

Optimal Energy-Efficient Transmission for Fading Channels with an Energy Harvesting Transmitter

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Abstract—This paper investigates the optimal energy-efficient transmission policy of multi-channels in energy harvesting systems. We configure the transmitter with the active mode in which the energy cost includes the basic operation cost and transmission cost and signal processing cost, while with the sleep mode only counting the basic operation cost. Then the energy efficiency maximization problem of joint transmission time and power allocation is formulated and studied in the *off-line* manner. Based on the fractional optimization theory, we transform the original fractional optimization problem into a series of subtractive-form optimization problems which are then further transformed into convex optimization problems. Then characteristics of the optimal solution are described based on the analysis of transmission time and power allocation. Finally, a special case without considering the basic operation cost as previous works assumed is studied. We find that the optimal policy results a best subchannel scheduling which can be viewed as the *peaky transmission*. Through this, the energy cost of the sleep mode in previous case can be interpreted as the switching operation cost.

I. INTRODUCTION

Energy harvesting (EH) has emerged as an important technology in green communications due to its capability of collecting ambient energy sources, such as solar energy, vibration energy, thermoelectric energy, radio-frequency (RF) energy and so on [1]–[3]. Apart from that, the efficient utilization of harvested energy is particularly crucial towards energy-efficient communications. In this work, we aim to study energy-efficient transmission in energy harvesting communication systems.

Recently, a few attempts have been reported on transmission strategies in energy harvesting systems. The optimal packet scheduling for delay minimization and throughput maximization have been considered in an energy harvesting system with different settings including finite battery capacity, fading channel, broadcast channel, etc [2]–[6]. However, all these works only consider the energy consumption for the over-the-air transmission. In order to make the system assumptions more realistic, the optimal packet scheduling for energy harvesting systems considering the signal processing cost has been investigated in [7], [8]. Specifically in [8], the authors showed that the optimal transmission policy is an on/off

switching transmission instead of a continuous one in [2]–[6]. In other words, the system only transmits for a fractional time during each scheduling interval instead of transmitting all the time because when the transmission energy and signal processing energy are both drained from the harvested energy, it is not always profitable to transmit for a long time due to the additional processing cost. All these works did not explicitly consider the energy efficiency metric, i.e. bits per joule. Only in [9], the authors considered the energy efficiency of energy harvesting systems. Therein, the energy-efficient resource allocation was investigated in OFDMA systems with a hybrid powered base station. However, they still assume a continuous transmission all along each time interval which does not fully exploit the time domain from the perspective of energy efficiency. Nevertheless, the joint time and power control for the energy-efficient transmission in energy harvesting systems has not been studied.

In this paper, we study the *off-line* energy-efficient transmission policy in multichannel energy harvesting communication systems. The transmitter is configured with the active mode and the sleep mode. A stable power source (such as conventional batteries or electrical power grids) supplies constant energy to maintain the basic operation in both modes while the energy harvested from the ambience covers the energy demands for transmission and signal processing in the active mode. Then the energy efficiency maximization problem is formulated under energy causality constraints. We transform the fractional objective function into a parameterized subtractive-form equivalently which are then solved by the dual decomposition method. Additionally, characteristics of the optimal solution are described based on the analysis of the transmission time and the power allocation. Finally, we investigate the special case without considering the basic operation cost and we find that the optimal energy-efficient transmission policy results in a best subchannel transmission with a fixed power over a time region.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A. System Model

Consider an energy harvesting system with a transmitter and a receiver between which there exists N independent and parallel fading subchannels, which can be formed by some orthogonal techniques such as frequency division. The energy harvesting process in the transmitter is characterized by a

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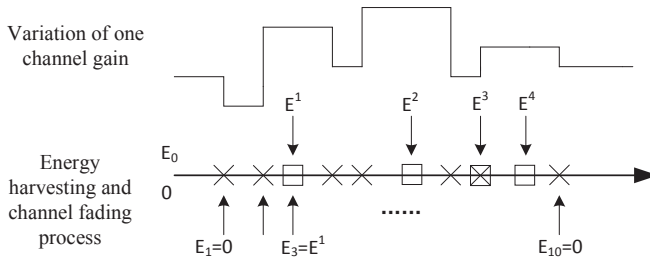


Fig. 1. An illustration of combined energy harvesting and channel fading process. Energy packet arrival events and channel fading change events are denoted as \square and \times , respectively. E^* represents the real energy packet and E_* represents the combined energy packets and channel changes.

packetized model with a deadline of T [2]–[6]. As previous works, we assume that the energy packet arrival times are given and denoted as $t_1^e = 0 < t_2^e \dots < t_k^e < T$ and the energy packets will be collected in a infinite battery [8], [10] before being used for transmission.

The scenario of finite battery capacity can also be solved by a slight modification with the algorithm developed in this paper. We denote the time instants where the fading channel changes as $t_1^f = 0 < t_2^f \dots < t_m^f < T$. For the convenience of expression, we combine all energy packet arrival events and the fading channel change events into a single time series as $t_1 = 0 < t_2 \dots < t_L < T$. Note that in the case that the channel changes without the energy packet arrival, the corresponding amount of virtual energy packet is zero, and in the case that the energy packet arrives without fading channel changes, the channel gain remains constant as the previous time interval. The time interval between two consecutive events is denoted as an epoch, i.e., $\tau_\ell = t_{\ell+1} - t_\ell$ and $t_{L+1} = T$. The corresponding energy packet arriving at instant t_ℓ and the channel gain during the epoch ℓ at the channel n are denoted as E_ℓ and $g_{\ell,n}$, respectively. Since the physical layer is the fading channels with additive white Gaussian noise of unit variance, the instantaneous achievable rate of the channel n during the epoch ℓ can be expressed as [2]

$$r_{\ell,n} = \frac{1}{2} \log_2(1 + p_{\ell,n} g_{\ell,n}), \quad (1)$$

where $p_{\ell,n}$ is the transmission power during the epoch ℓ on the subchannel n .

B. Energy Supply and Consumption Model

Previous works have shown that much energy can be saved if the transmitter can switch between the active and the sleep mode [1], [11]. Thus, we assume that the transmitter has these two modes. In the sleep mode, there is only constant power consumption for basic operations of the harvesting transmitter denoted as p_b . While in the active mode, the power consumption includes three parts, namely, the transmission power, the signal processing power and the basic operation power. Without loss of generality, the basic operational power in the active mode and in the sleep mode are assumed to be same. The signal processing power denoted as p_s is also

assumed to be constant for each subchannel in the active mode. Since the energy harvesting process is random, which makes it possible that the energy harvested from nature can not support the basic operation energy consumption, a hybrid energy supply is assumed for the harvesting transmitter. Specifically, the energy consumption for the basic operation power p_b is supported by the stable non-renewable power source [1], which also facilitates the management of it if there are large numbers of devices. In addition, the energy consumption for the transmission and the signal processing is supplied by the energy harvested from the ambience.

On the contrary to previous works, the optimal transmission policy may be not to transmit continuously during the whole epoch [9]. This is due to that the signal processing power as well as transmission power is also drained from the harvesting energy, which makes it non-profitable to transmit for a long time. Thus, we denote the transmission time during the epoch ℓ for subchannel n as $\theta_{\ell,n}$, i.e., $0 \leq \theta_{\ell,n} \leq \tau_\ell$. Due to the fading independence of parallel channels, the active time for one subchannel is not necessarily the same as others during the same epoch. Therefore, the overall throughput of all subchannels during the time T is

$$B_{tot} = \sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} r_{\ell,n}. \quad (2)$$

The fundamental characteristic of energy harvesting systems is that the energy can not be used before harvesting from nature and it can be described by the so called energy causality constraint

$$\sum_{i=1}^{\ell} \sum_{n=1}^N \theta_{i,n} (p_{i,n} + p_s) \leq \sum_{i=1}^{\ell} E_i, \ell = 1, 2, \dots, L. \quad (3)$$

In addition, the overall energy consumption of considered harvesting systems is

$$E_{tot} = \sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} (p_{\ell,n} + p_s) + \sum_{\ell=1}^L \tau_\ell p_b. \quad (4)$$

We denote the constant item $\sum_{\ell=1}^L \tau_\ell p_b$ as E_b in the following.

C. Problem Formulation

Energy efficiency is defined as the overall system bits B_{tot} transmitted over the overall system energy E_{tot} consumed, i.e.,

$$\eta_{ee} = \frac{B_{tot}}{E_{tot}}. \quad (5)$$

Mathematically, we can formulate the energy efficiency optimization problem as

$$\max_{\theta, p} \quad \eta_{ee} = \frac{\sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} r_{\ell,n}}{\sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} (p_{\ell,n} + p_s) + E_b} \quad (6a)$$

$$\text{s.t.} \quad \sum_{i=1}^{\ell} \sum_{n=1}^N \theta_{i,n} (p_{i,n} + p_s) \leq \sum_{i=1}^{\ell} E_i, \forall \ell, \quad (6b)$$

$$0 \leq \theta_{\ell,n} \leq \tau_\ell, \forall \ell, n, \quad (6c)$$

$$p_{\ell,n} \geq 0, \forall \ell, n. \quad (6d)$$

where $\boldsymbol{\theta} \triangleq \{\theta_{\ell,n} | \ell = 1, 2, \dots, L; n = 1, 2, \dots, N\}$ and $\mathbf{p} \triangleq \{p_{\ell,n} | \ell = 1, 2, \dots, L; n = 1, 2, \dots, N\}$.

Note that the work in [9] assumes $\theta_{\ell,n} = \tau_\ell$, which is only a suboptimal solution of problem (6). Clearly, problem (6) is not a convex optimization problem since the objective function is not jointly concave with respect to $\{\boldsymbol{\theta}, \mathbf{p}\}$ and energy causality constraints (6b) do not span a convex set either. Thus, there is no standard method to solve non-convex problems. However, thanks to the theory of fractional programming, we can first transform the objective function of problem (8) into an equivalent one, which make it possible for deriving efficient methods.

III. OPTIMAL ENERGY-EFFICIENT TRANSMISSION POLICY

A. Problem Transformation

Denote η_{ee}^* as the maximal energy efficiency of the considered system, i.e.,

$$\eta_{ee}^* = \frac{B_{tot}(\boldsymbol{\theta}^*, \mathbf{p}^*)}{E_{tot}(\boldsymbol{\theta}^*, \mathbf{p}^*)}. \quad (7)$$

According to the nonlinear fractional programming theory [12], for the problem with the following form,

$$\max_{\boldsymbol{\theta}, \mathbf{p}} \frac{B_{tot}(\boldsymbol{\theta}, \mathbf{p})}{E_{tot}(\boldsymbol{\theta}, \mathbf{p})}. \quad (8)$$

there exists a corresponding subtractive-form problem as follows:

$$T(\eta_{ee}) = \max_{\boldsymbol{\theta}, \mathbf{p}} B_{tot}(\boldsymbol{\theta}, \mathbf{p}) - \eta_{ee} E_{tot}(\boldsymbol{\theta}, \mathbf{p}) = 0. \quad (9)$$

It is easy to verify the equivalence of (8) and (9) at the optimal point $(\boldsymbol{\theta}^*, \mathbf{p}^*)$ with corresponding maximal value η_{ee}^* . Note that η_{ee}^* is also the optimal system energy efficiency to be determined. Dinkelbach [12] provides a method to iteratively update η_{ee} . In each iteration, it solves a subtractive-form maximization problem (9) with a given η_{ee} and then judge whether it converges. If not, update η_{ee} and repeat the maximization problem (9) until it converges or reaches the maximal iterations. Nevertheless, after this transformation, we can focus on the following problem with a given η_{ee} in each iteration. By taking (2) and (4) into (9), we can obtain

$$\begin{aligned} \max_{\boldsymbol{\theta}, \mathbf{p}} \quad & \sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} \frac{1}{2} \log_2(1 + p_{\ell,n} g_{\ell,n}) - \\ & \eta_{ee} \left(\sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} (p_{\ell,n} + p_s) + E_b \right) \\ \text{s.t.} \quad & (6b), (6c), (6d). \end{aligned} \quad (10)$$

B. Dual Decomposition based Solution

Note that problem (10) is still not convex programming problem due to the nonconcavity of the objection function. Here, we introduce an auxiliary variable $\tilde{p}_{\ell,n} = \theta_{\ell,n} p_{\ell,n}$, which is exactly the energy consumed for transmitting signals in channel n during the epoch ℓ . By substituting $p_{\ell,n}$ with

$\tilde{p}_{\ell,n}/\theta_{\ell,n}$, problem (11) is transformed into the following one,

$$\max_{\boldsymbol{\theta}, \mathbf{p}} \quad \sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} \frac{1}{2} \log_2(1 + \frac{\tilde{p}_{\ell,n}}{\theta_{\ell,n}} g_{\ell,n}) - \eta_{ee} \left(\sum_{\ell=1}^L \sum_{n=1}^N (\tilde{p}_{\ell,n} + \theta_{\ell,n} p_s) + E_b \right) \quad (11a)$$

$$\text{s.t.} \quad \sum_{i=1}^{\ell} \sum_{n=1}^N (\tilde{p}_{i,n} + \theta_{i,n} p_s) \leq \sum_{i=1}^{\ell} E_i, \quad \forall \ell, \quad (11b)$$

$$0 \leq \theta_{\ell,n} \leq \tau_\ell, \quad \forall \ell, n, \quad (11c)$$

$$\tilde{p}_{\ell,n} \geq 0, \quad \forall \ell, n, \quad (11d)$$

After this substitution, we can verify that problem (11) is a standard convex optimization problem, which implies that we can employ the KKT conditions to solve it. The Lagrangian function for (11) is

$$\begin{aligned} \mathcal{L}(\boldsymbol{\theta}, \tilde{\mathbf{p}}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = & \sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} \frac{1}{2} \log_2(1 + \frac{\tilde{p}_{\ell,n}}{\theta_{\ell,n}} g_{\ell,n}) \\ & - \eta_{ee} \left(\sum_{\ell=1}^L \sum_{n=1}^N (\tilde{p}_{\ell,n} + \theta_{\ell,n} p_s) + E_b \right) \\ & + \sum_{\ell=1}^L \lambda_\ell \left(\sum_{i=1}^{\ell} E_i - \sum_{i=1}^{\ell} \sum_{n=1}^N (\tilde{p}_{i,n} + \theta_{i,n} p_s) \right) \\ & + \sum_{\ell=1}^L \sum_{n=1}^N \mu_{\ell,n} (\tau_\ell - \theta_{\ell,n}), \end{aligned} \quad (12)$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the Lagrange dual vectors corresponding to (11b) and (11c). Note that the non-negative boundary constraints with respect to $\theta_{\ell,n}$ and $p_{\ell,n}$ will be absorbed into the optimal solution in the following. Then the associated dual function is

$$g(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \begin{cases} \max_{\boldsymbol{\theta}, \tilde{\mathbf{p}}} & \mathcal{L}(\boldsymbol{\theta}, \tilde{\mathbf{p}}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{s.t.} & \tilde{p}_{\ell,n} \geq 0, \theta_{\ell,n} \geq 0, \quad \forall \ell, n. \end{cases} \quad (13)$$

From (13), we have removed coupling constraints and $g(\boldsymbol{\lambda})$ is decomposed into N subproblems which can be solved in each sub-channel independently with given $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. The subproblem associated with the channel n is

$$\begin{aligned} \max_{\boldsymbol{\theta}, \tilde{\mathbf{p}}} \quad & \mathcal{L}_n = \sum_{\ell=1}^L \theta_{\ell,n} \frac{1}{2} \log_2(1 + \frac{\tilde{p}_{\ell,n}}{\theta_{\ell,n}} g_{\ell,n}) \\ & - \eta_{ee} \sum_{\ell=1}^L (\tilde{p}_{\ell,n} + \theta_{\ell,n} p_s) - \sum_{\ell=1}^L \lambda_\ell \sum_{i=1}^{\ell} (\tilde{p}_{i,n} + \theta_{i,n} p_s) \\ & - \sum_{\ell=1}^L \mu_{\ell,n} \theta_{\ell,n} \end{aligned} \quad (14a)$$

$$\text{s.t.} \quad \tilde{p}_{\ell,n} \geq 0, \theta_{\ell,n} \geq 0, \quad \forall \ell, n. \quad (14b)$$

Note that problem (14) now is a concave problem only with boundary constraints. By taking the derivative of \mathcal{L}_n with respect to $\theta_{\ell,n}$ and $\tilde{p}_{\ell,n}$, respectively, we can get the

transmission time and power allocation, i.e.,

$$\frac{\partial \mathcal{L}_n}{\partial \theta_{\ell,n}} = \frac{1}{2} \log_2 \left(1 + \frac{\tilde{p}_{\ell,n}^*}{\theta_{\ell,n}^*} g_{\ell,n} \right) - \frac{\tilde{p}_{\ell,n}^* g_{\ell,n}}{2(\theta_{\ell,n}^* + \tilde{p}_{\ell,n}^* g_{\ell,n}) \ln 2} - \eta_{ee} p_s - \sum_{i=\ell}^L \lambda_i p_s - \mu_{\ell,n} = 0. \quad (15)$$

$$\frac{\partial \mathcal{L}_n}{\partial \tilde{p}_{\ell,n}} = \frac{\theta_{\ell,n}^* g_{\ell,n}}{2(\theta_{\ell,n}^* + \tilde{p}_{\ell,n}^* g_{\ell,n}) \ln 2} - \eta_{ee} - \sum_{i=\ell}^L \lambda_i = 0. \quad (16)$$

With given λ_ℓ , $\mu_{\ell,n}$ and η_{ee} , $p_{\ell,n}^*$ and $\theta_{\ell,n}^*$ can be obtained by (15) and (16). Note that if the solution $p_{\ell,n}^* < 0$ in (16), then $p_{\ell,n}^* = 0$ and similarly for $\theta_{\ell,n}^*$ in (15) due to the concavity of \mathcal{L}_n with respect to $p_{\ell,n}$ and $\theta_{\ell,n}$ [13].

After computing $p_{\ell,n}$ and $\theta_{\ell,n}$, we now solve the standard dual optimization problem which is

$$\min_{\lambda, \mu} g(\lambda, \mu) \quad (17a)$$

$$\text{s.t.} \quad \lambda_\ell \geq 0, \mu_{\ell,n} \geq 0, \quad \forall \ell, n. \quad (17b)$$

Since the dual problem is always a convex optimization problem by definition, the commonly used ellipsoid method can be employed to update λ and μ toward the optimal solution with global convergence. According to (13), the required gradient for the ellipsoid method is given as follows

$$\Delta \lambda_\ell = \sum_{i=1}^{\ell} E_i - \sum_{i=1}^{\ell} \sum_{n=1}^N (\tilde{p}_{i,n} + \theta_{i,n} p_s), \quad \forall \ell. \quad (18)$$

$$\Delta \mu_{\ell,n} = \tau_\ell - \theta_{\ell,n}, \quad \forall \ell, n. \quad (19)$$

The details of the updating the ellipsoid and the step size can be found in [13], [14]. Since the transformed subtractive-form problem (11) is a standard concave optimization problem and satisfies the Slater's qualification [13], the duality gap between (11) and (17) is zero, which leads to the optimal solution of the primal problem. The details of the energy-efficient transmission policy is finally summarized in Algorithm 1.

C. Characteristics of the Optimal Transmission Policy

In subsection A and B, we provide the procedure summarized in Algorithm 1 to obtain the optimal solution to problem (6). In order to reveal more insight into the energy-efficient transmission policy, two theorems are given in the following based on the analysis of (14), (15) and (16).

a) When $\theta_{\ell,n}^* = 0$, then $\tilde{p}_{\ell,n}^* = 0$ and $p_{\ell,n}^* = 0$, which means that the channel n during the epoch ℓ is not scheduled.

b) When $0 < \theta_{\ell,n}^* < \tau_\ell$, from the complementary slackness condition [13] we know that the associated $\mu_{\ell,n} = 0$. Then by simple manipulations of (15) and (16), we have

$$\log_2 \left(1 + \frac{\tilde{p}_{\ell,n}^*}{\theta_{\ell,n}^*} g_{\ell,n} \right) = \frac{g_{\ell,n} (\tilde{p}_{\ell,n}^* + \theta_{\ell,n}^* p_s)}{(\theta_{\ell,n}^* + \tilde{p}_{\ell,n}^* g_{\ell,n}) \ln 2}. \quad (20)$$

Replacing $\tilde{p}_{\ell,n}^*$ with $\theta_{\ell,n}^* p_{\ell,n}^*$, we can get

$$\log_2 (1 + p_{\ell,n}^* g_{\ell,n}) = \frac{g_{\ell,n} (p_{\ell,n}^* + p_s)}{(1 + p_{\ell,n}^* g_{\ell,n}) \ln 2}. \quad (21)$$

Algorithm 1 Optimal energy-efficient transmission policy for energy harvesting systems

- 1: **Initialization:** Given the initial $\eta_{ee} = 0$, $n = 0$ and the maximal tolerance ϵ ;
- 2: **Repeat**
- 3: Initialize λ and μ ;
- 4: Compute θ^* and \tilde{p}^* by (15) and (16) with given η_{ee} , λ and μ ;
- 5: Update dual variables λ , μ and the ellipsoid via ellipsoid method with (18) and (19);
- 6: Repeat steps 4-5 until convergence;
- 7: **If** $T(\eta_{ee}) > \epsilon$
- 8: Update $\eta_{ee} = \frac{B_{tot}(\theta^*, p^*)}{E_{tot}(\theta^*, p^*)}$;
- 9: **else** Convergence
- 10: $\theta^*, p^*, \eta_{ee}^*$.
- 11: **return**
- 12: **EndIf**
- 13: **End**

Denote this special solution as $p_{\ell,n}^* = v_{\ell,n}^*$. From (20), we can explicitly observe that the optimal transmission power $p_{\ell,n}^*$ only depends on the signal processing power p_s , the channel gain $g_{\ell,n}$. This is due to that if the energy available in this epoch is not sufficient enough to activate the channel n all along the whole epoch with $p_{\ell,n}^*$, i.e., $\mu_{\ell,n} = 0$, it is preferable to transmit for a fractional time $\theta_{\ell,n}^*$ followed by the sleep mode rather than all along the epoch in the active mode with a lower transmission power, which leads to much energy cost on signal processing.

c) When $\theta_{\ell,n}^* = \tau_\ell$, then $\mu_{\ell,n} > 0$. Similarly by (15) and (16), we have

$$\frac{\log_2 (1 + p_{\ell,n}^* g_{\ell,n})}{2} = \frac{g_{\ell,n} (p_{\ell,n}^* + p_s)}{2(1 + p_{\ell,n}^* g_{\ell,n}) \ln 2} + \mu_{\ell,n}. \quad (22)$$

Comparing (21) and (22), we can conclude that the optimal transmission power $p_{\ell,n}^*$ in this case is larger than $v_{\ell,n}^*$ in the case b) due to their monotonic property of $p_{\ell,n}^*$. Additionally, from (16) $p_{\ell,n}^*$ is given as

$$p_{\ell,n}^* = \frac{1}{2(\eta_{ee} + \sum_{i=\ell}^L \lambda_i) \ln 2} - \frac{1}{g_{\ell,n}}. \quad (23)$$

The interpretation is that the energy available in this case is sufficient enough to activate the channel n all along the epoch ℓ with $v_{\ell,n}^*$, and the additional energy is used to enhance the power level leading to a larger $p_{\ell,n}^*$.

Theorem 1: In the optimal energy-efficient transmission policy, all channels with positive energy allocated between two positive energy packet arrival instants have the same water level.

Proof: From (23), we can see that if $p_{\ell,n}^* > 0$ we have

$$p_{\ell,n}^* + \frac{1}{g_{\ell,n}} = \frac{1}{2(\eta_{ee} + \sum_{i=\ell}^L \lambda_i) \ln 2}. \quad (24)$$

Firstly, since all channels in the same epoch are associated

with the same λ_ℓ in (12), then from (24) we know that they must have the same water level. Secondly from Fig. 1, there may be many fading events, i.e., many epochs, between two energy packets events. For those epochs, since $E_\ell = 0$ then the corresponding λ_ℓ is also zero. From (24), we can see all channels among epochs of fading events (zero energy arrival events) also have the same water level. ■

Theorem 2: In the optimal energy-efficient transmission policy, whenever the water level of any channel n , i.e., $p_{\ell,n} + \frac{1}{g_{\ell,n}}$ increases from one epoch to the next, $p_{\ell,n}$, the energy depletes up to that instant.

Proof: When

$$p_{\ell+1,n} + \frac{1}{g_{\ell+1,n}} > p_{\ell,n} + \frac{1}{g_{\ell,n}}. \quad (25)$$

From the right side of (24), we can see that there must be $\lambda_\ell = 0$. Due to the complementary slackness condition, we conclude that the energy causality constraint in the epoch ℓ must be strictly satisfied which means that the energy depletes up completely in the epoch ℓ . ■

IV. A SPECIAL CASE : WHEN $p_b = 0$

In this section, we investigate the energy efficiency in energy harvesting systems when the basic power component $p_b = 0$ or it is not taken into account in the problem formulation as what in [2]–[10]. By exploring the special structure of (6), we have the following theorem.

Theorem 3: If the channel j in the epoch k has the highest channel gain among all the channels during all epochs, then only activating this subchannel in the active mode with transmission power $v_{k,j}^*$ for a time region $(0, \min(\tau_\ell, \frac{\sum_{i=1}^L E_i}{v_{k,j}^* + p_s})]$ must be one of optimal solutions and $v_{k,j}^*$ is given by (21) and the maximal system energy efficiency can be achieved by the energy efficiency of this subchannel.

Proof: Please refer to Appendix A. ■

We can see that if there is no basic energy consumption, it is more profitable to solely activate the best subchannel. Similar to the *bursty transmission* [15] via considering the processing cost in throughput maximization systems, this kind of phenomenon in energy efficient harvesting systems can be viewed as the *peaky transmission*. The optimal transmission time of this scenario is not a fixed value but a variable one, which means that we can turn on or turn off the transmitter many times without changing the energy efficiency. However, in practical scenario, the on/off switching will also lead to a certain energy cost, then the energy cost in the sleep mode configured in this paper can be viewed as a representation of it.

Theorem 4: If there are multiple subchannels with the same channel gain g_{\max} in some epochs, then problem (6) has multiple solutions and the optimal solutions are to transmit over these subchannels with a fixed power v^* among a polyhedron time region composed of the transmission time of each subchannel.

Note that if there are some subchannels during all L epochs having the same highest channel gain, it is also optimal to

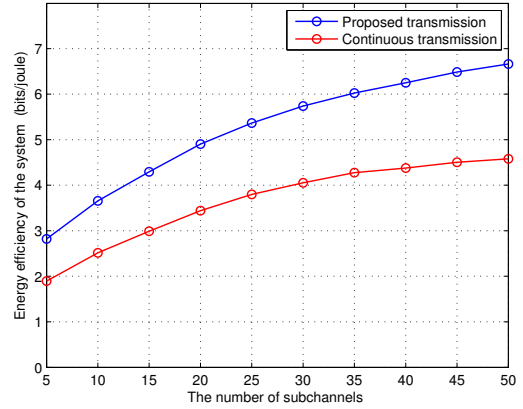


Fig. 2. The performance comparison vs the number of subchannels.

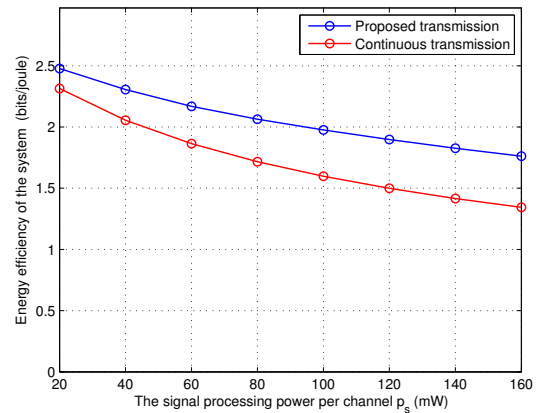


Fig. 3. The performance comparison vs the signal processing power p_s .

activate a *subset* of all the subchannels in a time region with the power level v^* . Due to the limited space, the rigorous proof of this theorem will be put into the journal version of this paper.

V. NUMERICAL RESULTS

In this section, we provide numerical results to compare the system energy efficiency of the proposed transmission with the continuous transmission [9] in different scenarios. We consider the Rayleigh fading among four independent subchannels by default. The energy packet arrival events and fading events are uniformly distributed along $T = 10$ seconds. The energy packets are randomly generated in a uniform distribution with the parameter of 10 Joule. The signal processing power and basic operation power are 80 mW and 20 mW without specific explanation. The performance comparison between the proposed transmission policy and the continuous transmission policy is illustrated in Fig. 2. We can clearly see that the system energy efficiency increases with the number of channels goes up due to the frequency diversity.

In Fig. 3, the system optimal energy efficiency of both methods decreases as the signal processing power increases as

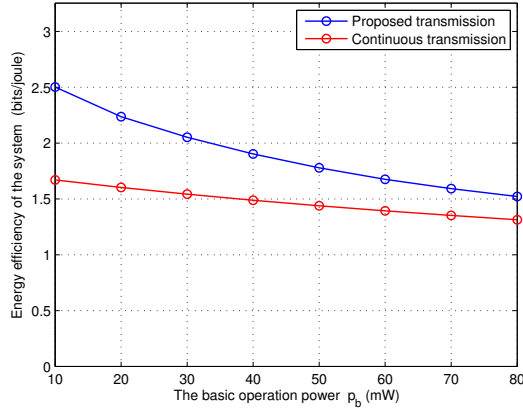


Fig. 4. The performance comparison vs the basic operation power p_b .

expected. Additionally, we can observe that the gap between them increases with p_s . The reason is that with increasing p_s , the signal processing consumption becomes more dominant in the system power consumption which requires more energy-efficient utilization of transmission time and power, i.e., energy.

In Fig. 4, both of their optimal system energy efficiency decreases with the increasing of basic operation power p_b . Unlike the results in Fig. 3, the gap between these two transmission policies decreases as p_b increases. This is because the larger basic operation cost entices more transmission time to transmit signals in order to improve the system energy efficiency which leads the convergence of both methods.

VI. CONCLUSIONS

In this paper, we have investigated the energy efficiency maximization problem of the joint transmission time and power control in energy harvesting systems. Based on the equivalent transformation, we solve the energy efficiency maximization problem by dual decomposition method. Then characteristics of the optimal solution are presented based on the analysis of transmission time and power allocation. We show that whenever a subchannel is activated for a strictly fractional time, its corresponding transmission power is a unique value only dependent of signal processing power and channel gain. As a special case, we finally study the energy efficiency of energy harvesting systems without considering the basic operation cost. We have found it results in a *peaky transmission* policy where only the best subchannel will be activated by a fixed power for a variable time region in achieving the maximal system energy efficiency.

APPENDIX A PROOF OF THEOREM 3

Assuming that the channel n in epoch ℓ is with the largest channel gain among all channels during all epochs, i.e., $g_{i,j} = \max_{\ell,n} g_{\ell,n}$ for $\ell \in \{1, 2, \dots, L\}$ and $n \in \{1, 2, \dots, N\}$. Then, the

maximal system energy efficiency

$$\begin{aligned}
 \eta_{ee} &= \frac{\sum_{\ell=1}^L \sum_{n=1}^N \frac{\theta_{\ell,n}}{2} \log_2(1 + p_{\ell,n} g_{\ell,n})}{\sum_{\ell=1}^L \sum_{n=1}^N \theta_{\ell,n} (p_{\ell,n} + p_s)} \\
 &\leq \max_{\ell,n} \frac{\frac{\theta_{\ell,n}}{2} \log_2(1 + p_{\ell,n} g_{\ell,n})}{\theta_{\ell,n} (p_{\ell,n} + p_s)} \\
 &\leq \max_{\ell,n} \frac{\frac{1}{2} \log_2(1 + p_{\ell,n} g_{\ell,n})}{(p_{\ell,n} + p_s)} \\
 &= \frac{\frac{1}{2} \log_2(1 + v_{k,j}^* g_{k,j})}{v_{k,j}^* + p_s} \quad (26)
 \end{aligned}$$

where $v_{k,j}^*$ can be obtained by taking the derivative of $\frac{\frac{1}{2} \log_2(1 + p_{k,j} g_{k,j})}{(p_{k,j} + p_s)}$ with respect to $p_{k,j}$, which results the same expression as (21). In addition, it is interesting to note that $\frac{\frac{1}{2} \log_2(1 + v_{k,j}^* g_{k,j})}{(v_{k,j}^* + p_s)}$ is exactly the energy efficiency of the subchannel j during epoch k . Then the energy causality constraints (6b) shrink to only one constraint and we can easily get the transmission time region as $(0, \min(\tau_\ell, \frac{\sum_{i=1}^{\ell} E_i}{v_{k,j}^* + p_s})]$.

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