Diversity-Multiplexing Tradeoff Analysis for Wireless Multicast Systems with Amplify-and-Forward Scheme

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***Abstract*—In this paper, we consider a Amplify-and-Forward (AF) cooperative multicast system in a Rayleigh-fading environment, where two-sources communicate with two destinations assisted by N relays ($2-N-2$ system). We consider two cooperative schemes, i.e., the conventional Round-Robin (RR) Scheme and the proposed relay selection (RS) scheme in the multicast system. The diversity-multiplexing tradeoff (DMT) analysis is performed over the two cooperative schemes, by which we find both cooperative schemes can provide N order diversity. The simulation not only validates the theoretical prediction, but also shows that RR offers better performance than RS in the sense of system outage probability (SOP). But RS just employs one relay in one time frame. This offers a tradeoff between the outage probability and the number of relays in service during one time frame.**

***Index Terms*—Amplify-and-Forward(AF), Round-Robin, Relay-Selection, Diversity-Multiplexing Tradeoff (DMT).**

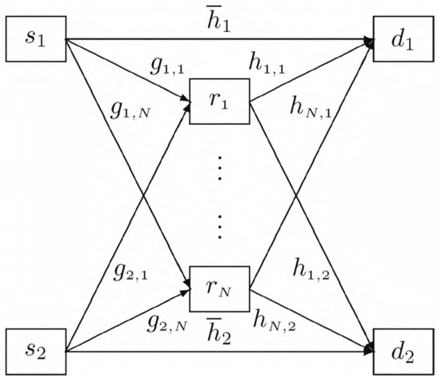

Fig. 1. Multicast cooperative channels with N relays.

I. Introduction

Recently, there has been a growing interest in wireless cooperative relaying schemes. There are several cooperation schemes being widely used, i.e., the Amplify-and-Forward (AF) scheme, the Decode-and-Forward (DF) scheme and the Compress-and-Forward (CF) scheme. For AF scheme, the relay retransmits a scaled version of its soft observation [2]. For DF scheme, however, the relay first attempts to decode the information stream and then re-encodes it before transmitting [1][5]. In a CF scheme, the relay transmits a quantized and compressed observation of its received signal to the destination, and the destination decodes the information by combining the observations from the source and the relay [6][8]. In [9], the authors also proposed the so-called NAF scheme, which is proved to be optimal in the sense of diversity-multiplexing tradeoff for the one source and one destination pair with N relays ($1-N-1$ system) cooperated in Round-Robin way [5].

Diversity-multiplexing tradeoff (DMT) is an efficient tool to measure the performance of MIMO system, which was introduced by Zheng [7]. Then Azarian [4] introduced this metric into cooperative system. Recently, Li prosed, a two source and two destination pair with one relay ($2-1-2$) system, and used DMT to analyze the system [3]. In this paper we consider the two source and two destination pair with N relays ($2-N-2$) system and give DMT analysis for the multicast system.

As N parallel relays are employed, several relay-cooperative schemes can be used in the system. In [5], Azarian used Round-Robin scheme, in which the relays take turns repeating the signals that they previously observed. In [10], the authors considered a relay-selection scheme based on whether the the fading coefficients between source to relay lie above a threshold. In [11], the authors decided to employ the relay based on a modified version of the harmonic mean function of its source-relay and relay-destination instantaneous channel gains among N relays. In this paper, we consider the conventional Round-Robin (RR) and the newly proposed Relay-Selection (RS) schemes. In our proposed RS, a scheduler (that exists physically or logically) decides which relay to employ based on max-min values of the source-destination and relay-destination channel gains. We find both schemes can provide N order diversity gain. The simulation not only validates the theoretical prediction, but also shows that RR offer better performance than RS in the sense of system outage probability (SOP).

The notations used in this paper are defined as follows. We use $[\cdot]^\dagger$ denote the matrix's conjugated transposition. $|\cdot|$ means

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$$
\begin{aligned}
|\,\mathbf{A}\,| &= d_N \left(\begin{array}{cc|c} \multicolumn{2}{c|}{\text{diag}(\mathbf{a})} & \begin{array}{c}\text{diag}(\mathbf{b}') \\ 0\end{array} \\ \hline \text{diag}(\mathbf{c}') & 0 & \text{diag}(\mathbf{d}') \end{array} \right) + c_N(-1)^N \left(\begin{array}{c|cc} \begin{array}{c}\text{diag}(\mathbf{a}') \\ 0\end{array} & \multicolumn{2}{c}{\text{diag}(\mathbf{b})} \\ \hline \text{diag}(\mathbf{c}') & \text{diag}(\mathbf{d}') & 0 \end{array} \right) \\
&= a_N d_N \begin{pmatrix} \text{diag}(\mathbf{a}') & \text{diag}(\mathbf{b}') \\ \text{diag}(\mathbf{c}') & \text{diag}(\mathbf{d}') \end{pmatrix} - c_N b_N \begin{pmatrix} \text{diag}(\mathbf{a}') & \text{diag}(\mathbf{b}') \\ \text{diag}(\mathbf{c}') & \text{diag}(\mathbf{d}') \end{pmatrix} \\
&= \prod_{i=1}^{N} (a_i d_i - b_i c_i).
\end{aligned} \tag{3}
$$

the determinant of a matrix. $(x)^+$ denotes $\max\{0, x\}$, and $\mathbb{R}^N$, $\mathbb{C}^N$ denote the set of real and complex N-tuples respectively. $\mathbb{R}^{N+}$ denotes the set of non-negative N-tuples. $\Lambda_{\mathbf{x}}$ denotes the auto-covariance matrix of vector $\mathbf{x}$.

II. Preliminaries and System Model

A. Preliminaries

Let R(SNR) denote the transmission rate of the system and P_e(SNR) denote the frame error probability (FEP) as the functions of Signal-to-Noise Ratio (SNR). Then the multiplexing gain r and the corresponding diversity $d(r)$ are defined as

$$
\lim_{SNR\to\infty} \frac{R(SNR)}{\log SNR} = r, \lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR} = -d(r). \tag{1}
$$

So a scheme's DMT means that at multiplexing gain r, the diversity gain that the scheme obtains should not exceed $d(r)$ as a whole system.

We denote that as $\rho \to \infty$, $f(\rho) \doteq \rho^a$, if function $f(\rho)$ is exponentially equal to ρ^a, that is

$$
\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log \rho} = a.
$$

The $\dot{\leq}$ and $\dot{\geq}$ are similarly defined. From the definitions, Eq. (1) can be written as

$$
R(SNR) \doteq r \log SNR, \quad P_e(SNR) \doteq SNR^{-d}.
$$

Let g denote a Gaussian random variable with zero mean and unit variance, and z denote the exponential order of $1/|g|^2$. Then

$$
z = -\lim_{\rho\to\infty} \frac{\log |g|^2}{\log \rho}.
$$

We now present two lemmas which will be used in the following sections.

Lemma 1: (See [5]) For independent random variables $\{v_j\}_{j=1}^N$ distributed identically, the probability P_O that $(v_1, \cdots, v_N)$ belongs to set $\mathcal{O}^+$ can be characterized by

$$
P_O \doteq \rho^{-d_o}, \quad \text{for} \quad d_o = \inf_{(v_1,\cdots,v_N)\in\mathcal{O}^+} \sum_{j=1}^{N} v_j,
$$

that is, the exponential order of P_O only depends on $\mathcal{O}^+$.

Lemma 2: Let $\mathbf{A}$ be a $2N \times 2N$ matrix, i.e.,

$$
\mathbf{A} = \begin{pmatrix} \text{diag}(\mathbf{a}) & \text{diag}(\mathbf{b}) \\ \text{diag}(\mathbf{c}) & \text{diag}(\mathbf{d}) \end{pmatrix},
$$

where $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathbb{C}^N$, whose entries are a_i, b_i, c_i, d_i, for $i = 1, \cdots, N$, respectively. Then

$$
|A| = \prod_{i=1}^{N} (a_i d_i - b_i c_i). \tag{2}
$$

Proof: We prove it by induction. Obviously, (2) holds for $N = 1$. Assume that it holds for a $2(N-1) \times 2(N-1)$ matrix $\tilde{\mathbf{A}}$, where

$$
\tilde{\mathbf{A}} = \begin{pmatrix} \text{diag}(\mathbf{a}') & \text{diag}(\mathbf{b}') \\ \text{diag}(\mathbf{c}') & \text{diag}(\mathbf{d}') \end{pmatrix},
$$

and $\mathbf{a}', \mathbf{b}', \mathbf{c}', \mathbf{d}' \in \mathbb{C}^{N-1}$, whose entries are a_i, b_i, c_i, d_i, for $i = 1, \cdots, N-1$, respectively. Then $|\tilde{\mathbf{A}}| = \prod_{i=1}^{N-1} (a_i d_i - b_i c_i)$. As shown in Eq. (3), the lemma is proved. ■

B. System model

Fig. 1 depicts a multicast model with two sources, two destinations and N half-duplex relays ($2 - N - 2$ system). We assume that d_1 (or d_2) is out of the transmission range of s_2 (or s_1, respectively). Thus the shared relays must help s_1 (or s_2) to reach its destination. The transmission strategy of the system is as follows:

1) $s_1 \to \{\mathcal{T},\ d_1\}$ with signal X_{S1} , $s_2 \to \{\mathcal{T},\ d_2\}$ with signal X_{S2};
2) $r \to \{d_1,\ d_2\}$ with signal X_R, where $\mathcal{T} \subseteq \{r_1, \cdots, r_N\}$.

The channel gains are indicated in Fig. 1, where the $\overline{h}_k, g_{k,i}, h_{i,k}$ denote the channel's coefficients between the k-th source and the k-th destination, the k-th source and the i-th relay, the i-th relay and the k-th destination respectively. All channels are assumed to be non-frequency selective, and quasi-static during at least one system frame period with independently identically distributed Rayleigh distribution of unit variance. We assume that all the nodes are equipped with single antenna, and each relay is isolated from the others. The noise observed by all the receivers and the relays are assumed to have a Gaussian distribution with zero mean and unit variance. Finally note that joint maximum likelihood (ML) decoding is performed at all the receivers.

III. Diversity-Multiplexing Tradeoff Analysis

In this section, we give the diversity-multiplexing tradeoff (DMT) analysis of the multicast system with AF cooperative scheme at the relays. First we will consider the Round-Robin scheme. Then we will consider the multi-node Relay-Selection scheme.

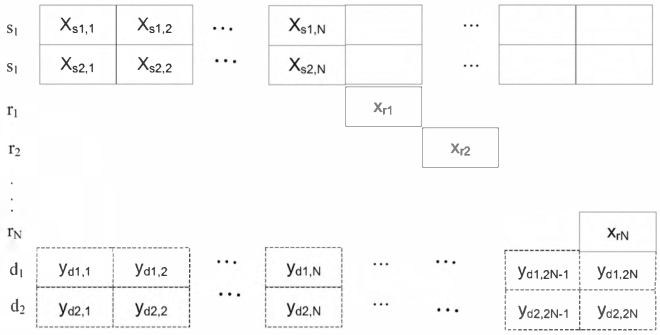

Fig. 2. Round-Robin scheme in multicast system

A. Round-Robin cooperative scheme

In Round-Robin cooperative scheme, at any time slot, only one relay is active. Assume that the frame length is $2N$. Fig. 2 depicts the transmission scheme in $2-N-2$ system. Note that the dashed boxes denote the signals' reception processes and the solid ones denote the signals' transmission processes.

We assume that the total power consumption during a frame period is NP. We denote $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{t}$ as power allocation factors for S_1, S_2, R respectively in one frame, where $\mathbf{k}_k = [\kappa_{k,1}, \cdots, \kappa_{k,N}]^T$, for $k=1,2$ is the power allocation vector of S_k, in which, the i-th entry is the power allocation factor for the i-th symbol of $\mathbf{X}_{S_k}$, and $\mathbf{t} = [\tau_1, \cdots, \tau_N]^T$ is the power allocation vector of relays, in which, the i-th entry is the power allocation factor for the i-th symbol of $\mathbf{X}_R$. We define $\mathbf{X}_{S_1}$, $\mathbf{X}_{S_2}$, $\mathbf{X}_R$ as the transmission signals at node S_1, S_2, R, respectively. $\mathbf{X}_{s_k}$, $\mathbf{X}_R$ are composed of N symbols, i.e., $\mathbf{X}_{s_k} = [x_{s_k,1}, x_{s_k,2}, \cdots, x_{s_k,N}]$, for $k=1,2$, $\mathbf{X}_R = [x_{r,1}, x_{r,2}, \cdots, x_{r,N}]$. We define $\mathbf{Y}_R$, $\mathbf{Y}_{d1}$, $\mathbf{Y}_{d2}$ as the received signals at node R, D_1, D_2 respectively. Similarly, we have $\mathbf{Y}_{D_k} = [y_{d_k,1}, y_{d_k,2}, \cdots, y_{d_k,2N}]$, for $k=1,2$, and $\mathbf{Y}_R = [y_{r,1}, y_{r,2}, \cdots, y_{r,N}]$. Then the power constraint satisfies

$$\sum_{i=1}^{N} (\kappa_{2,i} + \kappa_{2,i} + \tau_i) = N.$$

We measure the performance by system outage probability (SOP). If any one of the destinations is in outage, then the system is in outage. The outage is defined as the event that the mutual information of the channel does not support the target data R, i.e., in a channel defined as $\mathbf{Y} = H\mathbf{X} + Z$, the outage event can be written as

$$\mathcal{O} \triangleq \{H : I(\mathbf{X}; \mathbf{Y} | \mathbf{H} = H) \leq R\}.$$

Let Λ_x and Λ_z denote the covariance matrix of the input and noise, respectively. The mutual information between $\mathbf{X}$ and $\mathbf{Y}$ on the condition $\mathbf{H} = H$ is

$$I(\mathbf{X}; \mathbf{Y} | \mathbf{H} = H) = \log\det\left(I + H\Lambda_x H^\dagger SNR \Lambda_z^{-1}\right). \quad (4)$$

In DMT analysis, without loss of generality [7], we can always assume the input distribution to have covariance matrix $\Lambda_x = SNR I_N$. Then

$$I(\mathbf{X}; \mathbf{Y} | \mathbf{H} = H) = \log\det\left(I + HH^\dagger SNR \Lambda_z^{-1}\right).$$

In [7], it is proved that outage probability provides a lower bound of the optimal error probability, up to the SNR exponent, no matter what coding and decoding techniques are used. However, as long as the block length is long enough, this lower bound is tight enough, up to the scale of the SNR exponent. Hence, in the following we will only consider outage probability. According to the definition of SOP,

$$P_{SOP} = P_{o,d_1} + P_{o,d_2} - P_{o,d_1 d_2} \doteq P_{o,d_1} + P_{o,d_2},$$

where $P_{o,d_k}, k=1,2$ denote the outage probability in one of the two destinations, and $P_{o,d_1 d_2}$ in both destinations. By the symmetry of the system, let $R^{(S_1)} = R^{(S_2)} = R/2$. The $s_1, s_2 \to d_k$ link can be seen as a multi-access channel. Focus on D_1, $\mathcal{O}_{d_1}$ is formalized as

$$\mathcal{O}_{d_1} = \mathcal{O}_{d_1}^{\mathbf{x}_{s1}} \cup \mathcal{O}_{d_1}^{\mathbf{x}_{s2}} \cup \mathcal{O}_{d_1}^{\mathbf{x}_{s_1 s_2}}, \quad (5)$$

where

$$\mathcal{O}_{d_1}^{\mathbf{x}_{s_1}} = \left\{\frac{1}{2} I(\mathbf{X}_{S_1}; \mathbf{Y}_{D_1} \mid \mathbf{X}_{S_2}) < NR/2\right\}, \quad (6)$$

$$\mathcal{O}_{d_1}^{\mathbf{x}_{s_2}} = \left\{\frac{1}{2} I(\mathbf{X}_{S_2}; \mathbf{Y}_{D_1} \mid \mathbf{X}_{S_1}) < NR/2\right\}, \quad (7)$$

$$\mathcal{O}_{d_1}^{\mathbf{x}_{s_1 s_2}} = \left\{\frac{1}{2} I(\mathbf{X}_{S_1}\mathbf{X}_{S_2}; \mathbf{Y}_{D_1}) < NR\right\}. \quad (8)$$

Then we have the following theorem, which shows the RR can provide N order diversity gain.

Theorem 1: The diversity-multiplexing tradeoff achieved by the AF scheme in $2-N-2$ system, utilizing N relays in Round-Robin way, is characterized by

$$d^{RR}(\overline{r}) = N(1-r)^+, \quad (9)$$

where, $\overline{r} = (r/2, r/2)$.

Proof: The signals received at the relay node is

$$y_{r_i} = g_{1,i}\sqrt{\kappa_{1,i}} x_{s_1,i} + g_{2,i}\sqrt{\kappa_{2,i}} x_{s_2,i} + v_{r,i}, \quad (10)$$

for $i = 1, \cdots, N$, $k = 1, 2$. The signals received at the destination can be expressed as:

$$\begin{cases} y_{d_k,i} = \overline{h}_k \sqrt{\kappa_{k,i}} x_{s_k,i} + v_{d_k,i}, \\ y_{d_k,N+i} = h_{i,k}\sqrt{\tau_i} x_{r_i} + v_{d_k,N+i}. \end{cases} \quad (11)$$

Let $x_{r_i} = y_{r_i}$, $\mathbf{K}_k = diag(\mathbf{k}_k)$, for $k=1,2$. Focusing on destination 1, we rewrite (10)-(11) in matrix form as

$$Y_{d_1} = H_1 \mathbf{X}_{S_1} + H_2 \mathbf{X}_{S_2} + \mathbf{z}_{d_1}, \quad (12)$$

where

$$H_1 = \begin{bmatrix} \overline{h}_1 I_N \\ diag(\mathbf{c}) diag(\mathbf{b}_1) \end{bmatrix} \mathbf{K}_1,$$

$$H_2 = \begin{bmatrix} \boldsymbol{O}_N \\ diag(\mathbf{c}) diag(\mathbf{b}_2) \end{bmatrix} \mathbf{K}_2,$$

$$\mathbf{z}_{d_1} = \begin{bmatrix} \boldsymbol{O}_N & \\ & diag(\mathbf{c}) \end{bmatrix} \mathbf{v}_r + \mathbf{v}_{d_1},$$

where $\boldsymbol{O}_N$ is the $N \times N$ matrix with all 0 entries, $\mathbf{v}_r = [0, \cdots, 0, v_{r_1}, \cdots, v_{r_N}]^T$ and $v_{d_1} = [v_{d_1,1}, \cdots, v_{d_1,2N}]^T$ are the noise vector observed by the relay and D_1, respectively, and $\mathbf{c}$, $\mathbf{b}_1$, $\mathbf{b}_2 \in \mathbb{C}^N$, whose entries are defined by

$$c_i \triangleq h_{i,1}\sqrt{\tau_i}, \quad b_{1,i} \triangleq g_{1,i}, \quad b_{2,i} \triangleq g_{2,i}.$$

For simplicity, we ignore the terms $\mathbf{K}_1, \mathbf{K}_2$ in DMT analysis since it does not influence the final results [2]. Then refer to (2) and (4), we have

$$\begin{aligned} I(\mathbf{X}_{S_1};\mathbf{Y}_{D_1}|\mathbf{X}_{S_2}) &= \log\left(I_{2N} + H_1 H_1^{\dagger} SNR \Lambda_{z_{d_1}}^{-1}\right) \\ &= \sum_{i=1}^{N} \log\left(1 + SNR|\overline{h}_1|^2 + \frac{SNR|h_{i,1}|^2\tau_i|g_{1,i}|^2}{1+|h_{i,1}|^2\tau_i}\right). \end{aligned} \tag{13}$$

Similarly, we have

$$\begin{aligned} I(\mathbf{X}_{S_2};\mathbf{Y}_{D_1}|\mathbf{X}_{S_1}) &= \log\left(I_{2N} + H_2 H_2^{\dagger} SNR \Lambda_{z_{d_1}}^{-1}\right) \\ &= \sum_{i=1}^{N} \log\left(1 + \frac{SNR|h_{i,1}|^2\tau_i|g_{2,i}|^2}{1+|h_{i,1}|^2\tau_i}\right), \end{aligned} \tag{14}$$

and

$$\begin{aligned} I(\mathbf{X}_{S_1}\mathbf{X}_{S_2};\mathbf{Y}_{D_1}) = \sum_{i=1}^{N} \log\Bigg(&1 + SNR|\overline{h}_1|^2 \\ &+SNR^2\frac{|\overline{h}_1|^2|h_{i,1}|^2|g_{2,i}|^2}{1+|h_{i,1}|^2\tau_i} + SNR\frac{|h_{i,1}|^2\tau_i(|g_{1,i}|^2+|g_{2,i}|^2)}{1+|h_{i,1}|^2\tau_i}\Bigg). \end{aligned} \tag{15}$$

By substituting (13)-(15) into (6)-(8), we can easily get

$$\mathcal{O}_{d_1}^{\mathbf{X}_{S_1}} = \left\{(z_{g_{1,i}}, z_{h_{i,1}}, z_{\overline{h}_1}) \in \mathbb{R}^{3+} \mid \sum_{i=1}^{N} \max(1 - z_{g_{1,i}} - z_{h_{i,1}}, 1 - z_{\overline{h}_1})^+ < Nr\right\},$$

$$\mathcal{O}_{d_1}^{\mathbf{X}_{S_2}} = \left\{(z_{g_2,i}, z_{h_{i,1}}) \in \mathbb{R}^{2+} \mid \sum_{i=1}^{N} (1 - z_{g_{2,i}} - z_{h_{i},1})^+ < Nr\right\},$$

$$\begin{aligned} \mathcal{O}_{d_1}^{\mathbf{x}_{s1x2}} = \Bigg\{&(z_{g1,i}, z_{g2,i}, z_{hi,1}, z_{\overline{h}_1}) \in \mathbb{R}^{4+} \mid \\ &\sum_{i=1}^{N} \max(1 - z_{\overline{h}_1}, 1 - z_{h_{i,1}} - z_{g_{1,i}}, \\ &1 - z_{h_{i,1}} - z_{g_{2,i}}, 2 - z_{\overline{h}_1} - z_{h_{i,1}} - z_{g_{2,i}})^+ < 2Nr\Bigg\}, \end{aligned}$$

where z with different subscripts denote the negative order of the corresponding channels, i.e, $z_{h1} = -\lim_{\rho\to\infty} \frac{\log|h_1|^2}{\log \rho}$. By applying Lemma 1, we conclude that

$$\begin{aligned} d_{O^{x_{s_1}}} &= \inf_{\mathcal{O}_{d_1}^{x_{s_1}}} z_{\overline{h}_1} + \sum_{i=1}^{N} (z_{g_{1,i}} + z_{h_{i,1}}) \\ &= \inf_{\mathcal{O}_{d_1}^{x_{s_1}}} z_{\overline{h}_1} + \sum_{i=1}^{N} z_{q_i}. \end{aligned}$$

Note that we used the symbol substitution $z_{q_i} \triangleq z_{g_{1,i}} + z_{h_{i,1}}$, for $i = 1, \cdots, N$. By symmetry, the optimum $d_{O^{x_{s_1}}}$ must satisfy $z_{q_1} = \cdots = z_{q_N}$. Let $P(\mathcal{O}_{d_1}^{\mathbf{X}_{s_1}}) = SNR^{-d_1}$. Then we get $d_1 = (N+1)(1-r)^+$. By the similar reason, we have $p(\mathcal{O}_{d_1}^{\mathbf{X}_{S2}}) \doteq SNR^{-N(1-r)}$ and $p(\mathcal{O}_{d_1}^{\mathbf{X}_{s1x2}}) \doteq SNR^{-2N(1-r)}$. Then we can conclude that the outage probability at D_1 is

$$\begin{aligned} P_{O,d1} &= P(\mathcal{O}_{d_1}^{\mathbf{X}_{s_1}}) + p(\mathcal{O}_{d_1}^{\mathbf{X}_{S2}}) + p(\mathcal{O}_{d_1}^{\mathbf{X}_{s1x2}}) \\ &\doteq SNR^{-\min(N(1-r),(1+N)(1-r),2N(1-r))^+} \\ &= SNR^{-N(1-r)}. \end{aligned}$$

The outage probability of the whole system is two times outage probability of d_1,

$$P_{o,sys} = 2P_{o,d1} \doteq P_{o,d1},$$

which proves Theorem 1. ■

B. Multi-node Relay-Selection cooperative scheme

As conventional Round-Robin scheme will use all the N-relays to cooperate in one frame period, it may leads to a waste of resources to some extent. Since all the channel gains remain constant during a frame period, one can always choose the *best* relay for information transmission at each frame. Then other relay nodes can be used for other missions.

In our proposed Relay-Selection (RS) scheme, at the each beginning time of a frame period, a metric will be used to determine the optimal relay. Any relay node that has the largest value of

$$\min(|h_{i,1}|^2|g_{2,i}|^2, |h_{i,1}|^2|g_{1,i}|^2, |h_{i,2}|^2|g_{1,i}|^2, |h_{i,2}|^2|g_{2,i}|^2)$$

will be used for forwarding the information data to the destinations. Then the $2-N-2$ system become a $2-1-2$ system, and just one relay during one time frame in service, in which the other relay nodes can used in other missions. We will show that our proposed RR scheme provide the N order diversity gain as well.

As discussed in Section III-A, we first focus on destination D_1, and formalize $\mathcal{O}_{d_1}$ as in (5). Let

$$\beta_i = \min\left(|h_{i,1}|^2|g_{1,i}|^2, |h_{i,1}|^2|g_{2,i}|^2, |h_{i,2}|^2|g_{2,i}|^2, |h_{i,2}|^2|g_{1,i}|^2\right)$$

and $\beta_0 = \max(\beta_1, \cdots, \beta_N)$. Then there is

$$\beta_0 = \min\left(|h_{0,1}|^2|g_{1,0}|^2, |h_{0,1}|^2|g_{2,0}|^2, |h_{0,2}|^2|g_{2,0}|^2, |h_{0,2}|^2|g_{1,0}|^2\right).$$

Also we have $z_{\beta_0} = \min(z_{\beta_1}, \cdots, z_{\beta_N})$, and

$$z_{\beta_i} = \max\left(z_{h_{i,1}} + z_{g_{1,i}}, z_{h_{i,1}} + z_{g_{2,i}}, z_{h_{i,2}} + z_{g_{1,i}}, z_{h_{i,2}} + z_{g_{2,i}}\right).$$

Then the outage probability in X_{S_2} can be calculated as

$$\begin{aligned} P(\mathcal{O}_{d_1}^{X_{s_2}}) &= P\{I(\mathbf{X}_{S_2};\mathbf{Y}_{D_1}|\mathbf{X}_{S_1}) < NR\} \\ &= P\left\{N\log\left(1 + \frac{SNR|h_{0,1}|^2|g_{2,0}|^2\tau_i}{1+|h_{0,1}|^2\tau_i}\right) < NR\right\} \\ &\doteq P\{z_{h_{0,1}} + z_{g_{2,0}} > 1 - r\} \\ &\leq P\{z_{\beta_0} > 1 - r\} \\ &= P\{\min(z_{\beta_1}, \cdots, z_{\beta_N}) > 1 - r\} \\ &= SNR^{-N(1-r)}, \end{aligned}$$

$$
\begin{aligned}
&P(\min(z_{\beta_1},\cdots,z_{\beta_N})>1-r)\\
&=(P(\max(z_{h_{i,1}}+z_{g_{1,i}},z_{h_{i,1}}+z_{g_{2,i}},z_{h_{i,2}}+z_{g_{2,i}},z_{h_{i,2}}+z_{g_{1,i}})>1-r))^N\\
&=(1-P(\max(z_{h_{i,1}}+z_{g_{1,i}},z_{h_{i,1}}+z_{g_{2,i}},z_{h_{i,2}}+z_{g_{2,i}},z_{h_{i,2}}+z_{g_{1,i}})<1-r))^N\\
&=(1-P(z_{h_{i,1}}+z_{g_{1,i}}<1-r)P(z_{h_{i,1}}+z_{g_{2,i}}<1-r)P(z_{h_{i,2}}+z_{g_{2,i}}<1-r)P(z_{h_{i,2}}+z_{g_{1,i}}<1-r))^N\\
&=(1-(1-SNR^{-(1-r)})^4)^N\\
&\doteq SNR^{-N(1-r)}.
\end{aligned}
\tag{16}
$$

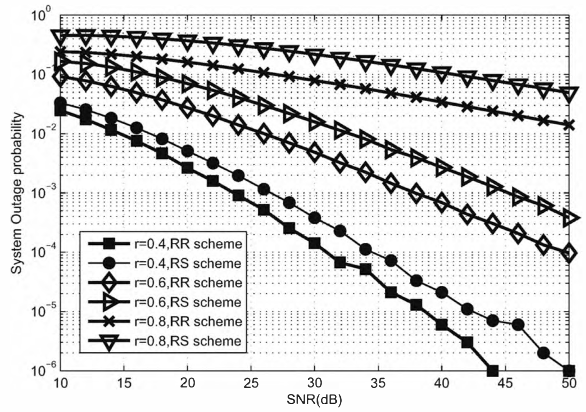

Fig. 3. system outage probability vs. SNR when multiplexing gain varies

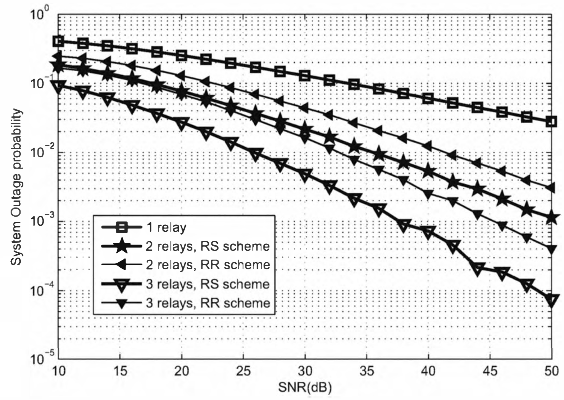

Fig. 4. system outage probability vs. SNR when the number of relays varies

where we calculate $P\{\min(z_{\beta_1},\cdots,z_{\beta_N})>1-r\}$ as in (16). The outage probability in X_{S_1}, $X_{S_1S_2}$ can be similarly calculated as

$$
\begin{aligned}
P(\mathcal{O}_{d_1}^{X_{s_1}}) &\le P\left(\log\left(1+\frac{P|h_{0,1}|^2|g_{1,0}|^2\tau_i}{1+|h_{0,1}|^2\tau_i}\right)<R\right)\\
&\doteq SNR^{-N(1-r)},\\
P(\mathcal{O}_{d_1}^{X_{s_1s_2}}) &\dot{\le} SNR^{-N(1-r)}.
\end{aligned}
$$

Following the same steps in Theorem 1, we get the following theorem.

Theorem 2: The diversity-multiplexing tradeoff achieved by the AF $2-N-2$ multicast system, utilizing N relays in the proposed Relay-Selection (RS) scheme, is characterized by

$$d^{RS}(\overline{r})=N(1-r)^+,$$

where, $\overline{r}=(r/2,r/2)$.

IV. Numerical Results

In this section, we give the simulation results to show the DMT analysis derived in the previous section. All channel coefficients are set up as in Section. II. The power allocation factors are set as $\kappa_{1,i}=\kappa_{2,i}=\tau_i=1/3$. We show the results by Monte Carlo simulations.

Theorem 1 and Theorem 2 have conluded that the two cooperative schemes show the same diversity-multiplexing tradeoff as $d(\overline{r})=N(1-r)^+$. Fig. 3 and Fig. 4 verified the prediction. It presents the system outage probability versus the SNR from 10 dB to 50 dB for the RR scheme and the RS scheme. Three scenarios, i.e., $r=0.4$, $r=0.6$, $r=0.8$, are presented in Fig. 3 for comparison. We have set $N=4$ in the simulation. From Fig. 3, we can find that both curves obtained for the RR scheme and the RS scheme have the same slope, which verifies the fact that the two schemes have the same DMT feature. From Fig. 3, we can also find that outage probability achieved by RR scheme is lower than that by the RS scheme. But the RS scheme only employs one relay in cooperation during one time frame, while the RR employs all N relays in cooperative during one time frame.

Fig. 4 presents a comparison for the different number of relays as $N=1$, $N=2$, and $N=3$, when $r=0.4$. As we can see, the $2-N-2$ multicast system has a great advantage over the $2-1-2$ system as it greatly reduces the system outage probability. As the number of relay increases, the diversity gain (the slope of the curves) of the system raises simultaneously.

V. Conclusion

In this paper, we considered Amplify-and-Forward $2-N-2$ multicast system. We derived the DMT analysis for the Round-Robin scheme and our proposed Relay Selection scheme. Although, our proposed Relay Selection scheme only employs one relay in cooperation during one time frame, it can offer the same diversity order as the Round Robin scheme. But the simulations show that the Round Robin Scheme has lower outage probability than the Relay Selection Scheme. This offers a tradeoff between the outage probability and the number of relay in service during one time frame.

VI. ACKNOWLEDGEMENT

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