# Generalized Channel Inversion Methods for Multiuser Two-Way MIMO Relay Channels

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***Abstract*—In this paper, we propose a novel linear beamforming scheme for multiuser two-way MIMO relay channels to maximize the system sum rate. To achieve a low complexity, our scheme is designed by using the non-iterative method based on generalized channel inversion (GCI). Both perfect and imperfect channel estimations are considered in the design. Simulation results shown that the proposed scheme outperforms the existing scheme in the scenarios with or without channel estimation error**

***Index Terms*—Generalized channel inversion, multiuser MIMO, two-way relay, beamforming.**

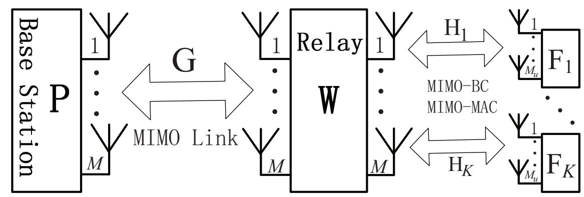


Fig. 1. The multiple-access and broadcast two-way MIMO relay system with $K$ mobile users, one relay, and one base station.

## I. Introduction

Recently, two-way multiple-input multiple-output (MIMO) relaying technique has attracted considerable interest from academic community due to its higher spectral efficiency compared to one-way relay techniques [1]–[8]. A two-way MIMO relaying network can be the scenarios of point-to-point (P2P) or point-to-multipoint (P2MP). Many works have been done on the optimal relay beamforming matrix (BM) design for the P2P two-way MIMO relaying network [1]–[4]. For the P2MP two-way MIMO relaying network, which may correspond to a cellular system with a base station (BS), a relay station (RS) and multiple users (MU), the P2P's property does not hold any more. Prior efforts on MU two-way MIMO relaying networks are reported in [5]–[8]. In [5], [6], MU two-way MIMO relay protocols with decode-and-forward (DF) RS have been considered. In [7], [8], the same beamforming scheme is proposed independently to design the beamforming matrix (BM) at RS and BS based on zero-forcing (ZF) which can cancel unknown co-channel interferences for each user.

In this paper, we consider the MU two-way MIMO relay channel in cellular systems by extending the work in [8] to multiple-antenna users. Furthermore, instead of simple ZF beamforming scheme at RS and BS in [8], we here provide a new linear beamforming scheme based on generalized channel inversion (GCI) scheme [9] to design the BM at RS and precoding matrix (PM) at BS associated with each user's PM for maximizing the sum-rate. To avoid complexity, we consider the non-iterative design method. At the same time, the effect of channel estimation error is also considered in the proposed scheme in term of the sum-rate. Numerical results demonstrate the effectiveness and robustness of our scheme.

*Notations:* $\mathrm{E}(\cdot)$, $\mathrm{tr}(\cdot)$, $(\cdot)^{-1}$, $(\cdot)^T$, $(\cdot)^\dagger$, and $|\cdot|$ denote expectation, trace, inverse, transpose, conjugate transpose, and determinant, respectively. $\mathbf{I}_N$ stands for an $N \times N$ identity matrix. $\mathbf{A}_{(i,j)}$ denotes the entry at the $i$th row and the $j$th column of the matrix $\mathbf{A}$, $\mathrm{diag}(a_1, \ldots, a_N)$ is a diagonal matrix with the $i$th diagonal entry $a_i$. log is of base 2. $\mathcal{C}^{M\times N}$ represents the set of $M \times N$ matrices over complex field.

## II. System Model

We consider an MU two-way MIMO relay channel in cellular systems, which involves one BS, one RS and $K$ mobile users, as shown in Fig. 1. It is assumed that both BS and RS are quipped with $M$ antennas, and each user with $M_u$ antennas. When the BS implements spatial multiplexing (SM) for each user with $M_u$ independent substreams, which requires $KM_u \le M$. Here, we only consider $KM_u = M$ for compact presentation, which is different from [10]. In practice, from the viewpoint of the cost for RS, the number of antennas at RS should be at least equal to or less than that of BS especially in cellular systems. We consider a non-regenerative half-duplex relaying scheme applied at the relay to process and forward the received signals for simplicity [2]. Thus, an exchange information transmission between BS and users is composed of two phases, which may correspond to two frequency channels or two time slots. Since the uplink channels and the downlink channels can be assumed to be reciprocal in the time division duplex (TDD) systems due to channel reciprocity, the TDD mode is used for its simplicity in this paper. We focus on a single carrier with a narrow enough bandwidth so that all channels involved are assumed to be quasi-static independent, identically distributed (i.i.d) Rayleigh fading.

During the first time-slot, the BS transmits $KM_u$ precoded data streams to the relay after applying a linear PM to the original data symbol vector. At the same time, each user transmits $M_u$ precoded data streams to the relay after applying a linear PM to its own original symbol vector. Let $\mathbf{P}_k$ be the

 

PM for the $k$th user at BS, $\mathbf{F}_k$ be the PM at the $k$th user, $\mathbf{s}_k$ and $\mathbf{d}_k$ be the symbol vectors to and from the $k$th user, respectively. Suppose that $\mathbf{G} \in \mathcal{C}^{M\times M}$ is the channel matrix from BS to relay, $\mathbf{H} = [\mathbf{H}_1 \cdots \mathbf{H}_K]$ with $\mathbf{H}_k \in \mathcal{C}^{M\times M_u}$ is the channel matrix from the $k$th user to relay, and $\mathbf{z}_r$ is the noise vector at relay. Then, the signal vector received at relay can be expressed as

$$\mathbf{y}_r = \mathbf{GPs} + \mathbf{HFd} + \mathbf{z}_r, \tag{1}$$

where

$$\begin{aligned}\mathbf{P} &= [\mathbf{P}_1 \cdots \mathbf{P}_K], \quad \text{and} \quad \mathbf{F} = \mathrm{diag}(\mathbf{F}_1, \cdots, \mathbf{F}_K),\\ \mathbf{s} &= [\mathbf{s}_1^T \cdots \mathbf{s}_K^T]^T \quad \text{with} \quad \mathrm{E}(\mathbf{ss}^\dagger) = \mathbf{I}_M,\\ \mathbf{d} &= [\mathbf{d}_1^T \cdots \mathbf{d}_K^T]^T \quad \text{with} \quad \mathrm{E}(\mathbf{dd}^\dagger) = \mathbf{I}_M,\end{aligned}$$

For fairness consideration, we assume

$$\mathrm{E}\{\mathbf{s}_k^\dagger \mathbf{P}_k^\dagger \mathbf{P}_k \mathbf{s}_k\} = \mathrm{tr}(\mathbf{P}_k^\dagger \mathbf{P}_k) \le P_{bk} = P_b/K, \tag{2}$$

and

$$\mathrm{E}\{\mathbf{d}_k^\dagger \mathbf{F}_k^\dagger \mathbf{F}_k \mathbf{d}_k\} = \mathrm{tr}(\mathbf{F}_k^\dagger \mathbf{F}_k) \le P_u. \tag{3}$$

We assume that both channel matrices and noise vector are ergodic and stationary, and their entries are i.i.d and zero mean circularly symmetric complex Gaussian (ZMCSCG) with unit variance.

During the second time-slot, relay broadcasts the $\mathbf{y}_r' = \rho\mathbf{W}\mathbf{y}_r$ to BS and all users under the power constraint $P_r$, where $\mathbf{W} \in \mathcal{C}^{M\times M}$ is the relay BM. Thus, the power control factor at relay is

$$\rho = \sqrt{P_r/\mathrm{tr}(\mathbf{W}(\mathbf{GPP}^\dagger\mathbf{G}^\dagger + \mathbf{HFF}^\dagger\mathbf{H}^\dagger + \mathbf{I}_M)\mathbf{W}^\dagger)}. \tag{4}$$

Hence, during the second time-slot, the signal vector received at the $k$th user is

$$\mathbf{y}_k^* = \rho\mathbf{H}_k^\dagger\mathbf{W}\left(\mathbf{GPs} + \mathbf{HFd} + \mathbf{z}_r\right) + \mathbf{z}_k, \tag{5}$$

and the signal vector received at BS is

$$\mathbf{y}_b^* = \rho\mathbf{G}^\dagger\mathbf{W}\left(\mathbf{GPs} + \mathbf{HFd} + \mathbf{z}_r\right) + \mathbf{z}_b, \tag{6}$$

where $\mathbf{z}_k$ and $\mathbf{z}_b$ are the noise vectors at the $k$th user and BS, respectively, and their entries are i.i.d ZMCSCG with unit variance.

However, channel estimation errors are inevitable due to the presence of background noise in the estimated signals. Thus, considering the channel estimation error model introduced in [11], we model the channel state information (CSI) as

$$\mathbf{H}_k = \widehat{\mathbf{H}}_k + \mathbf{H}_{e,k}, \quad \mathbf{G} = \widehat{\mathbf{G}} + \mathbf{G}_e, \tag{7}$$

where $\mathbf{A}$, $\widehat{\mathbf{A}}$, and $\mathbf{A}_e$ represent the true channel matrix, the estimated channel matrix and the estimation error matrix, respectively. Moreover, we assume that $\mathbf{A}_e$ is uncorrelated with $\widehat{\mathbf{A}}$, and entries of $\mathbf{A}_e$ are i.i.d ZMCSCG with variance $\sigma_{e,\mathrm{A}}^2$.

Since BS and each user know its transmitted signals, respectively, BS and each user can cancel their own signal from the received mixed signal. Then, the signal vector received at the $k$th user can be adjusted as

$$\mathbf{y}_k = \rho\widehat{\mathbf{H}}_k^\dagger\mathbf{W}\widehat{\mathbf{G}}\mathbf{P}_k\mathbf{s}_k + \rho\mathbf{n}_k + \rho\mathbf{H}_k^\dagger\mathbf{W}\mathbf{z}_r + \mathbf{z}_i, \tag{8}$$

where

$$\begin{aligned}\mathbf{n}_k = \widehat{\mathbf{H}}_k^\dagger\mathbf{W}(\mathbf{G}_e\mathbf{P}_k\mathbf{s}_k + \mathbf{H}_{e,k}\mathbf{F}_k\mathbf{d}_k + \mathbf{G}\sum_{j\neq k}\mathbf{P}_j\mathbf{s}_j +\\ \sum_{j\neq k}\mathbf{H}_j\mathbf{F}_j\mathbf{d}_j) + \mathbf{H}_{e,k}^\dagger\mathbf{W}\left(\mathbf{GPs} + \mathbf{HFd}\right)\end{aligned} \tag{9}$$

is the interference term due to channel estimation error and co-channel interferences. The signal vector received at BS can be rewritten as

$$\begin{aligned}\mathbf{y}_b = \rho\widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}\mathbf{Fd} + \rho\widehat{\mathbf{G}}^\dagger\mathbf{W}(\mathbf{H}_e\mathbf{Fd} + \mathbf{G}_e\mathbf{Ps}) +\\ \rho\mathbf{G}_\mathrm{e}\mathbf{W}\left\{\mathbf{GPs} + \mathbf{HFd}\right\} + \rho\mathbf{G}^\dagger\mathbf{W}\mathbf{z}_r + \mathbf{z}_b.\end{aligned} \tag{10}$$

Overall, we can model the MU two-way MIMO relay channel as an MU downlink MIMO relay channel in (8) and an MU uplink MIMO relay channel in (10) with the power constrains at BS, relay, and each user as $P_b$, $P_r$ and $P_u$, respectively.

Our aim is to find the PM $\mathbf{P}$ at BS, BM $\mathbf{W}$ at relay and PM $\mathbf{F}$ at users to maximize the sum-rate of the network under the individual power constrains. Therefore, the optimization problem can be formulated as

$$\{\mathbf{P}_\mathrm{o}, \mathbf{W}_\mathrm{o}, \mathbf{F}_\mathrm{o}\} = \arg\max_{\mathbf{P},\mathbf{W},\mathbf{F}} \left(R_\Sigma^{\mathrm{dl}}(\mathbf{P},\mathbf{W}) + R_\Sigma^{\mathrm{ul}}(\mathbf{F},\mathbf{W})\right),$$

where $R_\Sigma^{\mathrm{dl}}(\mathbf{P},\mathbf{W})$ and $R_\Sigma^{\mathrm{ul}}(\mathbf{F},\mathbf{W})$ are respectively the sum-rate of the downlink relay channel and the sum-rate of the uplink relay channel, respectively, which will be discussed in detail in the following sections. However, it is difficult to directly obtain the optimum closed-form solution because the optimization problem is not convex. In fact, there's no optimal closed-form solution even for MU MIMO relay channel with perfect CSI [12]. In the following sections, we propose an efficient linear beamforming scheme based on block GCI algorithm [9]. The simulation results demonstrate that the proposed scheme outperforms other conventional beamforming schemes in terms of the network sum-rate.

## III. The Relay Processing Matrix and Base Station Precoding Matrix Design

In this section, we first assume that the relay consists of $K$ virtual relays and each virtual relay is quipped with $M_u$ antennas to group the message vectors from and desired to the same user, i.e., group $\mathbf{s}_i$ and $\mathbf{d}_i$ together [8]. Then, we propose an efficient linear beamforming scheme based on block GMI algorithm [9] to design the BM at relay and PM at BS.

From (8) and (10), each user not only receives the desired signal, but also receives the interference signals from or destined to other users due to the existence of co-channel interference. But there is no co-channel interference signals at BS. Hence, it is very important to design the $\mathbf{P}$ and $\mathbf{W}$ meticulously to suppresses the undesired signals to increase the desired signal-to-interference-pulse-noise ratio (SINR) at each user under antenna constraint at relay.

### A. The Design of Beamforming Matrix at the Relay

We first divide the BM $\mathbf{W}$ into two parts: 1) a receiving matrix (RM) $\mathbf{W}_1$, and 2) a transmitting matrix (TM) $\mathbf{W}_2$, i.e., $\mathbf{W} = \mathbf{W}_2\mathbf{W}_1$. Due to the number of antenna constraint at relay ($KM_u = M$), it is difficult to design the BM $\mathbf{W}$ by considering all the signals from and to both BS and all users at the same time like in [10]. To suppress the undesired signals at each user, we design the BM $\mathbf{W}$ based on the users' channel matrices. Moreover, without loss of generality, we assume that the first virtual relay includes the first to the $M_u$th antennas at the whole relay, and so on. To design the RM $\mathbf{W}_1$, we first define the $\widetilde{\mathbf{W}}_1$ as

$$\widetilde{\mathbf{W}}_1 = \widehat{\mathbf{H}}^\dagger(\widehat{\mathbf{H}}\widehat{\mathbf{H}}^\dagger + \alpha_1\mathbf{I}_M)^{-1} = [\widetilde{\mathbf{W}}_{11}^T\ \widetilde{\mathbf{W}}_{12}^T \cdots \widetilde{\mathbf{W}}_{1K}^T]^T, \quad (11)$$

where $\alpha_1 = M\sigma_{e,\mathbf{H}}^2 + M_u/P_u$ [9]. To orthogonalize $\widetilde{\mathbf{W}}_{1k}$ [9], we employ the QR decomposition as

$$\widetilde{\mathbf{W}}_{1k}^T = \mathbf{Q}_k^T\mathbf{R}_k^T. \quad (12)$$

Then

$$\widetilde{\mathbf{W}}_{1k} = \mathbf{R}_k\mathbf{Q}_k, \quad \text{for} \quad k = 1, \cdots, K, \quad (13)$$

where $\mathbf{R}_k$ is an $M_u \times M_u$ upper triangular matrix and $\mathbf{Q}_k$ is an $M_u \times M$ matrix whose rows form an orthonormal basis for $\widetilde{\mathbf{W}}_{1k}$. Hence, we construct the RM $\mathbf{W}_1$ as

$$\mathbf{W}_1^T = [\mathbf{Q}_1^T\ \mathbf{Q}_2^T\ \cdots\ \mathbf{Q}_K^T], \quad (14)$$

where $\mathbf{Q}_k$ is an RM at the $k$th virtual relay.

After deciding the RM $\mathbf{W}_1$, the relay transmitting scheme can be modeled as an MU MIMO broadcast scheme. However, it is very intractable to find an optimal TM even if there is only a single receiving antenna at each user [13]. Here, we use GCI scheme [9] to design the TM $\mathbf{W}_2$ at relay again. We denote $\widetilde{\mathbf{W}}_2$ as

$$\widetilde{\mathbf{W}}_2 = \left(\widehat{\mathbf{H}}\widehat{\mathbf{H}}^\dagger + \alpha_2\mathbf{I}_M\right)^{-1}\widehat{\mathbf{H}} = [\widetilde{\mathbf{W}}_{21}\ \widetilde{\mathbf{W}}_{22} \cdots \widetilde{\mathbf{W}}_{2K}], \quad (15)$$

where $\alpha_2 = M\sigma_{e,\mathbf{H}}^2 + M/P_r$ [9]. After employing the QR decomposition to $\widetilde{\mathbf{W}}_{2k}$ as

$$\widetilde{\mathbf{W}}_{2k} = \bar{\mathbf{Q}}_k\bar{\mathbf{R}}_k, \quad (16)$$

the relay TM $\mathbf{W}_2$ can be constructed as

$$\mathbf{W}_2 = [\bar{\mathbf{Q}}_1\ \ \bar{\mathbf{Q}}_2\ \ldots\ \bar{\mathbf{Q}}_K], \quad (17)$$

where $\bar{\mathbf{Q}}_k$ is an $M \times M_u$ TM at the $k$th virtual relay.

### B. The Design of Precoding Matrix $\mathbf{P}$ at the Base Station

Due to the design of the BM $\mathbf{W}$ at relay without considering the channel from BS, all virtual relays must be regarded as separated to finish the grouping work to suppress the undesired signals for each user. Thus, the transmission from BS to relay can also be modeled as an MU MIMO broadcast channel. Here, we also use the GCI scheme [9] to design the PM $\mathbf{P}$. But, we here treat the

$$\widetilde{\mathbf{G}}_k = \mathbf{Q}_k\widehat{\mathbf{G}} \quad (18)$$

as the equivalent channel from BS to the $k$th virtual relay, and denote

$$\widetilde{\mathbf{G}} = \mathbf{W_1}\widehat{\mathbf{G}} = [\widetilde{\mathbf{G}}_1^T\ \widetilde{\mathbf{G}}_2^T \cdots \widetilde{\mathbf{G}}_K^T]^T, \quad (19)$$

and

$$\widetilde{\mathbf{P}} = \left(\widetilde{\mathbf{G}}^\dagger\widetilde{\mathbf{G}} + \alpha_b\mathbf{I}_M\right)^{-1}\widetilde{\mathbf{G}}^\dagger = [\widetilde{\mathbf{P}}_1\ \widetilde{\mathbf{P}}_2\ \cdots\ \widetilde{\mathbf{P}}_K], \quad (20)$$

where $\alpha_b = M\sigma_{e,\widetilde{\mathbf{G}}}^2 + M/P_b$ [9]. After employing the QR decomposition to $\widetilde{\mathbf{P}}_k$ as

$$\widetilde{\mathbf{P}}_k = \bar{\bar{\mathbf{Q}}}_k\bar{\bar{\mathbf{R}}}_k \quad (21)$$

where $\bar{\bar{\mathbf{Q}}}_k$ is an $M \times M_u$ matrix whose rows form an orthonormal basis for $\widetilde{\mathbf{P}}_k$ with $\bar{\bar{\mathbf{Q}}}_k^\dagger\bar{\bar{\mathbf{Q}}}_k = \mathbf{I}_{M_u}$. Furthermore, to maximize sum-rate of the downlink channel from BS to all users, we construct PM $\mathbf{P}_k$ using a linear combination of columns of $\bar{\bar{\mathbf{G}}}_k$. Thus, we denote the PM $\mathbf{P}_k$ as

$$\mathbf{P}_k = \bar{\bar{\mathbf{Q}}}_k\mathbf{T}_k, \quad (22)$$

where $\mathbf{T}_k$ is an $M_u \times M_u$ transmit combining matrix, which will be discussed further in the sequel. Hence, during the first slot, the observations at the $k$th virtual relay can be expressed as

$$\mathbf{y}_{rk} = \mathbf{Q}_k\mathbf{H}_k\mathbf{F}_k\mathbf{d}_k + \mathbf{Q}_k\mathbf{G}\bar{\bar{\mathbf{Q}}}_k\mathbf{T}_k\mathbf{s}_k + \Delta + \mathbf{Q}_k\mathbf{z}_r, \quad (23)$$

where

$$\Delta = \sum_{j\neq k}\mathbf{Q}_k\mathbf{H}_j\mathbf{F}_j\mathbf{d}_j + \sum_{j\neq k}\widetilde{\mathbf{G}}_k\mathbf{P}_j\mathbf{s}_j \quad (24)$$

is very small compared to the addition of the first and second terms in (23). Recognize $\Delta = 0$, when $\alpha_1 = 0$ in (11), $\alpha_b = 0$ in (20), and the CSIs are perfect. Thus, the received signal vector of the $k$th user can be rewritten from (8) as

$$\mathbf{y}_k = \rho\widehat{\mathbf{H}}_k^\dagger\mathbf{W}\widehat{\mathbf{G}}\bar{\bar{\mathbf{Q}}}_k\mathbf{T}_k\mathbf{s}_k + \rho\mathbf{n}_k + \rho\mathbf{H}_k^\dagger\mathbf{W}\mathbf{z}_r + \mathbf{z}_i, \quad (25)$$

where the term $\widehat{\mathbf{H}}_k^\dagger\mathbf{W}\widehat{\mathbf{G}}\bar{\bar{\mathbf{Q}}}_k$ can be treated as the block equivalent channel from BS to the $k$th user. In order to decouple this block channel into $M_u$ parallel subchannels, we apply the SVD to $\widehat{\mathbf{H}}_k^\dagger\mathbf{W}\widehat{\mathbf{G}}\bar{\bar{\mathbf{Q}}}_k$ as

$$\widehat{\mathbf{H}}_k^\dagger\mathbf{W}\widehat{\mathbf{G}}\bar{\bar{\mathbf{Q}}}_k = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{V}_k^\dagger, \quad (26)$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are unitary matrices, and $\mathbf{\Lambda}_k = \mathrm{diag}(\lambda_{k,1}, \cdots, \lambda_{k,M_u})$ is a diagonal matrix. Then, after applying

$$\mathbf{P}_k = \bar{\bar{\mathbf{Q}}}_k\mathbf{T}_k = \bar{\bar{\mathbf{Q}}}_k\mathbf{V}_k\mathbf{\Pi}_k \quad (27)$$

at BS and an RM $\mathbf{U}_k^\dagger$ at the $k$th user in (25), the received signal vector can be reexpressed from (25) as

$$\begin{aligned}\bar{\mathbf{y}}_k &= \rho\mathbf{U}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{W}\widehat{\mathbf{G}}\bar{\bar{\mathbf{Q}}}_k\mathbf{T}_k\mathbf{s}_k + \rho\mathbf{U}_k^\dagger(\mathbf{n}_k + \mathbf{H}_k^\dagger\mathbf{W}\mathbf{z}_r) + \mathbf{U}_k^\dagger\mathbf{z}_i \\ &= \rho\mathbf{\Lambda}_k\mathbf{\Pi}_k\mathbf{s}_k + \rho\mathbf{U}_k^\dagger\mathbf{n}_k + \rho\mathbf{U}_k^\dagger\mathbf{H}_k^\dagger\mathbf{W}\mathbf{z}_r + \mathbf{U}_k^\dagger\mathbf{z}_i, \quad (28)\end{aligned}$$

where

$$\mathbf{\Pi}_k^2 = \mathrm{diag}(P_{k,1}, P_{k,2}, \cdots, P_{k,M_u}) \quad (29)$$

is a power allocation matrix for the $k$th user at BS. Hence, as a function of $\mathbf{\Pi}_k$, the individual rate $R_k^{\mathrm{dl}}$ for the $k$th user is given by

$$R_k^{\mathrm{dl}} = \frac{1}{2}\log\frac{|\mathbf{\Lambda}_k^2\mathbf{\Pi}_k^2+\Theta|}{|\Theta|} \simeq \frac{1}{2}\sum_{i=1}^{M_u}\log\left(1+\frac{P_{k,i}\lambda_{k,i}^2}{(\Theta)_{(i,i)}}\right) \quad (30)$$

where

$$\Theta = \mathrm{E}(\mathbf{U}_k^\dagger\mathbf{n}_k\mathbf{n}_k^\dagger\mathbf{U}_k)+\mathbf{U}_k^\dagger\mathbf{H}_k^\dagger\mathbf{W}\mathbf{W}^\dagger\mathbf{H}_k\mathbf{U}_k+\rho^{-2}\mathbf{I}_{M_u}$$

and use $\tilde{\rho}$ calculated by (4) with

$$\mathbf{\Pi}_i^2 = \frac{P_b}{M}\mathbf{I}_{M_u} \quad (\text{for } \ i=1,\cdots K) \ \text{ and } \ \mathbf{F} = \frac{P_u}{M_u}\mathbf{I}_M$$

to approximate the $\rho$. The problem of the power allocation to maximize the $R_k^{\mathrm{dl}}$ under the power constraint $\sum_{i=1}^{M_u} P_{k,i} = P_b/K$ can be solved by water-filling [14].

In comparison to the ZF beamforming scheme [8] or block ZF beamforming scheme [10], the proposed beamforming scheme takes the noise into account, which is able to overcome the noise enhancement issue observed in the conventional ZF or block ZF schemes, and this leads to a linear growth of the sum-rate with the number of users and receive antennas.

## IV. Precoding design at users

After deciding the BM $\mathbf{W}$ and PM $\mathbf{P}$, the communication from users to BS can be modeled as an MU MIMO uplink relay system. Thus, we first attach a virtual linear receiver at BS to set up an equivalent channel gain from each user to BS. The virtual linear receiver at BS can be reformulated as

$$\mathbf{A}_b = \mathbf{H}_{bu}^\dagger(\mathbf{H}_{bu}\mathbf{H}_{bu}^\dagger+\alpha_u\mathbf{I}_M)^{-1} = [\mathbf{A}_1^T\ \mathbf{A}_2^T\cdots\mathbf{A}_K^T]^T, \quad (31)$$

where $\mathbf{H}_{bu} = \widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}$ and $\alpha_u = M\sigma_{e,\mathbf{H}}^2 + M_u/P_u$. After employing the QR decomposition to $\mathbf{A}_k$ as

$$\mathbf{A}_k = \mathbf{R}_{a,k}\mathbf{Q}_{a,k}, \quad (32)$$

where $\mathbf{R}_{a,k}$ is an $M_u\times M_u$ upper triangular matrix and $\mathbf{Q}_{a,k}$ is an $M_u\times M$ matrix whose rows form an orthonormal basis for $\mathbf{A}_k$. then, we can treat the $\mathbf{Q}_{a,k}\widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}_k$ as the equivalent channel gain from the $k$th user to BS. Thus, we can employ the SVD decomposition to the $\mathbf{Q}_{a,k}\widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}_k$ as

$$\mathbf{Q}_{a,k}\widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}_k = \mathbf{U}_k\mathbf{\Xi}_k\mathbf{V}_k^H, \quad \text{for} \quad k=1,\cdots,K, \quad (33)$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are unitary matrices, $\mathbf{\Xi}_k = \mathrm{diag}(\xi_{k1},\xi_{k2},\cdots,\xi_{kM_u})$. Then, we can set the users' precoding matrix $\mathbf{F}$ as

$$\mathbf{F} = \mathrm{diag}(\mathbf{V}_1,\mathbf{V}_2,\cdots,\mathbf{V}_K)\mathrm{diag}(\mathbf{\Psi}_1,\mathbf{\Psi}_2,\cdots,\mathbf{\Psi}_K)^{\frac{1}{2}}, \quad (34)$$

i.e., the $k$th user's precoding matrix $\mathbf{F}_k$ is

$$\mathbf{F}_k = \mathbf{V}_k\mathbf{\Psi}_k^{\frac{1}{2}}, \quad \text{for} \quad k=1,2,\cdots,K, \quad (35)$$

where $\mathbf{\Psi}_k = \mathrm{diag}(\psi_{k,1},\psi_{k,2},\cdots,\psi_{k,M_u})$ is a power allocation diagonal matrix. Assume that the BS uses $\mathbf{U}_k^H\mathbf{Q}_{a,k}$ as a linear receiver for the $k$th user's signal. Then, the signal vector received at BS after applying the linear receiver for the $k$th user can be expressed as

$$\begin{aligned}\mathbf{y}_{b,k} = \rho\mathbf{\Xi}_k\mathbf{\Psi}_k^{\frac{1}{2}}\mathbf{d}_k + \rho\sum_{i=1,i\neq k}^{K}\mathbf{U}_k^H\mathbf{Q}_{a,k}\widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}_k\mathbf{F}_k\mathbf{d}_k + \\ \rho\mathbf{U}_k^H\mathbf{Q}_{a,k}\Big(\widehat{\mathbf{G}}^\dagger\mathbf{W}(\mathbf{H}_e\mathbf{F}\mathbf{d}+\mathbf{G}_e\mathbf{P}\mathbf{s})+ \\ \mathbf{G}_e\mathbf{W}\left\{\mathbf{G}\mathbf{P}\mathbf{s}+\mathbf{H}\mathbf{F}\mathbf{d}\right\}+\mathbf{G}^\dagger\mathbf{W}\mathbf{z}_r\Big)+\mathbf{U}_k^H\mathbf{Q}_{a,k}\mathbf{z}_b. \quad (36)\end{aligned}$$

Hence, according the point-to-point MIMO channel precoding technique [15], the power allocation diagonal matrix $\mathbf{\Psi}_k$ can be solved by water-filling algorithm [14], i.e.,

$$\psi_{k,i} = \left(\mu - \frac{(\mathbf{\Omega}_k)_{(i,i)}}{\xi_{k,i}}\right) \quad \text{s.t.} \quad \mathrm{tr}(\mathbf{\Psi}_k) = P_u, \quad (37)$$

where $\mathbf{\Omega}_k = \mathrm{E}(\mathbf{\Upsilon}\mathbf{\Upsilon}^H)$, and

$$\begin{aligned}\mathbf{\Upsilon} \triangleq \mathbf{U}_k^H\mathbf{Q}_{a,k}\Big(\sum_{i=1,i\neq k}^{K}\widehat{\mathbf{G}}^\dagger\mathbf{W}\widehat{\mathbf{H}}_k\mathbf{F}_k\mathbf{d}_k + \widehat{\mathbf{G}}^\dagger\mathbf{W}(\mathbf{H}_e\mathbf{F}\mathbf{d}+ \\ \mathbf{G}_e\mathbf{P}\mathbf{s})+\mathbf{G}_e\mathbf{W}\left\{\mathbf{G}\mathbf{P}\mathbf{s}+\mathbf{H}\mathbf{F}\mathbf{d}\right\}+\mathbf{G}^\dagger\mathbf{W}\mathbf{z}_r+\frac{\mathbf{z}_b}{\rho}\Big). \quad (38)\end{aligned}$$

## V. Numerical results

In this section, the performance of the proposed linear beamforming scheme (PLBS) in term of sum-rate will be evaluated by using Monte Carlo simulation with 2000 random channel realizations. We compare the proposed scheme with other two different schemes, which are:

1) Simple-ZF scheme [8]:

$$\mathbf{P} = \widehat{\mathbf{G}}^{-1}\widehat{\mathbf{H}}\mathbf{D}_s, \quad \text{and} \quad \mathbf{W} = (\widehat{\mathbf{H}}^\dagger)^{-1}\mathbf{D}_r\widehat{\mathbf{H}}^{-1}, \quad (39)$$

where $\mathbf{D}_s$ and $\mathbf{D}_r$ are power allocation diagonal matrices at BS and RS, respectively.

2) Block-ZF scheme: let $\alpha_1 = 0$ in (11), $\alpha_2 = 0$ in (15) and $\alpha_b = 0$ in (20), and other conditions are the same as the proposed scheme.

All schemes are compared under the same design method for user's PM as the Section IV for fair comparison.

Fig. 2 shows the average sum-rate versus the transmit power with the perfect CSI. It is observed that the block-ZF scheme is close to the proposed scheme at high SNR regime, which is like that in [9] [16]. But a gap always remains compared to the simple-ZF at all SNR regime, because the interior gains of each user is with multiple antennas.

Fig. 3 shows the average sum-rate versus the variance of the channel estimation error. It is observed that the proposed scheme is a better robust scheme.

Fig. 4 shows sum-rate versus the number of the users. Unlike the block-ZF and the simple-ZF schemes, the sum rate of the proposed scheme is linearly growing with $K$, which is like that in [9] [16]. These figures also show that the proposed scheme takes the noise into account, which is able to overcome the noise enhancement issue observed in the simple-ZF or the block ZF schemes, and leads to a linear growth of the sum-rate with the number of users and receive antennas.

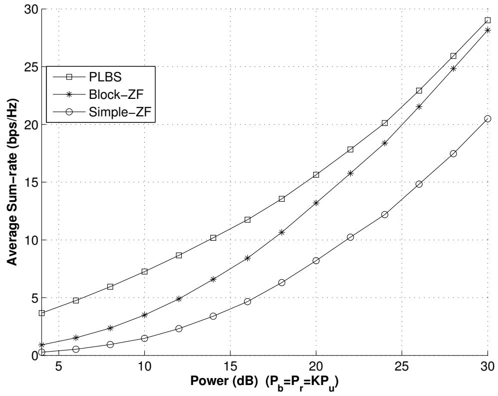


Fig. 2. Average sum-rate versus transmit power, where $P_b = P_r = KP_u$, $K = 4$, $M_u = 2$, and $\sigma^2_{e,\mathbf{H}} = \sigma^2_{e,\mathbf{G}} = 0$.

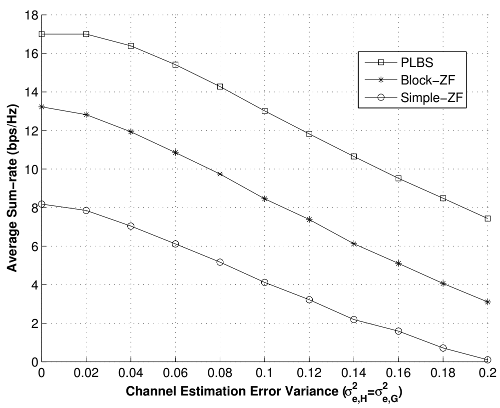


Fig. 3. Average sum-rate versus channel estimation error variance ($\sigma^2_{e,\mathbf{H}} = \sigma^2_{e,\mathbf{G}}$), where $P_b = P_r = KP_u = 20$dB, $K = 4$, $M_u = 2$.

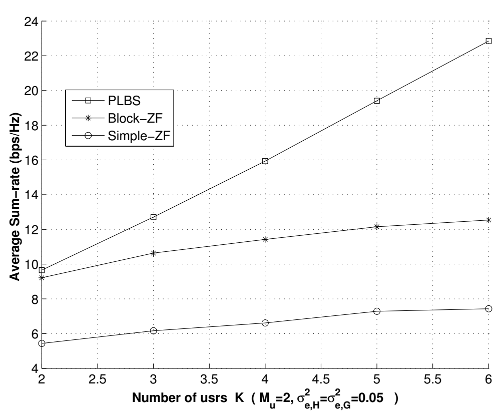


Fig. 4. Average sum-rate versus $K$, where $P_b = P_r = KP_u = 20$dB, $M_u = 2$, and $\sigma^2_{e,\mathbf{H}} = \sigma^2_{e,\mathbf{G}} = 0.05$.

## VI. Conclusion

In this paper, generalized channel inversion methods are developed for MU two-way MIMO non-regenerative half-duplex relay channel in cellular systems, in which a linear beamforming scheme is proposed to design the BM at relay, PM at BS, and PMs at users to maximize the sum-rate. We also consider the effect of channel estimation error in term of the sum-rate. Simulation results shown that the proposed scheme outperforms the existing scheme in the scenarios with or without channel estimation error.

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## References

[1] R. Zhang, Y. C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Select. Areas Commun.*, vol. 27, no.5, pp. 699–712, June 2009.

[2] S. Xu and Y. Hua, "Optimal design of spatial source-and-relay matrices for a non-regenerative two-way MIMO relay system," *IEEE Trans. Wireless Commun.*, vol. 10, no.5, pp. 1645–1655, MAY 2011.

[3] G. A. S. Sidhu, F. Gao, W. Chen, and A. Nallanathan, "A joint resource allocation scheme for multiuser two-way relay networks," *IEEE Trans. Commun.*, vol. 59, no.11, pp. 2970–2975, Nov. 2011.

[4] G. Wang, F. Gao, W. Chen, and C. Tellambura, "Channel estimation and training design for two-way relay networks in time-selective fading environments," *IEEE Trans. Wireless Commun.*, vol. 10, no.8, pp. 2681–2691, August 2011.

[5] L. Weng and R. D. Murch, "Multi-user MIMO relay system with self-interference cancellation," in *Proc. of the IEEE Wireless Commun. Net. Conf.(WCNC)*, pp. 958–962, 2007.

[6] C. Eşli and A. Wittneben, "Multiuser MIMO two-way relaying for cellular communications," in *Proc. of the IEEE 19th Inte. Symp. on PIMR Commun.*, 2008.

[7] S. Toh and D. T. M. Slock, "A linear beamforming scheme for multi-user MIMO af two-phase two-way relaying," in *Proc. of the IEEE 20th Inter. Symp. on PIMR Commun.*, 2009.

[8] Z. Ding, I. Krikidis, J. Thompson, and K. K. Leung, "Physical layer network coding and precoding for the two-way relay channel in cellular systems," *IEEE Trans. Sig. Proc.*, vol. 59, no.2, pp. 696–712, Feb 2011.

[9] H. Sung, S.-R. Lee, and I. Lee, "Generalized channel inversion methods for multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 57, no.11, pp. 3489–3499, Nov 2009.

[10] J. Kim, J. Hwang, K.-J. Lee, and I. Lee, "Blockwise amplify-and-forward relaying strategies for multipoint-to-multipoint MIMO networks," *IEEE Trans. Wireless Commun.*, vol. 10, no.7, pp. 2028–2033, July 2011.

[11] T. Yoo and A. Goldsmith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inf. Theory*, vol. 52, no.5, pp. 2203–2214, May 2006.

[12] Y. Yu and Y. Hua, "Power allocation for a MIMO relay system with multiple-antenna users," *IEEE Trans. Sig. Proc.*, vol. 58, no.5, pp. 2823–2835, May 2010.

[13] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Tech.*, vol. 53, no.1, Jan. 2004.

[14] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, Inc., 2006.

[15] I. E. Telatar, "Capacity of multi-antenna gaussian channels," *Europ. Trans. Telecommun.*, vol. 10, pp. 585–596, Nov.-Dec 1999.

[16] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: Channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no.1, pp. 195–202, Jan. 2005.