by (12). Let $m = 2$.
S3	Consider $M - m + 1$ mode- m selections, i.e. $\{t_1, \dots, t_{m-1}, x_m\}$ where $x_m = 1, 2, \dots, M$ and $x_m \neq t_1, t_2, \dots, t_{m-1}$. Compute $\mathbf{y}_{m-1, x_m}^{\{t_{m-1}\}}$ by (11) from $\mathbf{F}_{m-1}^{\{t_{m-1}\}}$ and $\mathbf{v}_{m-1, x_m}^{\{t_{m-1}\}}$. Then compute the $(M - m + 1) p_{m, x_m}^{\{t_{m-1}\}}$ s by (19) from $\mathbf{y}_{m-1, x_m}^{\{t_{m-1}\}}$ and r_{x_m} . Find $t_m = \arg \min_{x_m} p_{m, x_m}^{\{t_{m-1}\}}$ to obtain $\{t_1, \dots, t_{m-1}, t_m\}$, i.e. the best selection for mode- m . Compute $\mathbf{F}_m^{\{t_m\}}$ by (20), (10b) and (9). Notice that $\mathbf{y}_{m-1, t_m}^{\{t_{m-1}\}}$ in (10b) has been obtained when computing $p_{m, t_m}^{\{t_{m-1}\}}$.
S4	If $m < M$, go back to step S3 with $m = m + 1$.

antenna t_m is detected and its interference is cancelled, the undetected $m-1$ antennas in Π_m has the same SNIRs as those in the mode- $(m-1)$ selection $\Pi_{m-1} = \{t_1, t_2, \dots, t_{m-1}\}$. This method is utilized to propose efficient implementations for both “S-PARC” [5] and the *all-combination* S-PARC, where we compute \mathbf{F}_m for Π_m from \mathbf{F}_{m-1} for Π_{m-1} , to further reduce the computational complexity.

In Table II we propose the efficient algorithm for “S-PARC” [5], to decide the best antenna selection for each mode.

$$\lambda_m = \sqrt{p_m} \quad (20)$$

will be utilized, which is deduced from (19).

For simplicity, assume $M = 4$, i.e. the maximum antenna number in 3GPP UTRA [5]. Then in Table III we propose an efficient algorithm for the *all-combination* S-PARC. For each of the $2^4 - 1 = 15$ antenna combinations, we compute the $p_{i,|...}^{\{...\}}$ s, from which the SNIRs are computed by (16) or (17).

The algorithm in Table III obtains the SNIRs of each antenna combination. For example, the combination of antennas 1, 3 and 4 corresponds to the mode-3 selection $\{3, 4, 1\}$. The SNIRs of that selection, i.e. those of antennas 1, 4 and 3, can be computed from $p_{3,|1}^{\{3,4\}}$, $p_{2,|4}^{\{3\}}$ and $\mathbf{P}_1^{\{3\}}$, respectively, which have been obtained in steps P4, P3 and P2. From the SNIRs

TABLE III
THE PROPOSED ALGORITHM FOR THE *all-combination* S-PARC

- P1)** The same as the above-mentioned step S1.
- P2)** There are 4 mode-1 selections, i.e., $\{x_1\}$ where $x_1 = 1, 2, 3, 4$. Obtain all $M \mathbf{P}_1^{\{x_1\}}$ s (i.e. $p_1^{\{x_1\}}$ s) from $\mathbf{R}_1^{\{x_1\}}$ s by (3b). Use (12) to compute the \mathbf{F}_1 s that are necessary for the next steps, e.g., $\mathbf{F}_1^{\{1\}}$, $\mathbf{F}_1^{\{2\}}$ and $\mathbf{F}_1^{\{3\}}$.
- P3)** There are 6 mode-2 selections, which can be assumed to be $\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2, 3\}$, $\{2, 4\}$ and $\{3, 4\}$. Use (11) and (19) to compute $p_{2,|2}^{\{1\}}$, $p_{2,|3}^{\{1\}}$ and $p_{2,|4}^{\{1\}}$ from $\mathbf{F}_1^{\{1\}}$, compute $p_{2,|3}^{\{2\}}$ and $p_{2,|4}^{\{2\}}$ from $\mathbf{F}_1^{\{2\}}$, and compute $p_{2,|4}^{\{3\}}$ from $\mathbf{F}_1^{\{3\}}$. Compute the necessary \mathbf{F}_{2s} , e.g., $\mathbf{F}_2^{\{1,2\}}$ and $\mathbf{F}_2^{\{3,4\}}$, by (20), (10b) and (9). Notice that $\mathbf{y}_{1,|\dots}^{\{\dots\}}$ in (10b) has been obtained when computing the corresponding $p_{2,|\dots}^{\{\dots\}}$.
- P4)** There are 4 mode-3 selections, which can be assumed to be $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{3, 4, 1\}$ and $\{3, 4, 2\}$. Use (11) and (19) to compute $p_{3,|3}^{\{1,2\}}$ and $p_{3,|4}^{\{1,2\}}$ from $\mathbf{F}_2^{\{1,2\}}$, and compute $p_{3,|1}^{\{3,4\}}$ and $p_{3,|2}^{\{3,4\}}$ from $\mathbf{F}_2^{\{3,4\}}$. Compute the necessary \mathbf{F}_{3s} , e.g., $\mathbf{F}_3^{\{1,2,4\}}$, by (20), (10b) and (9).
- P5)** The mode-4 selection can be assumed to be $\{1, 2, 4, 3\}$. Use (11) and (19) to compute $p_{4,|3}^{\{1,2,4\}}$ from $\mathbf{F}_3^{\{1,2,4\}}$.

of each combination, we can find the combination with the minimal total transmit power to implement the all-combination scheme in [4], or find the combination with the greatest total information rate to implement “PARC” in [5]. Notice that we assume the constant transmit power from each active antenna in any selection. If this assumption is not satisfied, each SNIR should be multiplied by a proper scale factor [5].

C. Improvement III: Algorithms to compute $\mathbf{y} = \mathbf{F}^H \mathbf{v}$ recursively for “S-PARC” [5] and the all-combination S-PARC

In step S3, we compute $\mathbf{y}_{m-1,|x_m}^{\{t_{m-1}\}}$ from $\mathbf{F}_{m-1}^{\{t_{m-1}\}}$ and $\mathbf{v}_{m-1,|x_m}^{\{t_{m-1}\}}$ by (11), to obtain $p_{m,|x_m}^{\{t_{m-1}\}}$ by (19). Substitute (9) into (11) to obtain

$$\mathbf{y}_{m-1,|x_m}^{\{t_{m-1}\}} = \begin{bmatrix} \mathbf{F}_{m-2}^{\{t_{m-2}\}} & \mathbf{u}_{m-2,|t_{m-1}}^{\{t_{m-2}\}} \\ \mathbf{0}_{m-2}^T & \lambda_{m-1,|t_{m-1}}^{\{t_1, \dots, t_{m-2}\}} \end{bmatrix}^H \begin{bmatrix} \mathbf{v}_{m-2,|x_m}^{\{t_{m-2}\}} \\ r_{t_{m-1},x_m} \end{bmatrix} = \\ \begin{bmatrix} \mathbf{y}_{m-2,|x_m}^{\{t_{m-2}\}} \\ (\mathbf{u}_{m-2,|t_{m-1}}^{\{t_{m-2}\}})^H \mathbf{v}_{m-2,|x_m}^{\{t_{m-2}\}} + \lambda_{m-1,|t_{m-1}}^{\{t_{m-2}\}} r_{t_{m-1},x_m} \end{bmatrix}, \quad (21)$$

where r_{t_{m-1},x_m} is in the t_{m-1}^{th} row and x_m^{th} column of \mathbf{R}_M . Then we can use (21) instead of (11) to compute $\mathbf{y}_{m-1,|x_m}^{\{t_{m-1}\}}$ from $\mathbf{y}_{m-2,|x_m}^{\{t_{m-2}\}}$, for $x_m = 1, \dots, M$ and $x_m \neq t_1, \dots, t_{m-1}$. The initial $\mathbf{y}_{1,|x_2}^{\{t_1\}}$ is still computed by (11) for $x_2 = 1, \dots, M$ and $x_2 \neq t_1$, in the first recursion of step S3.

In the proposed algorithm for the *all-combination* S-PARC, we can also use (21) instead of (11) to compute $\mathbf{y}_{m-1,|\dots}^{\{\dots\}}$ from $\mathbf{y}_{m-2,|\dots}^{\{\dots\}}$ recursively, for $m = 3, 4, \dots, M$.

V. COMPLEXITY EVALUATION

Let (j, k) denote the computational complexity of j complex multiplications and k complex additions, which take $6j$ and

TABLE IV
TOTAL COMPLEXITIES OF THE EQUATIONS IN STEP S3

(19)	$\left(\sum_{m=1}^M ((M-m+1)m) \right) \approx \left(\frac{M^3}{6} \right)$
(10b)	$\left(\sum_{m=1}^M \left(\frac{m^2}{2} \right) \right) \approx \left(\frac{M^3}{6} \right)$
(11)	$\left(\sum_{m=1}^M \left((M-m+1) \frac{(m-1)m}{2} \right) \right) \approx \left(\frac{M^4}{24} + \frac{M^3}{12} \right)$
(21)	$\left(\sum_{m=1}^M ((M-m+1)(m-1)) \right) \approx \left(\frac{M^3}{6} \right)$

$2k$ floating-point operations (flops), respectively. If $j = k$, simplify (j, k) to (j) .

The complexity of step S1 is $(\frac{1}{2}M^2N)$ [7]. The dominant computations of step S3 come from (19), (10b) and (11), or (11) is replaced by (21) when improvement III is adopted. The complexities of these equations are listed in Table IV. Then we can obtain the complexity of the proposed algorithm for “S-PARC” [5]. Moreover, it can be seen that the maximum average complexity of the proposed algorithm for the S-PARC scheme in [4] is identical to that of the V-BLAST algorithm in [7]. In Table V, we list the above-mentioned complexities.

To compute SNIRs from nulling vectors, we apply the proposed algorithms with improvements I and II to compute \mathbf{F}_{ms} . From \mathbf{F}_{ms} , nulling vectors are computed by [7]

$$\mathbf{g}_m^H = \lambda_m \left[\begin{array}{cc} \mathbf{u}_{m-1}^H & \lambda_m^* \end{array} \right] \mathbf{H}_m^H, \quad (22)$$

where \mathbf{u}_{m-1} and λ_m are in \mathbf{F}_m , as shown in (13). To the best of our knowledge, this implementation is more efficient than any existing one, when nulling vectors are required to compute SNIRs for S-PARC. For each considered mode- k selection, the S-PARC scheme in [4] compute k ZF nulling vectors by (22) for $m = k, k-1, \dots, 1$, from which k SNIRs are computed by (15). “S-PARC” [5] at least needs to compute M MMSE nulling vectors by (22), from which M SNIRs are computed by (14). In Table V, we list total complexities for the above-described computations of nulling vectors and SNIRs, and those for the corresponding full S-PARC algorithms.

Assume $M = N$. We compute the speedups (in the number flops) of the proposed S-PARC algorithms, with respect to the corresponding S-PARC algorithms mentioned above that compute SNIRs from nulling vectors. The results are listed in Table V. On the other hand, we carry out numerical experiments to count the average flops of the presented S-PARC algorithms, for different number of transmit/receive antennas. The results are shown in Fig. 1. It can be seen that they are consistent with the theoretical flops calculation.

When $M = N = 4$, the proposed algorithm for the *all-combination* S-PARC only requires about 10% more computational complexity than the proposed algorithm for the S-PARC scheme in [4], while with respect to the latter, the former has a performance gain that can be up to 0.4 dB [4].

VI. CONCLUSION

In this paper, we develop efficient algorithms for S-PARC schemes, e.g., the scheme in [4], “S-PARC” in [5], and the *all-combination* S-PARC. We compute SNIRs from diagonal entries of the estimation error covariance matrix \mathbf{P} , to avoid

TABLE V
TOTAL COMPLEXITIES OF THE PRESENTED ALGORITHMS (ALG.)

	The Scheme in [4]	“S-PARC” [5]
Proposed S-PARC Alg. with Improvements I and II	$(\frac{7}{9}M^3 + \frac{1}{2}M^2N, \frac{5}{9}M^3 + \frac{1}{2}M^2N)$	$(\frac{1}{24}M^4 + \frac{5}{12}M^3 + \frac{1}{2}M^2N)$
Proposed S-PARC Alg. with Improvements I, II and III	Null	$(\frac{1}{2}M^3 + \frac{1}{2}M^2N)$
Computations of Nulling Vectors	$(\sum_{k=1}^M (\sum_{m=1}^k (mN)) \approx (\frac{M^3N}{6} + \frac{M^2N}{2})$	$(\sum_{m=1}^M (mN)) \approx (\frac{M^2N}{2})$
Computations of SNIRs by Nulling Vectors	$(\sum_{k=1}^M kN) \approx (\frac{M^2N}{2})$	$(\sum_{m=1}^M (m(2N))) \approx (M^2N)$
S-PARC Alg. Computing SNIRs from Nulling Vectors	$(\frac{3N+14}{18}M^3 + \frac{3}{2}M^2N, \frac{3N+10}{18}M^3 + \frac{3}{2}M^2N)$	$(\frac{1}{24}M^4 + \frac{5}{12}M^3 + 2M^2N)$
Speedup in Flops of Proposed S-PARC Alg. over S-PARC Alg. Based on Nulling Vectors (for $M = N$)	$0.14M + 1.82$	$\frac{M+58}{24}$ for All 3 Improvements, $\frac{M+58}{M+22}$ for Improvements I and II

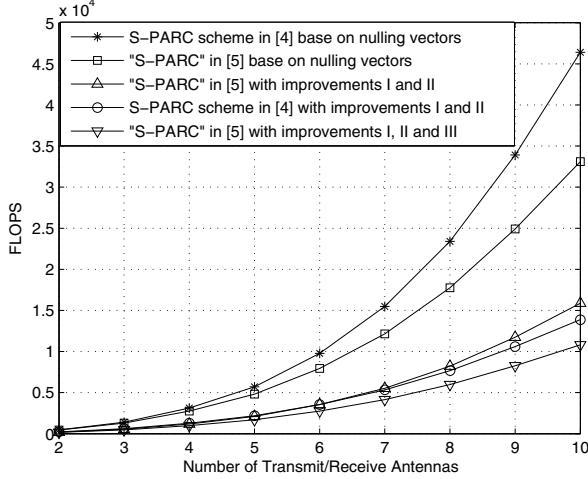


Fig. 1. Complexity comparison among the presented S-PARC algorithms.

computing nulling vectors. The diagonal entry of \mathbf{P} is computed from the diagonal entry of \mathbf{F} , i.e. the triangular square-root of \mathbf{P} . Then the square-root V-BLAST algorithm [7] is applied, to compute \mathbf{F} for an antenna set from \mathbf{F} for another set. When computing the required \mathbf{F} for “S-PARC” in [5] or the *all-combination* S-PARC, we can reuse the intermediate results to further reduce the computational complexity dramatically. If $M = N$, the proposed algorithm for the S-PARC scheme in [4] has the speedup of $0.14M + 1.82$, with respect to the corresponding S-PARC algorithm computing SNIRs from nulling vectors, while the proposed algorithm for “S-PARC” in [5] totally has the speedup of $\frac{M+58}{24}$. When $M = N = 4$, the proposed algorithm for the *all-combination* S-PARC only requires about 10% more complexity than the proposed algorithm for the S-PARC scheme in [4], while the former usually outperforms the latter [4].

APPENDIX A THE DERIVATION OF (16) AND (17)

To verify (16), we substitute (1) into (2) to get

$$\hat{\mathbf{a}} = \mathbf{PH}^H \mathbf{Ha} + \mathbf{PH}^H \mathbf{w} = \mathbf{a} - \alpha \mathbf{Pa} + \mathbf{PH}^H \mathbf{w}. \quad (23)$$

Let \mathbf{p}_i^H and \mathbf{p}_i denote the i^{th} row and column in the Hermitian matrix \mathbf{P} , respectively. Also let $p_{i,j}$ denote the entry in the i^{th}

row and j^{th} column of \mathbf{P} . Then the i^{th} entry in $\hat{\mathbf{a}}$ is

$$\hat{a}_i = a_i - \alpha \mathbf{p}_i^H \mathbf{a} + \mathbf{p}_i^H \mathbf{H}^H \mathbf{w} = (1 - \alpha p_{i,i}) a_i + \psi, \quad (24)$$

where ψ , the interference and noise, can be represented as

$$\psi = -\alpha [\begin{array}{cccccc} p_{i,1} & \cdots & p_{i,i-1} & p_{i,i+1} & \cdots & p_{i,M} \end{array}] \times [\begin{array}{cccccc} a_1 & \cdots & a_{i-1} & a_{i+1} & \cdots & a_M \end{array}]^T + \mathbf{p}_i^H \mathbf{H}^H \mathbf{w}. \quad (25)$$

From (25), we get

$$E\{\psi^* \psi\} = \alpha^2 \sigma_a^2 (\mathbf{p}_i^H \mathbf{p}_i - |p_{i,i}|^2) + \sigma_w^2 \mathbf{p}_i^H \mathbf{H}^H \mathbf{H} \mathbf{p}_i. \quad (26)$$

As $\mathbf{p}_i^H \mathbf{H}^H \mathbf{H} \mathbf{p}_i$ is in the i^{th} row and column of $\mathbf{P} \mathbf{H}^H \mathbf{H} \mathbf{P}^H = \mathbf{P}^H - \alpha \mathbf{P} \mathbf{P}^H$, we get

$$\mathbf{p}_i^H \mathbf{H}^H \mathbf{H} \mathbf{p}_i = p_{i,i} - \alpha \mathbf{p}_i^H \mathbf{p}_i. \quad (27)$$

Substitute (27) into (26) to yield

$$E\{\psi^* \psi\} = \sigma_w^2 p_{i,i} - (\sigma_w^4 / \sigma_a^2) |p_{i,i}|^2. \quad (28)$$

From (24) and (28), we can derive the SNIR of \hat{a}_i , i.e.,

$$\begin{aligned} E\{((1 - \alpha p_{i,i}) a_i)^* (1 - \alpha p_{i,i}) a_i\} / E\{\psi^* \psi\} \\ = |1 - \alpha p_{i,i}|^2 \sigma_a^2 / (\sigma_w^2 p_{i,i} - (\sigma_w^4 / \sigma_a^2) |p_{i,i}|^2). \end{aligned} \quad (29)$$

Then (29) can be simplified to (16).

When ZF filters are employed, $\alpha = 0$ in (26), (27), (23) and (25). Similarly we can also substitute (27) into (26), and then derive (17).

ACKNOWLEDGMENT

This work is supported by Major National S&T Program 2009ZX03003-002 of China.

REFERENCES

- [1] P.W. Wolniansky, G.J. Foschini, G.D. Golden, and RA. Valenzuela, “V-BLAST an architecture for realizing very high data rates over the rich-scattering wireless channel,” Proc. ISSSE’98, pp. 295 -300, 1998.
- [2] S. T. Chung, A. Lozano, and H. C. Huang, “Approaching eigenmode BLAST channel capacity using V-BLAST with rate and power feedback,” IEEE Veh. Tech. Conf. Fall, pp. 915-919, Sept. 2001.
- [3] RI-(01)0879, *Increasing MIMO throughput with per-antenna rate control*, Lucent Technologies, 3GPP TSG-RAN1 21, 28th August 2001.
- [4] H. Zhuang, L. Dai, S. Zhou, and Y. Yao, “Low Complexity Per-Antenna Rate and Power Control Approach for Closed-Loop V-BLAST”, IEEE Trans. on Communications, VOL. 51, NO. 11, Nov. 2003.
- [5] 3GPP TR 25.876 V7.0.0, *Multiple Input Multiple Output in UTRA*, 2007.
- [6] S.J. Grant, K.J. Molnar, and L. Krasny, “System-level performance gains of Selective per-antenna-rate-control (S-PARC),” IEEE VTC 2005-Spring.
- [7] H. Zhu, W. Chen, D. Chen, Y. Du, J. Lu, “Reducing the computational complexity for BLAST by using a novel fast algorithm to compute an initial square-Root matrix”, IEEE VTC 2008-Fall, Sept. 2008.
- [8] J. Chen, S. Jin, and Y. Wang, “Reduced Complexity MMSE-SIC Detector in V-BLAST Systems”, IEEE PIMRC 2007, 3-7 Sept. 2007.