Low Complexity Energy-Efficient Design for OFDMA Systems with an Elaborate Power Model

Qingqing Wu*, Wen Chen*, Jun Li† and Jinsong Wu‡
*Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, China.
†School of Electrical and Information Engineering, The University of Sydney, NSW, Australia.
‡Bell Laboratories, Alcatel-Lucent, Shanghai, China.
Emails: {wu.qq,wenchen}@sjtu.edu.cn, jun.li@sydney.edu.au, wujs@ieee.org.

Abstract*—In this paper, we investigate the resource allocation for joint transmitter and receiver energy efficiency maximization in orthogonal frequency division multiple access (OFDMA) systems. An elaborate power dissipation model is proposed for OFDMA systems considering the transmission power from the base station side, the signal processing power and radio frequency (RF) circuit power from both sides. Then we formulate the energy efficiency maximization problem and propose a two-step method based on the relationship analysis of the single subcarrier-user (SU) pair energy efficiency and system energy efficiency. Specifically, we first pair each subcarrier with the user resulting in highest SU pair energy efficiency, which is motivated by a special case study. Then, we propose a linear complexity scheme by exploring the inherent fractional structure of the system energy efficiency, which is proved to be optimal for the power allocation with given SU pairing in the first step. Finally, we provide a sufficient condition under which our proposed two-step method is globally optimal. Numerical results demonstrate the effectiveness of the proposed method and we also find that exploiting more user diversity is not always beneficial from the perspective of energy efficiency.

I. Introduction

Green technologies have inevitably become the design components of future communication systems. Energy efficiency defined as bits per joule has been accepted gradually as an important metric to assess the performance of communication systems besides system throughput [1] due to its nature of pursuing the most effective utilization of every joule. Meanwhile, orthogonal frequency division multiple access (OFDMA) has emerged as a promising candidate for the next generation wireless networks due to its high spectral efficiency and resistance to multipath fading [2]. Therefore, it is particularly crucial to investigate the energy efficiency of OFDMA systems towards transforming green concepts into future communication systems.

Recently, some attempts have been reported on the energy-efficient designs in OFDMA systems [3]–[9]. However, these previous works at least have three major limitations in terms of system assumptions. First, these works [3]–[8] only consider the power consumption either from the base station (BS) side or from the user side. Practically, the concept of energy efficiency should involve the overall system throughput and the overall system power consumption [1]. Second, most of these works [3]–[7] assume that the baseband signal processing power is a constant regardless of the bandwidth. However, according to recent investigations from industrial perspectives [8]–[10], the signal processing cost linearly scales with the bandwidth or the number of used subcarriers. Third, these works [3]–[9] assume the constant radio frequency (RF) circuit power model without considering the number of users, which does not realistically reflect the power consumption varying with the number of users in OFDMA systems. In addition, it has been reported that with inaccurate power consumption models, the associated energy-efficient designs may suffer from some performance loss and even lead to reverse conclusions sometimes [1], [11]. To the best of our knowledge, the energy efficiency of OFDMA systems under the general power consumption model considered in this paper has not been studied so far and we aim to cover all the above three issues.

Meanwhile, low complexity designs may be highly expected especially in *green-oriented* communication systems [1], [4], since the solutions with high computational complexities may result in additional energy costs in practice due to the huge computational loads and memory requirement, which contradicts with the green objectives. Authors in [4] study the low complexity scheduling but the proposed method is not applicable to the considered power consumption model here.

In this paper, we investigate the low complexity energy-efficient resource allocation in OFDMA systems under an elaborate power consumption model where the transmission power from BS side, the baseband processing power and the RF circuit power from both sides are taken into consideration. Based on a special case study, we design an efficient scheme to implement SU pairing. Then we further propose an optimal power allocation scheme under the given SU pairing in the first step. Finally, a sufficient condition which guarantees the global optimality of the proposed two-step method is provided.

II. System Description and Problem Formulation

A. System Model

Consider a downlink OFDMA network in a single cell with one BS and K users which are all equipped with one antenna. The total bandwidth W is equally divided into N

This work is supported by the National 973 Project #2012CB316106, by NSF China #61161130529 and #61328101, by the STCSM Science and Technology Innovation Program #13510711200, by the SEU National Key Lab on Mobile Communications #2013D11.

978-1-4799-3512-3/14/$31.00 ©2014 IEEE

subcarriers, each with a bandwidth of $B = W/N$. Besides, the channel is modeled to include both large scale fading and small scale fading. The large scale fading includes path loss and shadowing loss. The small scale fading is assumed to be frequency selective with block among subcarriers, i.e. the fading coefficient of each subcarrier among different users is an independent constant within each time slot [4]. We assume that perfect channel state information (CSI) is available for BS to optimize system energy efficiency. The SU pairing and power allocation settings are allowed to vary from one time slot to another according to the CSI [6]. We also assume that each subcarrier is exclusively assigned to at most one user in each scheduling slot, in order to avoid the interference among different users. Each user, on the other hand, can occupy more than one subcarrier to achieve the maximal energy efficiency. Denote $p_{i,j}$ and $h_{i,j}$ as the allocated power and channel gain for user j on subcarrier i, respectively. Then, the maximum corresponding achievable data rate $r_{i,j}$ of user j on subcarrier i is given by

$$r_{i,j} = B \log_2 \left(1 + \frac{p_{i,j} h_{i,j}}{N_0 B}\right), \tag{1}$$

where N_0 is the spectral density of the additive white Gaussian noise. Thus, the overall system data rate is

$$R_{tot} = \sum_{i=1}^{N} \sum_{j=1}^{K} \rho_{i,j} r_{i,j}, \tag{2}$$

where the pairing indicator $\rho_{i,j} \in \{0, 1\}$ indicates that the subcarrier i is paired to the user j if $\rho_{i,j} = 1$, otherwise $\rho_{i,j} = 0$.

B. An Elaborate Power Consumption Model

For the BS, its power consumption includes the transmission power, the constant RF circuit power, and the signal processing power which linearly scales with the number of used subcarriers [1], [8], [12]. Denote P_{s0} as the signal processing power per subcarrier in the BS side. The power consumption on subcarrier i is

$$P_{ti} = \sum_{j=1}^{K} \rho_{i,j} \left(\frac{p_{i,j}}{\xi} + \mathcal{I}(p_{i,j}) P_{s0}\right), \tag{3}$$

where $\xi \in (0, 1)$ is a constant accounting for for power efficiency of the amplifier [5]–[9] and the indicator function $\mathcal{I}(x)$ is defined as

$$\mathcal{I}(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

From (3), we can see that even if the subcarrier i is paired to the user j, P_{s0} should not be added to the power consumption of the BS side if the power allocation $p_{i,j}$ is zero. Then, the power consumption in the BS side is

$$P_t = \sum_{i=1}^{N} P_{ti} + P_b, \tag{5}$$

where P_b represents the constant RF circuit power of the BS.

For each user in the receiver side, similarly, its power consumption includes the constant RF circuit power, and the signal processing power which also linearly scales with the number of used subcarriers [1], [8], [13]. Denote P_{sj} as the per-subcarrier signal processing power and P_{uj} as the constant RF circuit power of the user j. Without loss of generality, P_{sj} and P_{uj} of different users can be different, which represents different types of terminals employed. Then, the power consumption of the user j is

$$P_{rj} = P_{uj} + \sum_{i=1}^{N} \rho_{i,j} \mathcal{I}(p_{i,j}) P_{sj}. \tag{6}$$

Therefore, the overall system power consumption is

$$P_{tot} = P_t + \sum_{j=1}^{K} P_{rj}. \tag{7}$$

After some manipulations, it can be expressed as

$$P_{tot} = \sum_{i=1}^{N} \sum_{j=1}^{K} \rho_{i,j} \underbrace{\left(\frac{p_{i,j}}{\xi} + \mathcal{I}(p_{i,j})(P_{s0} + P_{sj})\right)}_{P_{i,j}} + \underbrace{\sum_{j=1}^{K} P_{uj} + P_b}_{P_0}. \tag{8}$$

Note that $P_{i,j}$ denotes the power consumption of user j on subcarrier i and P_0 denotes the constant circuit power of the whole system which scales with the number of users.

C. Problem formulation

The energy efficiency considered is defined as the overall system rate R_{tot} over the overall system power P_{tot}. Thus, the average system energy efficiency can be expressed as

$$EE = \frac{R_{tot}}{P_{tot}}. \tag{9}$$

Mathematically, we can formulate the energy efficiency optimization problem as

$$\begin{aligned} \max_{\boldsymbol{\rho}, \boldsymbol{p}} \quad & \frac{\sum_{i=1}^{N} \sum_{j=1}^{K} \rho_{i,j} B \log_2 \left(1 + \frac{p_{i,j} h_{i,j}}{N_0 B}\right)}{\sum_{i=1}^{N} \sum_{j=1}^{K} \rho_{i,j} \left(\frac{p_{i,j}}{\xi} + \mathcal{I}(p_{i,j})(P_{s0} + P_{sj})\right) + P_0} \\ \text{s.t.} \quad & \sum_{j=1}^{K} \rho_{i,j} = 1, \quad \forall i, \quad \rho_{i,j} \in \{0, 1\}, \quad \forall i, j, \\ & p_{i,j} \geqslant 0, \qquad \forall i, j, \end{aligned} \tag{10}$$

where $\boldsymbol{\rho} \triangleq \{\rho_{i,j} | i = 1, 2, ...N; j = 1, 2, ...K\}$ and $\boldsymbol{p} \triangleq \{p_{i,j} | i = 1, 2, ...N; j = 1, 2, ...K\}$.

In the conventional spectral efficiency optimization, BS always transmits at the maximal power in order to achieve the date rate as high as possible. However, in the energy efficiency optimization, BS usually transmits within the allowed maximal power due to its conservative nature of saving power. Thus, the total power constraint in the downlink scenario is not necessary [14].

To obtain the optimal solution to problem (10) is especially challenging, due to the existence of the piecewise functions $\mathcal{I}(p_{i,j})$ and the combinatorial nature of the binary variables $\rho_{i,j}$. An alternative approach is the exhaustive search method whose complexity is prohibitively high. Moreover, the additional computations will introduce additional power consumption, which contradicts the expectation of high energy efficiency. Therefore, designing effective methods of low complexity for the energy efficiency optimization is necessary.

III. Low Complexity Designs For The Energy Efficiency Optimization

In this section, we propose a low complexity two-step method based on the relationship analysis of each SU pair energy efficiency $ee_{i,j}$ and the system energy efficiency EE. In the next, we firstly introduce the energy efficiency of a single SU pair.

A. Energy Efficiency of A Single SU Pair

Assume that the ith subcarrier is paired with the jth user. Define the energy efficiency of a single SU pair as

$$ee_{i,j} = \frac{r_{i,j}}{P_{i,j}} = \frac{B\log_2\left(1+\frac{p_{i,j}h_{i,j}}{N_0 B}\right)}{\frac{p_{i,j}}{\xi} + \mathcal{I}(p_{i,j})(P_{s0}+P_{sj})}. \tag{11}$$

Since the optimal transmission power $p_{i,j}$ can not be zero in maximizing the energy efficiency $ee_{i,j}$, we have the indicator function $\mathcal{I}(p_{i,j}) = 1$. It is easy to verify that $ee_{i,j}$ is a strictly quasiconcave function of $p_{i,j}$ [14] and this fractional type function has been proved to have the stationary point which is also the optimal point [14]. Due to this property, setting the derivative of $ee_{i,j}$ with respect to $p_{i,j}$ to zero, we get the optimal power allocation $p^*_{i,j}$ and the optimal energy efficiency as follows

$$ee^*_{i,j} = \frac{\xi B h_{i,j}}{(BN_0 + p^*_{i,j}h_{i,j})\ln 2}. \tag{12}$$

Based on (11) and (12), the numerical values of $ee^*_{i,j}$ and $p^*_{i,j}$ can be easily obtained. Moreover, from (11), we can clearly observe that the energy efficiency $ee^*_{i,j}$ of a single SU pair increases with the channel gain $h_{i,j}$ and decreases with the signal processing power P_{s0} and P_{sj}.

B. Subcarrier-User Pairing

In the following, we firstly investigate a special case of the optimal SU pairing, which will provide valuable insights into the design of the efficient pairing scheme. By separating the subcarrier m with the other subcarriers, the system energy efficiency can be written as

$$EE = \frac{\sum_{i\neq m}\sum_{j=1}^{K}\rho_{i,j}r_{i,j} + \sum_{j=1}^{K}\rho_{m,j}r_{m,j}}{\left(\sum_{i\neq m}\sum_{j=1}^{K}\rho_{i,j}P_{i,j}+P_0\right)+\sum_{j=1}^{K}\rho_{m,j}P_{m,j}}. \tag{13}$$

From (13) we can see that if the allocated power $p_{m,j}$ on the mth subcarrier is zero for all j, then the subcarrier m makes no contribution to the system energy efficiency, since the pairing rate $r_{m,j}$ are zeros for all j. On the other hand, if $p_{m,j}$ is nonzero for some j, then the indicator function $\mathcal{I}(p_{i,j}) = 1$. We should determine which user should be selected to pair with subcarrier m. Assume that user k is paired with subcarrier m, i.e., $\rho_{m,k} = 1$, and $\rho_{m,j} = 0$ for $j \neq k$. Substituting the pairing rate and pairing power of the subcarrier m into (13), we have

$$EE = \frac{\sum_{i\neq m}\sum_{j=1}^{K}\rho_{i,j}r_{i,j} + B\log_2\left(1+\frac{p_{m,k}h_{m,k}}{N_0 B}\right)}{\left(\sum_{i\neq m}\sum_{j=1}^{K}\rho_{i,j}P_{i,j}+P_0\right)+\left(\frac{p_{m,k}}{\xi}+P_{s0}+P_{sk}\right)}. \tag{14}$$

Based on the observation of (14), we have the following theorem.

Theorem 1: For subcarrier m, if there is a certain user k with the following property:

$$h_{m,k} = \max_{j=1,\ldots,K} h_{m,j} \text{ and } P_{sk} = \min_{j=1,\ldots,K} P_{sj}, \tag{15}$$

then user k is the globally optimal user paired with the subcarrier m in maximizing EE in (10).

Proof: Please see Appendix A. ■

The users with this kind of property can always contribute the highest data rate with the lowest power consumption among all the users on the subcarrier m. In this case, Theorem 1 implies that we can select the globally optimal user for each subcarrier independently. However in practice, the users on some subcarriers may not necessarily possess this property. This means that for some subcarrier m, there is no user simultaneously with the highest $h_{i,j}$ and the lowest P_{sj} among all users j. In those cases, Theorem 1 is not applicable. Besides, it is difficult to find an alternative way to directly get the optimal $\rho_{i,j}$, since it couples optimal power allocation $p_{i,j}$ which is unknown yet. On the other hand, the complexity of exhaustive search for the optimal $\rho_{i,j}$ is K^N. To facilitate practical applications of the energy-efficient design, we need to design the SU pairing scheme with low complexity.

Motivated by Theorem 1, we adopt the maximal single pair energy efficiency (SEE) $ee^*_{i,j}$ as the metric to select the user for each subcarrier. Then the SU pairing policy is

$$\rho^*_{i,j} = \begin{cases} 1, & j = \arg\max_{j=1,\ldots,K} ee^*_{i,j}, \\ 0, & \text{otherwise}, \ \forall i. \end{cases} \tag{16}$$

Note that on the same subcarrier, the different terms for the users are the channel gain $h_{i,j}$ and signal processing power P_{sj}. Clearly, the energy efficiency $ee^*_{i,j}$ of a single SU pair involves both of them. From Section III-A, we know that higher SEE implies that the user has higher $h_{i,j}$ or lower P_{sj} in some sense. Meanwhile, from Theorem 1, the user j with the higher $h_{i,j}$ and the lower P_{sj} is more preferable to occupy the ith subcarrier in order to achieve the highest energy efficiency. Thus, this SEE based scheme for the SU pairing is rational and the simulation results will demonstrate that the case of an optimal user with a lower SEE is very limited. When there is more than one user having the same $ee^*_{i,j}$ for some subcarrier, we can either choose the user with higher $h_{i,j}$ to increase the system throughput or the user with low P_{sj} to save the system power consumption.

C. Power Allocation

In this subsection, we propose an *optimal power allocation* scheme under the *given* $\boldsymbol{\rho}^*$ in Section III-B. Note that each subcarrier i has been paired with a unique user j by $\rho^*_{i,j}$, which means that the system is simplified to a parallel-channel system. For notation simplicity, the user index j is dropped in the subsequent discussion and denote $P_{ci} \triangleq P_{s0} + P_{sj}$. Then the optimization problem (10) is transformed into the following problem:

$$\max_{\boldsymbol{p}} \quad EE = \frac{\sum_{i=1}^{N} B\log_2(1+\frac{p_i h_i}{N_0 B})}{\sum_{i=1}^{N}(\frac{p_i}{\xi}+\mathcal{I}(p_i)P_{ci})+P_0}, \tag{17a}$$

$$\text{s.t.} \quad p_i \geqslant 0, \quad \forall i. \tag{17b}$$

From the problem analysis in Section II, the indicator function makes EE a piecewise function of p_i. Thus, to directly derive the closed-form power allocation solution seems impossible. Authors in [14] propose an optimal power allocation scheme for problem (17) with a *quadric* complexity. However, in our work, the optimal power allocation can be obtained with a linear complexity. In the next, we convert problem (17) into an equivalent one with the quasiconcave form as (11), which makes it easy to obtain the optimal power allocation.

Denote R as the set of SU pairs in which pairs are allocated with positive powers, i.e. $R = \{i \,|\, p_i > 0\}$. Then the problem (17) can be written as

$$\max_{\boldsymbol{p}} \quad EE = \frac{\sum_{i\in R} B\log_2\left(1+\frac{p_i h_i}{N_0 B}\right)}{\sum_{i\in R}(\frac{p_i}{\xi}+P_{ci})+P_0}. \tag{18}$$

Comparing (17) and (18), if we can obtain the set R which is composed of nonzero powers of optimal solutions to (17), then problem (18) can result in the same optimal power allocation as (17).

Given R, problem (18) is a standard quasiconcave optimization problem [15] and the optimal solution can be easily solved by setting the partial derivative of EE with respect to p_i to zero. After some manipulations, the optimal energy efficiency of (18) is given by

$$EE^* = \frac{\xi B h_i}{(BN_0 + p_i^* h_i)\ln 2}, \; i \in R. \tag{19}$$

Denote R^{opt} as the optimal set of problem (18). Our task is simplified to find the R^{opt} in which the allocated power for each SU pair is positive in (17). In the following, we propose an ordering based successive adding (OSA) scheme to find the set R^{opt}. The key idea of the OSA scheme is based on exploiting the inherent fractional property of the ee_i^* and EE^*. We firstly sort all SU pairs by their SEE ee_i^* in descending order according to (12), i.e., $ee_1^* \geqslant ee_2^* \geqslant ... \geqslant ee_N^*$. Then we add each SU pair to the set R successively according to the order. In the Lth round, we should determine whether the Lth SU pair should be added to the set R. Let $EE_L = (\sum_{l=1}^{L} r_l)/(\sum_{l=1}^{L} P_l + P_0)$. The maximal EE_L^* can be obtained by (19). Then we have the following theorem.

Algorithm 1 SEE Ordering based Successive Adding Method (SEEOSA)

For $i = 1:N$
  For $j = 1:K$
    Compute $p^*_{i,j}$ and $ee^*_{i,j}$ by (11) and (12);
  End
  Determine $\rho^*_{i,j}$ by (16);
End
Ordering all the N SU pairs in descending order and set $EE_0^* = 0$;
For $L = 1:N$
  Compare EE^*_{L-1} with ee^*_L;
  If $EE^*_{L-1} \leqslant ee^*_L$ **do**
    Add the Lth pair to R;
    Compute p^*_l and EE^*_{L-1} $(l \in R)$ by (18) and (19), respectively;
  else $EE^*_{L-1} > ee^*_L$ **do**
    $R^{opt} = R$;
    $\widetilde{p_l} = p^*_l \; (l \in R)$, $\widetilde{p_l} = 0 \, (l \notin R)$;
    $\widetilde{EE} = EE^*_{L-1}$;
    Return.
  End
End

Theorem 2: 1) If $EE^*_{L-1} \leqslant ee^*_L$, then there must be $EE^*_{L-1} \leqslant EE^*_L \leqslant ee^*_L$ and the Lth SU pair should be added to the set R; 2) If $EE^*_{L-1} > ee^*_L$, then there must be $EE^*_{L-1} > EE^*_L > ee^*_L$ and the Lth SU pair should not be added to the set R.

Proof: Please see Appendix B. ■

Theorem 2 implies that in the Lth round, the comparison result of EE^*_{L-1} and ee^*_L is sufficient to determine whether the Lth SU pair should be added to the set R. Note that the system energy efficiency and the power allocation will be updated by (18) and (19) whenever adding a SU pair to the set R. Based on Theorem 2 and the ordering property, we further have the following corollary.

Corollary 1: When $EE^*_{L-1} > ee^*_L$, the first $L-1$ SU pairs are allocated with positive powers and compose the optimal set R^{opt} and EE^*_{L-1} is the maximal system energy efficiency for given $\boldsymbol{\rho}^*$ in Section III-B.

Corollary 1 implies that SU pairs behind the Lth pair can not be added to the set R either, which means that the set including the first $L-1$ SU pairs is the optimal set R^{opt} and EE^*_{L-1} is the maximal system energy efficiency for given $\boldsymbol{\rho}^*$. Due to limited space, this proof and the following theorem's proof have been put in the journal version of this paper [16].

We call this two-step method for the energy-efficient OFDMA design as the SEE ordering based successive adding (SEEOSA), which is summarized in Algorithm 1. $\widetilde{EE}$ and $\widetilde{p_l}$ therein denote the solution of the proposed SEEOSA method to problem (10). In the next, we provide a sufficient condition which can guarantee the *optimality* of the proposed SEEOSA.

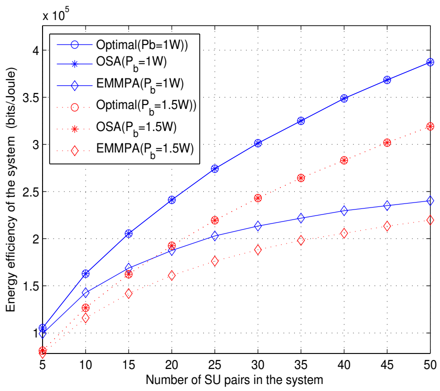

Fig. 1. The optimality validation for the OSA.

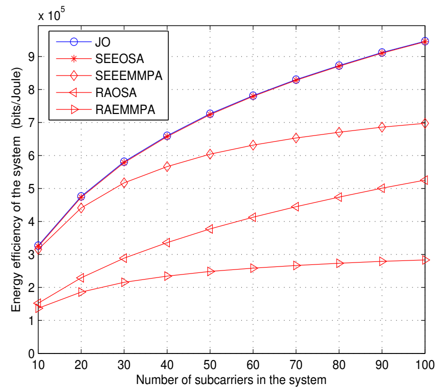

Fig. 2. Energy efficiency versus subcarriers ($K = 5$).

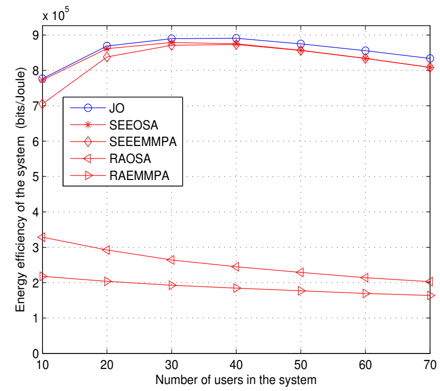

Fig. 3. Energy efficiency versus users ($N = 32$).

TABLE I
SYSTEM PARAMETERS

Parameter	Description
Subcarrier bandwidth, B	15kHz
Signal processing power of user j, P_{sj}	$10 - 30$ mW
Signal processing power of BS, P_{s0}	50 mW
Thermal noise density, N_0	-174 dBm/Hz
Base station power, P_b	1000mW
User RF circuit power, P_{uj}	20mW
Power amplifier efficiency, ξ	0.38
Path loss model	Okumura-Hata
Shadowing	20 dB
Fading model	Rayleigh fading

Theorem 3: If all users have the same signal processing power P_{sj}, then the SEEOSA method is globally optimal for problem (10).

For the scenario that terminals employ the same type of the signal processing component, our proposed SEEOSA method is optimal for the energy-efficient resource allocation and note that P_{uj} is not required the same for all terminals.

IV. Numerical Results

In this section, we conduct comprehensive simulation results to demonstrate the performance of the proposed method for the energy-efficient OFDMA designs. The cell of the network is hexagonal with a radius of 1000 meters, within which the users are uniformly distributed outside of the concentric circle with the radius of 100 meters. The main system parameters are listed in Table I [8] without specific explanation. The signal processing powers among users vary from 10 mW to 50 mW. We simulate 10000 channel realizations for all simulations.

In Fig. 1, we verify the optimality of the power allocation scheme in the second step, i.e., OSA, in two P_b cases. Another optimal scheme for problem (17) [14] which has a quadric complexity and a suboptimal scheme (EMMPA) in [14] are adopted as comparison. The channel gain of SU pairs are randomly generated without optimal SU pairing. It can be seen that the OSA scheme is optimal for the power allocation with given $\boldsymbol{\rho}^*$ in the first step, which validates the theoretical analysis in Section III-C.

In Fig. 2 and Fig. 3, the impacts of subcarriers and multiusers on the system energy efficiency and the performance of the proposed SEEOSA are presented. We adopt the following methods as comparison for SEEOSA: (1) Joint globally Optimal (JO) [16]; (2) SEEEMMPA: SEE pairing combining EMMPA; (3) RAOSA: random SU pairing combining OSA; (4) RAEMMPA: random SU pairing combining EMMPA. As we can see that in Fig. 2, the system energy efficiency considering this elaborate power consumption model increases with the number of subcarriers and SEEOSA of low complexity achieves *near-optimal* performance. However, it is interesting to note that in Fig. 3, the system energy efficiency first increases and then decreases with the number of the users in the system, which seems contradictory to conclusions in [3]–[9]. In fact, it is the *performance trade-off* between the multiuser diversity and the system power consumption. Specifically, increasing the number of users in the OFDMA system obviously benefits the system throughput due to the more multiuser diversity gain, but also results in the additional system power consumption (RF circuit power), which are two key components of the system energy efficiency. The user number of the corner point in Fig. 3 can provide valuable insights into the designs of energy-efficient systems. In addition, we can observe that SEEOSA only suffers from a slight performance loss due to more complicated SU pairing possibilities and outperforms the random pairing based methods without exploiting the multiuser diversity, which further demonstrates the effectiveness of the proposed method.

V. Conclusion

In this paper, we establish an elaborate power dissipation model in OFDMA systems by relaxing three system assumptions of the previous works [3]–[9], and formulate the energy efficiency maximization problem. In order to fulfill the expectation of *green-oriented* designs, we propose a low complexity two-step method i.e., SEEOSA, based on the relationship analysis of the single SU pair energy efficiency and the system energy efficiency, and also provide a sufficient condition under which the SEEOSA method is globally optimal. Simulation results demonstrate that this efficient method exhibits a near-optimal performance compared with the optimal method of high complexity and show that our proposed power consumption model realistically reveals the impacts of subcarriers and multiusers on the system energy efficiency.

Appendix A
Proof of Theorem 1

Assume that the user k satisfies the properties in Theorem 1. Denote $EE^*(m,j)$ as the optimal energy efficiency of selecting any user j $(j \neq k)$ for subcarrier m. Let $\hat{p}_{i,j}$ and $\hat{\rho}_{i,j}$ be the corresponding power allocation and pairing indicator of selecting user j for subcarrier m, respectively. Denote $EE^*(m,k)$ as the optimal energy efficiency of selecting user k for the subcarrier m. Let $\check{p}_{i,j}$ and $\check{\rho}_{i,j}$ be the corresponding power allocation and pairing indicator of selecting user j for m, respectively. Then

$$
\begin{aligned}
&EE^*(m,j)\\
&= \frac{\sum_{i\neq m}\sum_{j=1}^{K}\hat{\rho}_{i,j}r_{i,j}(\hat{p}_{i,j}) + B\log_2\left(1+\frac{\hat{p}_{m,j}h_{m,j}}{N_0B}\right)}{\left(\sum_{i\neq m}\sum_{j=1}^{K}\hat{\rho}_{i,j}P_{i,j}(\hat{p}_{i,j})+P_0\right)+\left(\frac{\hat{p}_{m,j}}{\xi}+P_{s0}+P_{sj}\right)}\\
&\leqslant \frac{\sum_{i\neq m}\sum_{j=1}^{K}\hat{\rho}_{i,j}r_{i,j}(\hat{p}_{i,j}) + B\log_2\left(1+\frac{\hat{p}_{m,k}h_{m,k}}{N_0B}\right)}{\left(\sum_{i\neq m}\sum_{j=1}^{K}\hat{\rho}_{i,j}P_{i,j}(\hat{p}_{i,j})+P_0\right)+\left(\frac{\hat{p}_{m,k}}{\xi}+P_{s0}+P_{sk}\right)}\\
&\leqslant \frac{\sum_{i\neq m}\sum_{j=1}^{K}\check{\rho}_{i,j}r_{i,j}(\check{p}_{i,j}) + B\log_2\left(1+\frac{\check{p}_{m,k}h_{m,k}}{N_0B}\right)}{\left(\sum_{i\neq m}\sum_{j=1}^{K}\check{\rho}_{i,j}P_{i,j}(\check{p}_{i,j})+P_0\right)+\left(\frac{\check{p}_{m,k}}{\xi}+P_{s0}+P_{sk}\right)}\\
&= EE^*(m,k)
\end{aligned}
\tag{20}
$$

Hence, for $\forall j\,(j \neq k)$, we have $EE^*(m,j) \leqslant EE^*(m,k)$. Therefore, user k is the optimal user pairing with subcarrier m in achieving maximal system energy efficiency.

Appendix B
Proof of Theorem 2

Denote $\hat{p}_l^L$ as the optimal power allocations corresponding to EE_L^*. p_L^* is the optimal power corresponding to the Lth pair's energy efficiency ee_L^*.

1) If $EE_{L-1}^* \leqslant ee_L^*$,

$$
\begin{aligned}
EE_L^* &= \frac{\sum_{l=1}^{L} r_l(\hat{p}_l^L)}{\sum_{l=1}^{L} P_l(\hat{p}_l^L)+P_0}\\
&= \max_{\boldsymbol{p}} \frac{\sum_{l=1}^{L} r_l(p_l)}{\sum_{l=1}^{L} P_l(p_l)+P_0}\\
&\geqslant \frac{\sum_{l=1}^{L-1} r_l(\hat{p}_l^L)+r_L(p_L^*)}{\sum_{l=1}^{L-1} P_l(\hat{p}_l^L)+P_0+P_L(p_L^*)}\\
&\geqslant \min\left\{\frac{\sum_{l=1}^{L-1} r_l(\hat{p}_l^L)}{\sum_{l=1}^{L-1} P_l(\hat{p}_l^L)+P_0}, \frac{r_L(p_L^*)}{P_L(p_L^*)}\right\}\\
&= \min\left\{EE_{L-1}^*, ee_L^*\right\}\\
&= EE_{L-1}^*
\end{aligned}
\tag{21}
$$

In this case, we know that adding the Lth SU pair can obviously increase the system energy efficiency. On the other hand,

$$
\begin{aligned}
EE_L^* &= \max_{\boldsymbol{p}} \frac{\sum_{l=1}^{L} r_l(p_l)}{\sum_{l=1}^{L} P_l(p_l)+P_0}\\
&= \frac{\sum_{l=1}^{L-1} r_l(\hat{p}_l^L)+r_L(\hat{p}_L^L)}{\sum_{l=1}^{L-1} P_l(\hat{p}_l^L)+P_0+P_L(\hat{p}_L^L)}\\
&\leqslant \max\left\{\frac{\sum_{l=1}^{L-1} r_l(\hat{p}_l^L)}{\sum_{l=1}^{L-1} P_l(\hat{p}_l^L)+P_0}, \frac{r_L(\hat{p}_L^L)}{P_L(\hat{p}_L^L)}\right\}\\
&\leqslant \max\left\{\frac{\sum_{l=1}^{L-1} r_l(\hat{p}_l^{L-1})}{\sum_{l=1}^{L-1} P_l(\hat{p}_l^{L-1})+P_0}, \frac{r_L(p_L^*)}{P_L(p_L^*)}\right\}\\
&= \max\left\{EE_{L-1}^*, ee_L^*\right\}\\
&= ee_L^*
\end{aligned}
\tag{22}
$$

Based on (21) and (22), we have $EE_{L-1}^* \leqslant EE_L^* \leqslant ee_L^*$.

2) If $EE_{L-1}^* > ee_L^*$, we can prove $EE_{L-1}^* > EE_L^* > ee_L^*$ by a similar argument in the case 1). In this case, adding the Lth SU pair can not increase the system energy efficiency.

References

[1] J. Wu, S. Rangan, and H. Zhang, *Green Communications: Theoretical Fundamentals, Algorithms and Applications*. CRC Press, 2012.

[2] Y. Liu, J. Mo, and M. Tao, "Qos-aware transmission policies for OFDM bidirectional decode-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2206–2216, 2013.

[3] G. Miao, N. Himayat, Y. Li, and D. Bormann, "Energy efficient design in wireless OFDMA," in *Proc. IEEE ICC*, 2008, pp. 3307–3312.

[4] G. Miao, N. Himayat, G. Y. Li, and S. Talwar, "Low-complexity energy-efficient scheduling for uplink OFDMA," *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 112–120, 2012.

[5] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation in OFDMA systems with large numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, 2012.

[6] C. Xiong, G. Y. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-and spectral-efficiency tradeoff in downlink OFDMA networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 11, pp. 3874–3886, 2011.

[7] X. Xiao, X. Tao, and J. Lu, "QoS-aware energy-efficient radio resource scheduling in multi-user OFDMA systems," *IEEE Commun. Lett.*, vol. 17, no. 1, pp. 75–78, 2013.

[8] Z. Xu, C. Yang, G. Y. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient configuration of spatial and frequency resources in MIMO-OFDMA systems," *IEEE Trans. Commun.*, vol. 61, no. 2, pp. 564–575, 2013.

[9] S. Zhang, Y. Chen, and S. Xu, "Joint bandwidth-power allocation for energy efficient transmission in multi-user systems," in *Proc. IEEE GLOBECOM*, 2010, pp. 1400–1405.

[10] J. Wu, "Green wireless communications: from concept to reality," *IEEE Wireless Commun.*, vol. 19, no. 4, pp. 4–5, 2012.

[11] E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debbah, "Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?" *CoRR*, vol. abs/1403.6150, 2014.

[12] O. Arnold, F. Richter, G. Fettweis, and O. Blume, "Power consumption modeling of different base station types in heterogeneous cellular networks," in *Future Network and Mobile Summit*, 2010, pp. 1–8.

[13] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1089–1098, 2004.

[14] G. Miao, "Energy-efficient uplink multi-user MIMO," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2302–2313, 2013.

[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[16] Q. Wu, W. Chen, M. Tao, J. Li, H. Tang, and J. Wu, "Resource allocation for joint transmitter and receiver energy efficiency maximization in downlink OFDMA systems," *submitted to IEEE Trans. Commun.*