

# Coordinated Beamforming for Wireless Multicast Cell with Nonregenerative Multi-Antenna Relay

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**Abstract**—This paper considers the wireless multicast cell, in which two source nodes with multiple antennas communicate with two destinations over a common medium simultaneously assisted by a multi-antenna relay node. Assume that the sources and relay have individual power constraints. The purpose of this work is to develop an optimal algorithm to compute the source beamforming vectors and the relay beamforming matrix to maximize the system sum-rate, subject to the power constraints in the sources and relay. The problem is nonconvex and apparently has no simple solution. In this paper, We first develop the beamforming vectors at the sources, and then explore the linear beamforming matrix for the nonregenerative relay. Finally, we present a coordinated optimal algorithm to jointly optimize the vectors and matrix, which performance is compared with the conventional methods. Simulations demonstrate that our algorithm outperforms the conventional methods.

## I. INTRODUCTION

Recently, there has been considerable interest in multiple-input, multiple-output (MIMO) communications in the wireless relay systems, due to their potential for dramatic gains in channel capacity and increasing the coverage of the wireless communications under power and spectral constraints. To date, most works on relay network have focused on the three nodes model [1], where the presence of other co-channel user is not considered. More recently, attention has been shifted to two-way relay channels, where two sources exchange informations via a relay based on network coding techniques [2] [3]. However, all these works focus on the point-to-point or multi-access model. The studies on multicast model [4] [5] are very rare in the literatures. However, the multicast topology is very popular in practical wireless networks, such as video on demand, and information gathering in wireless sensor networks. Hence, it is desirable to investigate the performance in a wireless relay multicast cell (see Fig.1).

A wireless relay can be classified as regenerative and nonregenerative relay. The regenerative relay requires digital decoding and re-encoding at the relay, which will cause a significant increase of delay and complexity. However, the nonregenerative relay does not need any digital decoding and re-encoding at the relay, and only performs linear processing (amplifying), which has advantage over the regenerative relay [6] in the term of complexity. In this paper, we consider the nonregenerative relay in our wireless multicast cell, where the relay applies a beamforming matrix to process the received signal and then forward it to both destinations. There have

been many research efforts on nonregenerative MIMO relay systems [6]. The MIMO two-way relay formulation in [3] includes the analog network coding relay problem as a special case. In [4], the authors have studied the power allocation for multicast cell with regenerative network coding in the high SNR region. To the best of our knowledge, there is no any research on the relay beamforming matrix in the wireless multicast cell.

In this paper, we focus on the design of beamforming vectors at the sources and the beamforming matrix at the non-regenerative relay, where the sources and relay are equipped with two antennas, while the destinations only have one receive antenna. We are going to find the optimal beamforming vectors at the sources and the beamforming matrix at the relay to maximize the system sum-rate subject to the power constraints at the sources and relay under the assumption that the channel state informations (CSIs) are available at the sources and relay. By making good use of the related CSIs, the sources and relay are able to carry out some further signal shaping. Consequently, the sources can carry out transmit beamforming, and the relay is no longer simple amplifying and forwarding the received signal, or decoding and re-encoding the signals transmitted from the sources. The processing at relay is a transceiving beamforming. To exploit the optimal beamforming vectors at sources and matrix at relay, we propose an algorithm in which the sources and relay coordinate to each other to maximize the system sum-rate. First, the sources process the signals as a multi-user multicast system [7]. Then the relay process the received signals as a point-to-point MIMO relay [8] combined with two-way MIMO relay [3] system treating sources' beamforming vectors as known. Finally, the sources and relay jointly optimize the vectors and matrix.

## II. SYSTEM MODEL

We first consider the downlink of the multicast relay system with two sources, one relay and two destinations (2-1-2 mode) [4] as illustrated in Fig. 1, where the sources and relay are equipped with two antennas, but each destination only has one antenna. It is easy to be extended to a general multiple antenna relay system.

Suppose that both  $S_1$  and  $S_2$  transmit their messages to the same destination set  $\{D_1, D_2\}$  simultaneously. We assume that the crossover direct links between the source  $S_i$  and

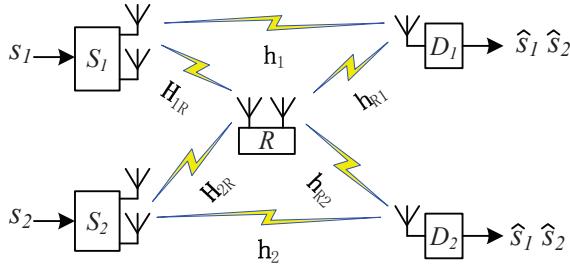


Fig. 1. The multicast cell with two sources, one relay, and two destinations

destination  $D_j$  ( $i, j = 1, 2, i \neq j$ ) are very weak and negligible due to the transmission range. The shared relay is used to assist the transmission from sources to destinations. When the relay is half-duplex, the transmission process takes two steps, i.e.,

1.  $S_1 \rightarrow \{R, D_1\}$  with  $X_{S_1}$ ;  $S_2 \rightarrow \{R, D_2\}$  with  $X_{S_2}$ ,
2.  $R \rightarrow \{D_1, D_2\}$  with  $f(X_{S_1}, X_{S_2})$ ,

where  $f(\cdot)$  denotes the processing protocol at the relay. There are two basic protocols: (A). Nonregenerative processing, in which the mixed signals from the two sources are not decoded before retransmission to the destinations [1], [4]; (B). Regenerative processing, in which the relay can decode both  $S_1$  and  $S_2$ 's signals, and then do network coding in Complex field or Galois field before retransmission [4]. The signal vector transmitted by source  $S_i$  is

$$\mathbf{x}_i(n) = \mathbf{w}_i s_i(n), \quad (1)$$

where  $s_i(n)$  is the complex symbol sent by source  $S_i$  using the beamforming vector  $\mathbf{w}_i \in \mathbb{C}^{2 \times 1}$ . We assume that  $s_i(n)$  is a zero mean, unit covariance circularly symmetric complex Gaussian signal,  $n$  is the symbol index, i.e.  $s_i(n) \sim \mathcal{CN}(0, 1)$ . The transmit power of the source  $S_i$  can be shown equal to

$$p_i = \mathbb{E}_n \{ \mathbf{x}_i(n) \mathbf{x}_i^H(n) \} = \| \mathbf{w}_i \|^2 \leq P_i. \quad (2)$$

Note that  $p_i$  is the allocated transmit power at  $S_i$ , and  $P_i$  is the power constraint at  $S_i$  ( $i = 1, 2$ ).

All the channels involved are assumed to be flat-fading over a common narrow-band. It is assumed that the transmission protocol uses two consecutive equal-duration time slots for one round of information transmission. It is also assumed that perfect synchronization has been established among  $S_1, S_2$  and  $R$  prior to data transmission. During the first time-slot, the transmission from the sources to destination, and to relay can be modeled as the standard point-to-point MISO, and MIMO channel, respectively. Therefore, the received baseband signal at the destination and relay in the first time-slot are respectively,

$$y_{11}(n) = \mathbf{h}_1 \mathbf{x}_1(n) + z_1(n), \quad (3a)$$

$$y_{21}(n) = \mathbf{h}_2 \mathbf{x}_2(n) + z_2(n), \quad (3b)$$

$$\mathbf{y}_R(n) = \mathbf{H}_{1R} \mathbf{x}_1(n) + \mathbf{H}_{2R} \mathbf{x}_2(n) + \mathbf{z}_R(n), \quad (3c)$$

where  $y_{ij}(n) \in \mathbb{C}$  is the received signal by  $D_i$  in the  $j$ th time-slot at symbol index  $n$ ,  $n = 1, \dots, N$  with  $N$  denoting the total number of transmitted symbols during the first time-slot,  $\mathbf{y}_R(n) \in \mathbb{C}^{2 \times 1}$  is the received signal vector by  $R$ ,  $\mathbf{h}_i \in \mathbb{C}^{1 \times 2}$ , and  $\mathbf{H}_{iR} \in \mathbb{C}^{2 \times 2}$  represent the channel vectors from  $S_i$  to  $D_i$ , and channel matrix from  $S_i$  to  $R$ , respectively, which are assumed to be constant during the first time-slot,  $z_i(n) \in \mathbb{C}$  and  $\mathbf{z}_R(n) \in \mathbb{C}^{2 \times 1}$  are the receiver noise, independent of  $n$ . Without loss of generality, assume that  $z_i(n) \sim \mathcal{CN}(0, 1)$ , ( $i = 1, 2$ ) and  $\mathbf{z}_R(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ ,  $\forall n$ .

As mentioned above, the relay have two options: nonregenerative processing or regenerative processing. In this paper, we only consider linear nonregenerative processing at relay, and hence the forward signal from  $R$  can be concisely represented as

$$\mathbf{x}_R(n) = \mathbf{A} \mathbf{y}_R(n), \quad n = 1, \dots, N, \quad (4)$$

where  $\mathbf{x}_R(n) \in \mathbb{C}^{2 \times 1}$  denotes the transmitted signal by  $R$ , and  $\mathbf{A} \in \mathbb{C}^{2 \times 2}$  is the relay processing matrix. Note that, the transmit power of  $R$  can be expressed as

$$\begin{aligned} p_R &= \text{tr}(\mathbb{E}_n [\mathbf{x}_R(n) \mathbf{x}_R(n)^H]) \\ &= \text{tr} \left( \mathbf{A} \left( \sum_{i=1}^2 \mathbf{H}_{iR} \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_{iR}^H + \mathbf{I} \right) \mathbf{A}^H \right) \leq P_R. \end{aligned} \quad (5)$$

The transmission from  $R$  to both destinations can be modeled as traditional MISO-BC. Thus, the received signals at  $D_1$  and  $D_2$  in the second time-slot can be expressed as

$$\begin{aligned} y_{12}(n) &= \mathbf{h}_{R1} \mathbf{x}_R(n) + \tilde{z}_1 \\ &= \mathbf{h}_{R1} \mathbf{A} \mathbf{H}_{1R} \mathbf{x}_1(n) + \mathbf{h}_{R1} \mathbf{A} \mathbf{H}_{2R} \mathbf{x}_2(n) + \\ &\quad \mathbf{h}_{R1} \mathbf{A} \mathbf{z}_R(n) + \tilde{z}_1(n), \end{aligned} \quad (6a)$$

$$\begin{aligned} y_{22}(n) &= \mathbf{h}_{R2} \mathbf{x}_R(n) + \tilde{z}_2 \\ &= \mathbf{h}_{R2} \mathbf{A} \mathbf{H}_{1R} \mathbf{x}_1(n) + \mathbf{h}_{R2} \mathbf{A} \mathbf{H}_{2R} \mathbf{x}_2(n) + \\ &\quad \mathbf{h}_{R2} \mathbf{A} \mathbf{z}_R(n) + \tilde{z}_2(n). \end{aligned} \quad (6b)$$

In this paper, we consider that both destinations decode both sources' data. Hence, in destination's opinion, the transmission from both sources to each destination can be modeled as a traditional MISO-MAC in the total two time-slots with the help of a relay. Assuming that all the channel information and beamforming vectors or matrix are perfectly known at  $D_1$  and  $D_2$  prior to data transmission. Hence, each destination can perform successive decoding in which the  $D_i$  first decodes  $S_i$ 's data and then subtract the  $S_i$ 's signal to decode the desired signal from the other source ( $S_j, j \neq i$ ). From (6a), subtracting the  $S_i$ 's data from  $y_{i2}$  yields

$$\tilde{y}_i = \mathbf{h}_{Ri} \mathbf{A} \mathbf{H}_{jR} \mathbf{x}_j(n) + \mathbf{h}_{Ri} \mathbf{A} \mathbf{z}_R(n) + \tilde{z}_i(n). \quad (7)$$

For the given  $\mathbf{w}_i$  and  $\mathbf{A}$ , the maximum instantaneous achievable rates  $r_{ii}$  and  $r_{ij}$  for the end-to-end links from  $S_i$  to both  $D_i$  and  $D_j$  ( $i, j = 1, 2, i \neq j$ ) with the help of the relay  $R$ , respectively, satisfies [9]

$$r_{ij} \leq \log \left( 1 + \frac{|\mathbf{h}_{Rj} \mathbf{A} \mathbf{H}_{iR} \mathbf{w}_i|^2}{||\mathbf{h}_{Rj} \mathbf{A}||^2 + 1} \right), \quad (8a)$$

$$r_{ii} \leq \log \left( 1 + |\mathbf{h}_i \mathbf{w}_i|^2 + T_i \right), \quad (8b)$$

with equality for zero-mean, circularly symmetric complex Gaussian signal used at sources, where

$$T_i \triangleq \frac{|\mathbf{h}_{Ri}\mathbf{A}\mathbf{H}_{iR}\mathbf{w}_i|^2}{|\mathbf{h}_{Ri}\mathbf{A}\mathbf{H}_{jR}\mathbf{w}_j|^2 + |\mathbf{h}_{Ri}\mathbf{A}|^2 + 1}. \quad (9a)$$

Note that the transmission rate  $r_i$  of the source  $S_i$  must be not more than  $\min\{r_{ii}, r_{ij}\}$  to make sure that both destinations can decode the  $S_i$ 's data successfully, i.e.  $r_i \leq \min\{r_{ii}, r_{ij}\}$ .

Next, we define the instantaneous capacity region  $\mathcal{C}(P_1, P_2, P_R)$ , of the multicast cell, where  $P_1, P_2$  and  $P_R$  denote the transmit powers subject to the constraints at  $S_1, S_2$ , and  $R$ , respectively. Then,  $\mathcal{C}(P_1, P_2, P_R)$  is defined as

$$\mathcal{C}(P_1, P_2, P_R) \triangleq \bigcup_{\|\mathbf{w}_i\| \leq P_i, p_R(\mathbf{A}) \leq P_R} \{r_1, r_2\}, \quad i = 1, 2. \quad (10)$$

Note that,  $\mathcal{C}(P_1, P_2, P_R)$  in (10) can be obtained by taking the union over all the achievable regions.

In the rest of this paper, we focus on the characterization of  $\mathcal{C}(P_1, P_2, P_R)$  for some fixed  $P_1, P_2$  and  $P_R$ . We design  $\mathbf{w}_1, \mathbf{w}_2$  and  $\mathbf{A}$  so as to maximize the instantaneous capacity  $\mathcal{C}_m$ . Let  $\mathcal{W} \triangleq \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{A}\}$  denote the collection of all the beam-vector or matrix. Then the problem formulation is :

$$\arg \max_{\mathcal{W}} \mathcal{C}_m = r_1 + r_2 \quad (11)$$

$$\text{s.t. : } p_i \leq P_i, \quad i = 1, 2, \quad (12)$$

$$p_R \leq P_R. \quad (13)$$

### III. ALGORITHM

In this section, we first relax the optimization problem for joint design of  $\mathbf{w}_1, \mathbf{w}_2$  and  $\mathbf{A}$  at the sources and relay, so as to maximize the sum-capacity of the multicast cell subject to a set of power constraints.

#### A. Beamforming Vectors

Toward solving our problem, we first consider the relay matrix  $\mathbf{A}$  is given, and at high SNR regime the equations (8a,8b) can be recast as follows:

$$r_{ij} = \log\left(1 + \frac{|\mathbf{G}_j\mathbf{H}_{iR}\mathbf{w}_i|^2}{1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2}}\right), \quad \left(\mathbf{G}_j \triangleq \frac{\mathbf{h}_{Rj}\mathbf{A}}{\|\mathbf{h}_{Rj}\mathbf{A}\|}\right) \quad (14a)$$

$$r_{ii} = \log\left(1 + |\mathbf{h}_i\mathbf{w}_i|^2 + \frac{|\mathbf{G}_i\mathbf{H}_{iR}\mathbf{w}_i|^2}{|\mathbf{G}_i\mathbf{H}_{jR}\mathbf{w}_j|^2 + 1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2}}\right), \quad (14b)$$

Consequently, the inequality  $r_i \leq \min\{r_{ii}, r_{ij}\}$ , is equivalent to  $\tilde{r}_i \leq \min\{|\mathbf{G}_j\mathbf{H}_{iR}\mathbf{w}_i|^2/(1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2}), |\mathbf{h}_i\mathbf{w}_i|^2 + |\mathbf{G}_i\mathbf{H}_{iR}\mathbf{w}_i|^2/(|\mathbf{G}_i\mathbf{H}_{jR}\mathbf{w}_j|^2 + 1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2})\}$ . Now, the instantaneous capacity maximization problem can be reformulated as following:

$$\arg \max_{\mathcal{W}} \tilde{\mathcal{C}}_m = \max_{\mathbf{w}_1} \tilde{r}_1 + \max_{\mathbf{w}_2} \tilde{r}_2, \quad (15a)$$

$$\text{s.t. : } p_i \leq P_i, \quad i = 1, 2. \quad (15b)$$

$$p_R \leq P_R, \quad (15c)$$

Obviously, the optimization problem in (15) is nonconvex, leading to the fact that no closed-form solutions are available. To maximize  $\tilde{r}_i$  ( $i = 1, 2$ ), there are three cases:

1) : if  $\tilde{r}_i$  is always equal to  $|\mathbf{h}_i\mathbf{w}_i|^2 + |\mathbf{G}_i\mathbf{H}_{iR}\mathbf{w}_i|^2/(|\mathbf{G}_i\mathbf{H}_{jR}\mathbf{w}_j|^2 + 1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2})$  for any  $\mathbf{w}_i$ , then, the optimal  $\mathbf{w}_i$  is  $\rho_s \mathbf{h}_i^H$  [10] where  $\rho_s$  is a power control factor which satisfied the power control equation (2).

2) : if  $\tilde{r}_i$  is always equal to  $|\mathbf{G}_j\mathbf{H}_{iR}\mathbf{w}_i|^2/(1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2})$  for any  $\mathbf{w}_i$ , then, the optimal  $\mathbf{w}_i$  is  $\rho_s (\mathbf{G}_j\mathbf{H}_{iR})^H$  [10] where  $\rho_s$  is a power control factor which satisfied the power control equation (2).

3) : if  $\tilde{r}_i$  is not aforementioned cases, we can set up an equation by adding a slack variable  $k_i \in \mathbb{R}, i \in \{1, 2\}$  since  $|\mathbf{h}_i\mathbf{w}_i|^2 \gg |\mathbf{G}_i\mathbf{H}_{iR}\mathbf{w}_i|^2/(|\mathbf{G}_i\mathbf{H}_{jR}\mathbf{w}_j|^2 + 1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2})$ . One obtains the relaxed equation:

$$\frac{|\mathbf{G}_j\mathbf{H}_{iR}\mathbf{w}_i|^2}{1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2}} = |\mathbf{h}_i\mathbf{w}_i|^2 + k_i, \quad (16)$$

where, we can modify  $\mathbf{w}_i$  and change the  $k_i$  to make the equation hold subject to the power constraints. Therefore, the following equality [7] can fulfill the constrain in (16) :

$$\mathbf{H}_i\mathbf{w}_i = \begin{bmatrix} \sqrt{c_i} e^{j\varphi_1} \\ \sqrt{d_i} e^{j\varphi_2} \end{bmatrix}, \quad \left(\mathbf{H}_i \triangleq \begin{bmatrix} \mathbf{h}_i \\ \mathbf{G}_j\mathbf{H}_{iR} \end{bmatrix}\right) \quad (17)$$

where  $c_i = |\mathbf{h}_i\mathbf{w}_i|^2$ , and  $d_i = (c_i + k_i) \times (1 + \frac{1}{\|\mathbf{h}_{Rj}\mathbf{A}\|^2})$ , for arbitrary real-valued  $\varphi_1$  and  $\varphi_2$ . We can solve  $\mathbf{w}_i$  by setting:

$$\mathbf{w}_i = \mathbf{H}_i^H (\mathbf{H}_i\mathbf{H}_i^H)^{-1} \begin{bmatrix} \sqrt{c_i} e^{j\varphi_1} \\ \sqrt{d_i} e^{j\varphi_2} \end{bmatrix} = n_1 \mathbf{h}_i^H + n_2 (\mathbf{G}_j\mathbf{H}_{iR})^H. \quad (18)$$

It shows that  $\mathbf{w}_i$  must be trade-off between  $\mathbf{h}_i$  and  $\mathbf{G}_j\mathbf{H}_{iR}$ . Then, the transmit power at source  $S_i$  can be computed as:

$$\begin{aligned} p_i &= \|\mathbf{w}_i\|^2 \\ &= c_i \tau_1 + d_i \tau_3 + 2\sqrt{c_i d_i} \Re\{e^{j(\varphi_2 - \varphi_1)} \tau_2\}, \end{aligned} \quad (19)$$

where,  $\tau_1, \tau_2$  and  $\tau_3$  are the entries at the upper left, upper right, and lower right of the  $2 \times 2$  matrix  $(\mathbf{H}_i\mathbf{H}_i^H)^{-1}$ . Since the equality in (19) is invariant with respect to a phase rotation, we can choose  $\varphi_1 = 0$ , without loss of generality. Maximizing  $c_i$  subject to power constraints  $p_i \leq P_i$  is therefore equivalent to minimizing  $\Re\{e^{j\varphi_2} \tau_2\}$  with regard to  $\varphi_2$ . Hence, the optimum value can be solved as [7]

$$\varphi_2 = \pi - \angle \tau_2, \quad (20)$$

that leads to  $\Re\{e^{j\varphi_2} \tau_2\} = -|\tau_2|$ . After inserting (20) into (19) and explicitly computing the the matrix  $(\mathbf{H}_i\mathbf{H}_i^H)^{-1}$ , we can obtain:

$$\|\mathbf{w}_i\|^2 = \frac{c_i \|\mathbf{h}_i\|^2 + d_i \|\mathbf{G}_j\mathbf{H}_{iR}\|^2 - 2\sqrt{c_i d_i} |\mathbf{h}_i^H \mathbf{G}_j \mathbf{H}_{iR}|}{\|\mathbf{h}_i\|^2 \|\mathbf{G}_j \mathbf{H}_{iR}\|^2 - |\mathbf{h}_i^H \mathbf{G}_j \mathbf{H}_{iR}|}. \quad (21)$$

Therefore, we can modify the  $c_i$  and  $d_i$  to satisfy the power constraint at the source  $S_i$  according to the channels.

#### B. Beamforming Matrix

Next, we first consider the solution to explore the optimal beamforming matrix  $\mathbf{A}$  for given  $\mathbf{w}_1$  and  $\mathbf{w}_2$ . Then jointly optimize the  $\mathbf{A}$  and  $\mathbf{w}_i$ , ( $i = 1, 2$ ).

In the previous works, the optimal beamforming based nonregenerative processing structure at relay to maximize the

end-to-end [8] [11] or two-way-relay [3] channel capacity have been studied. However, the model we considered in this paper is the combination of the end-to-end relay and two-way-relay structures. Therefore, the previous results can not be applied to our model directly.

*Lemma 1:* The optimal relay beamforming matrix  $\mathbf{A}$  at relay in our multicast cell, must be the matrix to maximize the sum-rate of the  $r_{12}$  and  $r_{21}$ , from  $S_1$  to  $D_2$  and from  $S_2$  to  $D_1$  via  $R$ , respectively.

*Proof:* Here, we only present a brief proof. For given  $\mathbf{w}_i (i = 1, 2)$ , if modify the beamforming matrix from  $\mathbf{A}$  to  $\mathbf{A}'$ , the  $r_{21}$  in (14) changes to  $r'_{21}$ , and the  $r_{11}$  changes to  $r'_{11}$ . But we can easily prove that  $\Delta_1 = |r'_{21} - r_{21}|$  is bigger than  $\Delta_2 = |r'_{11} - r_{11}|$ . So do the  $r_{12}$  and  $r_{22}$ . ■

According to *Lemma 1*, we can first not consider the channel links from  $S_i$  to  $D_i$  ( $i = 1, 2$ ), but only consider the cross-links from  $S_j$  to  $D_i$  ( $i, j = 1, 2; j \neq i$ ) via  $R$ . Thus, we can set up the following optimization problem w.r.t.  $\mathbf{A}$  as

$$\arg \max_{\mathbf{A}} \gamma = r_{12} + r_{21} \quad (22a)$$

$$\text{s.t. : (5).} \quad (22b)$$

Before solving this optimization problem, we first recast the  $r_{ij}$  as another expression and then set up a new equivalent problem by introducing a new lemma. Due to

$$\begin{aligned} r_{ij} &= \log \left( 1 + \frac{|\mathbf{h}_{Rj}\mathbf{A}\mathbf{H}_{iR}\mathbf{w}_i|^2}{||\mathbf{h}_{Rj}\mathbf{A}||^2 + 1} \right) \\ &= -\log \left( 1 - \mathbf{w}_i^H \mathbf{H}_{iR}^H \mathbf{A}^H \mathbf{h}_{Rj}^H \mathbf{F}_j^{-1} \mathbf{h}_{Rj} \mathbf{A} \mathbf{H}_{iR} \mathbf{w}_i \right) \\ &= -\log \left( 1 - \mathbf{m}_j \mathbf{h}_{Rj} \mathbf{A} \mathbf{H}_{iR} \mathbf{w}_i - \mathbf{w}_i^H \mathbf{H}_{iR}^H \mathbf{A}^H \mathbf{h}_{Rj}^H \mathbf{m}_j^H \right. \\ &\quad \left. + \mathbf{m}_j \mathbf{F}_j \mathbf{m}_j^H \right) \\ &\triangleq -\log e_j, \end{aligned} \quad (23)$$

where

$$\mathbf{F}_j \triangleq \mathbf{h}_{Rj} \mathbf{A} (\mathbf{H}_{iR} \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_{iR}^H + \mathbf{I}) \mathbf{A}^H \mathbf{h}_{Rj}^H + 1, \quad (24)$$

$$\mathbf{m}_i \triangleq \mathbf{w}_i^H \mathbf{H}_{iR}^H \mathbf{A}^H \mathbf{h}_{Rj}^H \mathbf{F}_j^{-1}. \quad (25)$$

Note that, we here used the Woodbury identity property:

$$(a + ubv)^{-1} = a^{-1} - a^{-1}u(b^{-1} + va^{-1}u)^{-1}va^{-1}.$$

*Lemma 2:* Let  $\mathbf{M} \triangleq \{\mathbf{m}_1, \mathbf{m}_2\}$ , and  $\mathbf{W} \triangleq \{\mathbf{w}_1, \mathbf{w}_2\}$ , where  $\mathbf{w}_k \succeq \mathbf{0}$  is a weight factor for the  $k$ th receiver. Then, the solution for the following problem is also the solution for the original problem formulated in (22):

$$\min_{\{\mathbf{M}, \mathbf{W}, \mathbf{A}\}} \sum_{k=1}^2 \text{Tr}(\mathbf{w}_k \mathbf{e}_k) - \log(\mathbf{w}_k), \quad (26a)$$

$$\text{s.t. : (5),} \quad (26b)$$

*Proof:* For any  $\mathbf{A}$  and  $\mathbf{M}$ , it can readily obtain the optimal  $\mathbf{w}_k$  which is equal to

$$\mathbf{w}_k = \mathbf{e}_k^{-1}, \quad k = 1, 2. \quad (27)$$

Then, substituting  $\mathbf{w}_k = \mathbf{e}_k^{-1}$  into (26), we can complete the proof [12]. ■

In fact, this new optimization problem is also a non-linear non-convex problem and its closed-form solution is still intractable. But, if fixing two of the three matrix variables ( $\mathbf{M}, \mathbf{W}, \mathbf{A}$ ), the problem is convex w.r.t. the remaining variable, and the solution has a closed-form. In particular, as in the proof *Lemma 2*, with given  $\mathbf{M}$  and  $\mathbf{A}$ , the solution for  $\mathbf{W}$  is given in (27). The solution for  $\mathbf{M}$  is given in (25) with given  $\mathbf{W}$  and  $\mathbf{A}$ .

Lastly, with given  $\mathbf{M}$  and  $\mathbf{W}$ , the optimization problem in (26) w.r.t.  $\mathbf{A}$  can be adjusted as following

$$\min_{\mathbf{A}} \sum_{k=1}^2 \text{tr}(\mathbf{w}_k \mathbf{e}_k), \quad (28a)$$

$$\text{s.t. : (5).} \quad (28b)$$

This optimization problem w.r.t.  $\mathbf{A}$  is a convex problem [13]. Thus, the Lagrangian function of (28) for  $\mathbf{A}$  is given as

$$\begin{aligned} \mathcal{L}(\mathbf{A}) &= \\ &\sum_{j=1}^2 \text{tr}(-\mathbf{w}_j \mathbf{m}_j \mathbf{h}_{Rj} \mathbf{A} \mathbf{H}_{iR} \mathbf{w}_i - \mathbf{w}_j \mathbf{w}_i^H \mathbf{H}_{iR}^H \mathbf{A}^H \mathbf{h}_{Rj}^H \mathbf{m}_j^H \\ &\quad + \mathbf{w}_j \mathbf{m}_j \mathbf{h}_{Rj} \mathbf{A} (\mathbf{H}_{iR} \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_{iR}^H + \mathbf{I}) \mathbf{A}^H \mathbf{h}_{Rj}^H \mathbf{m}_j^H) + \\ &\mu \left( \text{tr} \left( \mathbf{A} \left( \sum_{i=1}^2 \mathbf{H}_{iR} \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_{iR}^H + \mathbf{I} \right) \mathbf{A}^H \right) - P_R \right) \right). \end{aligned} \quad (29)$$

Then, the KKT conditions can be expressed as following

$$\frac{\partial \mathcal{L}}{\partial \mathbf{A}^*} = -\Delta + \sum_{j=1}^2 \Theta_j \mathbf{A} \Pi_j + \mu \mathbf{A} \Omega = 0, \quad (30)$$

$$\mu (\text{tr}(\mathbf{A} \Omega \mathbf{A}^H) - P_R) = 0, \quad (31)$$

$$\text{tr}(\mathbf{A} \Omega \mathbf{A}^H) \leq P_R, \quad (32)$$

where

$$\Delta \triangleq \sum_{j=1}^2 \mathbf{h}_{Rj}^H \mathbf{m}_j^H \mathbf{w}_j \mathbf{w}_i^H \mathbf{H}_{iR}^H, \quad \Theta_j \triangleq \mathbf{h}_{Rj}^H \mathbf{m}_j^H \mathbf{w}_j \mathbf{m}_j \mathbf{h}_{Rj},$$

$$\Pi_j \triangleq \mathbf{H}_{iR} \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_{iR}^H + \mathbf{I}, \quad \Omega \triangleq \sum_{i=1}^2 \mathbf{H}_{iR} \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_{iR}^H + \mathbf{I}.$$

Based on (30), we can obtain

$$\mathbf{A}(\mu) = \text{Mat} \left( \left[ \sum_{j=1}^2 \Pi_j^T \otimes \Theta_j + \mu \Omega \otimes \mathbf{I} \right]^{-1} \text{Vec}(\Delta) \right), \quad (33)$$

where  $\mu$  is the Lagrangian multiplier which can also be solved by a 1-D search method since  $\text{tr}(\mathbf{A}(\mu) \Omega \mathbf{A}(\mu)^H) \leq P_R$  is monotonically decreasing function of  $\mu$ .

### C. Joint Design of the Beamforming Vectors and Matrix

Based on the above observations, we can obtain the following algorithm for jointly design the optimal beamforming vectors  $\mathbf{w}_i$  at sources  $S_i$  ( $i = 1, 2$ ) and the matrix  $\mathbf{A}$  at relay  $R$ , respectively.

**Algorithm 1** Coordinated Beamforming for Multicast System

- 1: **given:**  $\mathbf{h}_i$ ,  $\mathbf{H}_{iR}$ ,  $\mathbf{h}_{Ri}$ ,  $i = 1, 2$ .
- 2: **initialization:**  $\mathbf{w}_i$ ,  $\mathbf{A}$  and  $\mathbf{k} = (k_1, k_2)$ .
- 3: **repeat**
- 4:   Update the  $\mathbf{w}_i$  using (18) for fixed  $\mathbf{A}$ .
- 5:   Update  $\mathbf{m}_i$  using (25) for fixed  $\mathbf{w}_i$  and  $\mathbf{A}$ .
- 6:   Update  $\mathbf{w}_i$  using (27) for fixed  $\mathbf{w}_i$  and  $\mathbf{A}$ .
- 7:   Update  $\mathbf{A}$  using (33) for fixed  $\mathbf{w}_i$ ,  $\mathbf{m}_i$  and  $\mathbf{w}_i$ .
- 8:   Update the  $\mathbf{k} = (T_1, T_2)$  using (9a).
- 9: **until** The termination criterion is satisfied.

**IV. SIMULATIONS**

In this section, we present the numerical result on the achievable system sum-rate of the proposed algorithm via Monte-Carlo simulations, and compare them with other two suboptimal beamforming structures for matrix at relay in [3], i.e., (A) Maximal-ratio reception and maximal-ratio transmission (MRR-MRT); (B) Zero-forcing reception and zero-forcing transmission (ZFR-ZFT). All channels' data are generated by assuming the entries of  $\mathbf{h}_i$ ,  $\mathbf{H}_{iR}$  and  $\mathbf{h}_{Ri}$  being independent complex Gaussian random variables with zero mean and unit variance. As mentioned earlier, we assume that the related CSIs are available at the sources and relay, i.e.,  $S_1$  knows the  $\mathbf{h}_1$ ,  $\mathbf{H}_{1R}$ ,  $\mathbf{h}_{R1}$  and  $\mathbf{h}_{R2}$ .  $S_2$  knows the  $\mathbf{h}_2$ ,  $\mathbf{H}_{2R}$ ,  $\mathbf{h}_{R1}$  and  $\mathbf{h}_{R2}$ .  $R$  knows  $\mathbf{H}_{jR}$  and  $\mathbf{h}_{Rj}$ , ( $j = 1, 2$ ). We assume that the maximum power constraints at the sources and relay are the same, i.e.  $P_1 = P_2 = P_R$ .

Fig. 2 compares the sum-rate of the coordinated beamforming algorithm, MRR-MRT, ZFR-ZFT, and average power matrix at relay, versus the same power constraints at sources and relay. It turns out that the proposed algorithm yields a significant larger sum-rate of overall system than average power matrix and the MRR-MRT and ZFR-ZFT beamforming structures for relay.

**V. CONCLUSION**

In this paper, we develop an optimal coordinated beamforming algorithm to generate the source beamforming vectors and the relay beamforming matrix for a wireless multicast cell with a nonregenerative multi-antenna relay. The optimal source beamforming vector and relay beamforming matrix maximize the system sum-rate between both sources and both destinations in the absence of the crossover direct links. Simulation results show that the proposed algorithm yields a significantly larger sum-rate of overall system than the average power vectors or matrix and the MRR-MRT and ZFR-ZFT beamforming structures for relay.

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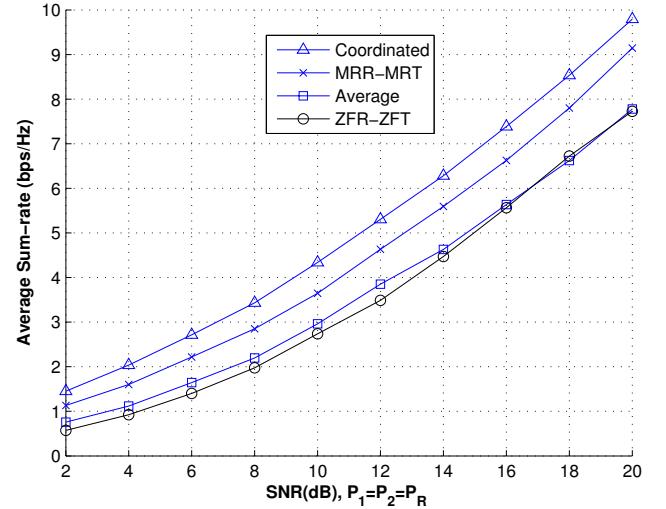


Fig. 2. Sum-rate versus system SNR for the coordinated beamforming algorithm, MRR-MRT, ZFR-ZFT, and average power matrix at relay

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