

# Robust Relay Precoding Design for Bidirectional Multi-User Multi-Relay Networks

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**Abstract**—<sup>1</sup> This paper studies the precoder design of a multi-relay multi-user bidirectional networks in which each relay is equipped with multiple antennas. We study the relay precoder design in the case of imperfect channel state information (CSI) and a norm-bounded error model is adopted. Then, we solve the power minimization problem under this scenario. Numerical results demonstrate that the proposed robust precoding algorithm outperforms the non-robust solution.

## I. INTRODUCTION

Ever since the emergence of multiple-input multiple-output (MIMO) technology, the physical layer research has gained considerable progress in the first decade of 21st century. During this era, E. Telatar derives the capacity expression of MIMO system [1]; L. Zheng gives a fundamental result revealing the tradeoff between multiplexing and diversity [2]. As time goes on, researchers realize that the gain through MIMO point-to-point communication might be severely degraded in cellular scenario due to the existence of practical factors such as large-scale fading, shadow-fading etc. Facing such pressures, researchers resort to relay technology which can efficiently enlarge the coverage and enhance the system throughput simultaneously [3].

Later on, people find that two way relay transmission can further improve the system performance. Although extensive studies on two way relay emerging the recent years, the initial concept can date back to 1960s [4]. In [5], R. Zhang proves that the optimal structure of precoding matrix should not be diagonal in two way scenario. With the aim of minimizing the sum mean squared error (MSE), iterative precoder design algorithms are proposed in [6].

However, all the studies mentioned above only consider the perfect channel state information (CSI). In practical scenario, due to estimation error, feedback error and other uncertain factors, the hypothesis of perfect CSI is an over-idealistic assumption. Therefore, it is necessary to study precoding design under imperfect CSI cases. In general, the models of imperfect CSI can be classified into two categories, the stochastic error model and norm-bounded error model. The stochastic error model assumes a random Gaussian complex error matrix or vector and usually caused by inaccuracy of channel estimation. In [7], it considers a non-regenerative two way relay system

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with stochastic channel estimation error, within which iterative algorithm and sub-optimal closed-form solutions are provided, respectively. Different from the stochastic model, the norm-bounded model assumes all the channel estimation error's norm within a fixed range which conforms the circumstance when the system utilizes limited feedback technique so as to feedback the channel direction information (CDI). The difference between original channel and quantized channel can be generally classified norm-bounded. This model has been widely used in current Long Term Evolution-Advanced (LTE-A) system and has the potential to be applied to large scale adhoc and sensor networks [8] [9]. Therefore, this offers more practical meanings for the research of norm-bounded error. The authors analyzes the MSE optimization method under a two way relay system in [10] with assumption that all channel are suffering from norm-bounded error. Furthermore, in [11], it studies the precoding design for a multiple single-antenna two way relay system with single user pair, and it assume that only the downlink channels have norm-bounded estimation error.

In this paper, we adopt a more general model that incorporates multiple two way relays and multiple users. We focus on the norm-bounded model. The whole system functions in a Time-division duplex (TDD) mode, and there exists a exterior central processor that compute the relay precoders by collecting all the CSI in the system. The rest of the paper is organized as follows. The system model and problem formulation are described in section II. Then, we discuss the power minimization precoder design in section III. The numerical results are presented in section IV. Finally, the conclusion is made in section V.

**Notation:** In this paper, we use bold uppercase and lowercase letters denote matrices and vectors, respectively;  $(\cdot)^T$ ,  $(\cdot)^H$ , denote the conjugate, transpose, conjugate transpose and inverses of a matrix or a vector, respectively;  $\text{blkdiag}\{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_N\}$  is the block diagonalized matrix formed by matrices  $\mathbf{X}_i, i = 1, 2, \dots, N$ ;  $\mathbf{I}_N$  is an  $N \times N$  identity matrix;  $\text{Tr}(\cdot)$  is the trace of a matrix;  $\text{vec}(\cdot)$  represents the matrix vectorization;  $\otimes$  denotes the Kronecker product;  $\|\cdot\|$  denotes the Frobenius norm;  $\succeq$  represents the property of semidefinite.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we will investigate a network incorporating  $R$  relays and  $2K$  users, or equivalently  $K$  user pairs that are

expected to exchange data with the help of the  $R$  relays as shown in Fig. 1. We assume that the  $r$ -th relay is equipped with  $N_r$  antennas. Let  $\bar{k}$  be the index of the user paired with user  $k$ . Additionally,  $\mathbf{h}_{rk} \in \mathbb{C}^{N_r \times 1}$  denotes the channel between the  $k$ -th user and the  $r$ -th relay in the first timeslot. Due to reciprocal characteristic of TDD system, we let  $\mathbf{h}_{rk}^T$  denote the channel between the  $r$ -th relay and the  $k$ -th user in the second timeslot. The two hop transmission processing is completed during two timeslots, i.e., in the first timeslot, users send their independent messages to the relays; while in the second timeslot, the relays will broadcast the precoded received signal to all users. The channel estimation error  $\mathbf{h}_{rk}$

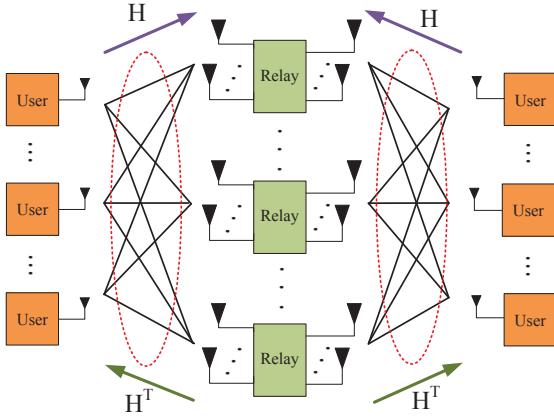


Fig. 1. Illustration of multi-relay multiuser bidirectional networks

is supposed to be norm-bounded which can be expressed as follows

$$\mathbf{h}_{rk} = \bar{\mathbf{h}}_{rk} + \Delta\mathbf{h}_{rk}, \|\Delta\mathbf{h}_{rk}\|^2 \leq \varepsilon_{rk}, \quad (1)$$

where  $\bar{\mathbf{h}}_{rk} \in \mathbb{C}^{N_r \times 1}$  is the estimated channel parameter and  $\Delta\mathbf{h}_{rk} \in \mathbb{C}^{N_r \times 1}$  denotes the norm-bounded error.

In the first time slot, the  $k$ -th user transmits precoded signal  $s_k$ . Without loss of generality, we assume  $\mathbf{E}\{s_k s_k^H\} = \sigma_s^2$ . The received signal at the  $r$ -th relay is

$$\mathbf{x}_r = \sum_{k=1}^{2K} \mathbf{h}_{rk} s_k + \mathbf{n}_r, \quad (2)$$

where  $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$  is circular complex additive white Gaussian noise vector at the  $r$ -th relay node with  $\mathbf{E}\{\mathbf{n}_r \mathbf{n}_r^H\} = \sigma_r^2 \mathbf{I}_{N_r}$ .

After multiplied by relay precoding matrix  $\mathbf{F}_r \in \mathbb{C}^{N_r \times N_r}$ , all the relays broadcast their post-processed signals at the beginning of the second time slot, i.e.,

$$\mathbf{y}_r = \sum_{i=1}^{2K} \mathbf{F}_r \mathbf{h}_{ri} s_i + \mathbf{F}_r \mathbf{n}_r. \quad (3)$$

The transmitting signal at each relay should also not violate the individual relay power constraint, which is

$$\text{Tr}[\mathbf{F}_r (\sigma_s^2 \sum_{i=1}^{2K} \mathbf{h}_{ri} \mathbf{h}_{ri}^H + \sigma_r^2 \mathbf{I}_{N_r}) \mathbf{F}_r^H] \leq P_{Rr}, \forall r \in 1, 2, \dots, R. \quad (4)$$

The received signal at the  $k$ -th user is expressed as

$$r_k = \sum_{i=1}^{2K} \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{h}_{ri} s_i + \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{n}_r + n_k, \quad (5)$$

where  $n_k$  is circular complex additive white Gaussian noise with  $\mathbf{E}\{n_k n_k^H\} = \sigma_x^2$ . After self-interference cancelation, we can express the expected decoded signal as

$$\begin{aligned} \hat{r}_k &= \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{h}_{r\bar{k}} s_{\bar{k}} + \sum_{r=1}^R \bar{\mathbf{h}}_{rk}^T \mathbf{F}_r \Delta\mathbf{h}_{rk} s_k \\ &\quad + \sum_{r=1}^R \Delta\mathbf{h}_{rk}^T \mathbf{F}_r \bar{\mathbf{h}}_{rk} s_k + \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{h}_{ri} s_i \\ &\quad + \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{n}_r + n_k. \end{aligned} \quad (6)$$

Therefore, the SINR of the  $k$ -th user can be written as

$$\text{SINR}_k = \frac{\psi_{4k}}{\psi_{1k} + \psi_{2k} + \psi_{3k} + \sigma_r^2}, \quad (7)$$

where

$$\psi_{1k} = \sigma_s^2 \left\| \sum_{r=1}^R (\bar{\mathbf{h}}_{rk}^T \mathbf{F}_r \Delta\mathbf{h}_{rk} + \Delta\mathbf{h}_{rk}^T \mathbf{F}_r \bar{\mathbf{h}}_{rk}) \right\|^2, \quad (8)$$

$$\psi_{2k} = \sigma_r^2 \sum_{r=1}^R \|\mathbf{h}_{rk}^T \mathbf{F}_r\|^2, \quad (9)$$

$$\psi_{3k} = \sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \left\| \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{h}_{ri} \right\|^2, \quad (10)$$

$$\psi_{4k} = \sigma_s^2 \left\| \sum_{r=1}^R \mathbf{h}_{rk}^T \mathbf{F}_r \mathbf{h}_{r\bar{k}} \right\|^2. \quad (11)$$

### III. POWER MINIMIZATION WITH INDIVIDUAL QOS CONSTRAINTS

Aiming at minimizing the total relay power while maintaining certain quality of service (QoS) constraints, our optimization problem can be formulated as

$$\begin{aligned} \min_{\mathbf{F}_r} \quad & \sum_{r=1}^R \text{Tr}[\mathbf{F}_r (\sigma_s^2 \sum_{i=1}^{2K} \mathbf{h}_{ri} \mathbf{h}_{ri}^H + \sigma_r^2 \mathbf{I}_{N_r}) \mathbf{F}_r^H], \\ \text{s.t.} \quad & \text{SINR}_k \geq \eta_k, \forall k \in 1, 2, \dots, 2K, \\ & \|\Delta\mathbf{h}_{rk}\|^2 \leq \varepsilon_{rk}, \forall r = 1, 2, \dots, R, \forall k = 1, 2, \dots, 2K, \end{aligned} \quad (12)$$

where  $\eta_k$  is the QoS threshold for the  $k$ -th user. Due to the fact that the expression of signal to interference plus noise ratio (SINR) comprising multiple norm-bounded vector variables, it is difficult to use traditional ways to solve this problem. Therefore, we resort to modifying the optimization problem and make it tractable. Our algorithm is to deal with the lower bound of SINR and upper bound of relay power consumption. Such method will eliminate the uncertainty parts of the optimization problem. The lower bound of SINR can be expressed as

$$\text{SINR}_k = \frac{\psi_{4k}}{\psi_{1k} + \psi_{2k} + \psi_{3k} + \sigma_r^2}, \quad (13)$$

where  $\bar{\psi}_{1k}$ ,  $\bar{\psi}_{2k}$ ,  $\bar{\psi}_{3k}$ ,  $\underline{\psi}_{4k}$  denote the upper bound of  $\psi_{1k}$ ,  $\psi_{2k}$ ,  $\psi_{3k}$  and lower bound of  $\psi_{4k}$ , respectively. First, we define  $\mathbf{F} = blkdiag\{\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_R\}$  and  $\mathbf{f}$  is defined as  $\mathbf{f} = [vec(\mathbf{F}_1)^T \dots vec(\mathbf{F}_R)^T]^T$ .

Thus, it is easy to verify that  $\mathbf{F}_r = \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T$ , where  $\mathbf{P}_r$  is the  $N_r \times \sum_{r=1}^R N_r$  permutation matrix with zeros and ones. Then, we define  $vec(\mathbf{F}) = \mathbf{T}\mathbf{f}$ ,  $vec(\mathbf{F}^* \otimes \mathbf{F}) = \mathbf{T}_f vec(\mathbf{f}\mathbf{f}^H)$  and  $vec(\mathbf{F}^* \otimes \mathbf{F}^T) = \mathbf{T}_{f^T} vec(\mathbf{f}\mathbf{f}^H)$  where  $\mathbf{T}$ ,  $\mathbf{T}_f$  and  $\mathbf{T}_{f^T}$  are the transformation matrix formed by ones and zeros, which can be built by observing the nonzero entries of  $vec(\mathbf{F})$ . Before explicit computation of the exact form of the lower bound, we state several useful norm inequalities [12] that will be used later, for the following two problems

$$\min_{\|\mathbf{x}\| \leq \delta} \mathcal{X}(\mathbf{x}) = \Re(\mathbf{x}^H \mathbf{y}), \quad (14)$$

$$\max_{\|\mathbf{x}\| \leq \delta} \mathcal{Y}(\mathbf{x}) = \Re(\mathbf{x}^H \mathbf{y}). \quad (15)$$

Their solutions can be given by  $\mathcal{X}(-(\delta/\|\mathbf{y}\|)\mathbf{y}) = -\delta\|\mathbf{y}\|$  and  $\mathcal{Y}((\delta/\|\mathbf{y}\|)\mathbf{y}) = \delta\|\mathbf{y}\|$ .

Furthermore, according to  $\|\mathbf{X}\| = \|vec(\mathbf{X})\|$  and  $vec(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})vec(\mathbf{B})$  [13] where  $\mathbf{X}_1 \in \mathbb{C}^{N_1 \times N_2}$ ,  $\mathbf{F} \in \mathbb{C}^{N_2 \times N_3}$ ,  $\mathbf{X}_2 \in \mathbb{C}^{N_3 \times N_3}$ ,  $\mathbf{X}_3 \in \mathbb{C}^{N_2 \times N_5}$ , we have

$$\begin{aligned} & \|\mathbf{X}_1 \mathbf{F} \mathbf{X}_2 \mathbf{F}^H \mathbf{X}_3\| \\ = & \left\| \left( vec(\mathbf{X}_2)^T \otimes (\mathbf{X}_3^T \otimes \mathbf{X}_1) \right) \mathbf{T}_f vec(\mathbf{f}\mathbf{f}^H) \right\|. \end{aligned} \quad (16)$$

Similarly, the following equations stand

$$\begin{aligned} & \|\mathbf{X}_1 \mathbf{F}^T \mathbf{X}_2 \mathbf{F}^H \mathbf{X}_3\| \\ = & \left\| \left( vec(\mathbf{X}_2)^T \otimes (\mathbf{X}_3^T \otimes \mathbf{X}_1) \right) \mathbf{T}_{f^T} vec(\mathbf{f}\mathbf{f}^H) \right\|, \end{aligned} \quad (17)$$

$$\begin{aligned} & \|\mathbf{X}_1 \mathbf{F}^H \mathbf{X}_2 \mathbf{F} \mathbf{X}_3\| \\ = & \left\| \left( vec(\mathbf{X}_2)^T \otimes (\mathbf{X}_3^T \otimes \mathbf{X}_1) \right) \mathbf{T}_{f^H} vec(\mathbf{f}\mathbf{f}^H) \right\|. \end{aligned} \quad (18)$$

During the following deductions, we will omit second-order terms of  $\Delta \mathbf{h}_{rk}$ . According to this rule, the term of  $\bar{\psi}_{1k}$  can be neglected. After that, we will use the equations mentioned above to calculate  $\bar{\psi}_{2k}$ ,  $\bar{\psi}_{3k}$  and  $\underline{\psi}_{5k}$ .

Firstly, we examine the term of  $\bar{\psi}_{2k}$  which can be further expressed in (19). It can be observed that  $\bar{\psi}_{2k}$  incorporates two terms named (a) and (b).

$$\begin{aligned} \bar{\psi}_{2k} \approx & \underbrace{\sigma_r^2 \sum_{r=1}^R Tr(\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T)(\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T)^H}_{(a)} \\ + & \underbrace{2\sigma_r^2 \sum_{r=1}^R \Re \left\{ \Delta \mathbf{h}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T)^H \right\}}_{(b)}. \end{aligned} \quad (19)$$

By using the equation  $vec(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})vec(\mathbf{B})$ , term

(a) can be expressed as follows

$$\begin{aligned} & \sigma_r^2 \sum_{r=1}^R Tr(\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T)(\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T)^H \\ = & \sigma_r^2 Tr \left( \mathbf{T}^H \left( \sum_{r=1}^R (\mathbf{P}_r \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r)) \right)^H (\mathbf{P}_r \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r)) \right) \\ & \mathbf{T} \mathbf{f} \mathbf{f}^H, \end{aligned} \quad (20)$$

With help of (15), term (b) can be upper bounded as follows

$$\begin{aligned} & 2\sigma_r^2 \sum_{r=1}^R \Re \left\{ \Delta \mathbf{h}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T)^H \right\} \\ \leq & 2\sigma_r^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \left\| \left( vec(\mathbf{P}_r^T \mathbf{P}_r)^T \otimes ((\mathbf{P}_r^T \bar{\mathbf{h}}_{rk}^*)^T \otimes \mathbf{P}_r) \right) \right. \\ & \left. \mathbf{T}_f vec(\mathbf{f}\mathbf{f}^H) \right\|. \end{aligned} \quad (21)$$

Thus, we have the upper bound of  $\bar{\psi}_{2k}$  expressed as

$$\bar{\psi}_{2k} \leq Tr(\mathbf{Q}_{2k} \mathbf{f}\mathbf{f}^H) + 2\sigma_r^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \left\| \mathbf{A}_r^k vec(\mathbf{f}\mathbf{f}^H) \right\| \triangleq \bar{\psi}_{2k}, \quad (22)$$

where

$$\mathbf{Q}_{2k} = \mathbf{T}^H \left( \sum_{r=1}^R (\mathbf{P}_r \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r)) \right)^H (\mathbf{P}_r \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r)) \mathbf{T}, \quad (23)$$

$$\mathbf{A}_r^k = \left( vec(\mathbf{P}_r^T \mathbf{P}_r)^T \otimes ((\mathbf{P}_r^T \bar{\mathbf{h}}_{rk}^*)^T \otimes \mathbf{P}_r) \right) \mathbf{T}_f. \quad (24)$$

Similarly, by utilizing similar approach, we have the upper bound of  $\bar{\psi}_{3k}$  as

$$\begin{aligned} \bar{\psi}_{3k} \leq & Tr(\mathbf{Q}_{3k} \mathbf{f}\mathbf{f}^H) + 2\sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \left\| \mathbf{B}_{ri}^k vec(\mathbf{f}\mathbf{f}^H) \right\| \\ + & 2\sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \sqrt{\varepsilon_{ri}} \left\| \mathbf{C}_{ri}^k vec(\mathbf{f}\mathbf{f}^H) \right\| \triangleq \bar{\psi}_{3k}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} \mathbf{Q}_{3k} = & \sigma_s^2 \mathbf{T}^H \left( \sum_{i=1, i \neq k, \bar{k}}^{2K} \left( \sum_{r=1}^R (\mathbf{P}_r^T \bar{\mathbf{h}}_{ri})^T \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r) \right) \right)^H \\ & \left( \sum_{r=1}^R (\mathbf{P}_r^T \bar{\mathbf{h}}_{ri})^T \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r) \right) \mathbf{T}, \end{aligned} \quad (26)$$

$$\mathbf{B}_{ri}^k = \sum_{j=1}^R \left( vec(\mathbf{P}_r^T \bar{\mathbf{h}}_{ri} \bar{\mathbf{h}}_{ji}^H \mathbf{P}_j)^T \otimes ((\mathbf{P}_j^T \bar{\mathbf{h}}_{jk}^*)^T \otimes \mathbf{P}_r) \right) \mathbf{T}_f, \quad (27)$$

$$\mathbf{C}_{ri}^k = \sum_{j=1}^R \left( vec(\mathbf{P}_r^T \bar{\mathbf{h}}_{rk} \bar{\mathbf{h}}_{ji}^H \mathbf{P}_j)^T \otimes ((\mathbf{P}_j^T \bar{\mathbf{h}}_{jk}^*)^T \otimes \mathbf{P}_r) \right) \mathbf{T}_{f^T}. \quad (28)$$

Then, we have to deduct the lower bound of  $\underline{\psi}_{4k}$  which is numerator of SINR<sub>k</sub>.  $\underline{\psi}_{4k}$  can be reformulated in (29) which involving three terms named (f), (g) and (h). Term (f) can

$$\begin{aligned}
\psi_{4k} &\approx \underbrace{\sigma_s^2 \left( \sum_{r=1}^R (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}}) \right) \left( \sum_{r=1}^R (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}}) \right)^H}_{(f)} + \underbrace{2\sigma_s^2 \Re \left\{ \sum_{r=1}^R \left( \Delta \mathbf{h}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}} \left( \sum_{r=1}^R \bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}} \right)^H \right) \right\}}_{(g)} \\
&+ \underbrace{2\sigma_s^2 \Re \left\{ \sum_{r=1}^R \left( \Delta \mathbf{h}_{r\bar{k}}^T \mathbf{P}_r \mathbf{F}^T \mathbf{P}_r^T \bar{\mathbf{h}}_{rk} \left( \sum_{r=1}^R \bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}} \right)^H \right) \right\}}_{(h)}. \tag{29}
\end{aligned}$$


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be rewritten in the following form

$$\begin{aligned}
&\sigma_s^2 \left( \sum_{r=1}^R (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}}) \right) \left( \sum_{r=1}^R (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r \mathbf{F} \mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}}) \right)^H \\
&= \sigma_s^2 T r \left( \left( \sum_{r=1}^R (\mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}})^T \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r) \right)^H \left( \sum_{r=1}^R (\mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}})^T \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r) \right) \mathbf{f}^H \right). \tag{30}
\end{aligned}$$

According to equation (14), we can find the lower bounds of term (g) and (h). Thus  $\psi_{4k}$  can be lower bounded as

$$\begin{aligned}
\psi_{4k} &\geq Tr(\mathbf{Q}_{5k} \mathbf{f}^H) - 2\sigma_s^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{D}_r^k vec(\mathbf{f}^H)\| \\
&- 2\sigma_s^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{E}_r^k vec(\mathbf{f}^H)\| \triangleq \underline{\psi}_{4k}, \tag{31}
\end{aligned}$$

where

$$\mathbf{Q}_{4k} = \sigma_s^2 \mathbf{T}^H \left( \sum_{r=1}^R (\mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}})^T \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r) \right)^H \left( \sum_{r=1}^R (\mathbf{P}_r^T \bar{\mathbf{h}}_{r\bar{k}})^T \otimes (\bar{\mathbf{h}}_{rk}^T \mathbf{P}_r) \right) \mathbf{T}. \tag{32}$$

$$\mathbf{D}_r^k = \sum_{j=1}^R \left( vec(\mathbf{P}_r^T \bar{\mathbf{h}}_{rk} \bar{\mathbf{h}}_{jk}^H \mathbf{P}_j)^T \otimes ((\mathbf{P}_j^T \bar{\mathbf{h}}_{jk}^*)^T \otimes \mathbf{P}_r) \right) \mathbf{T}_f, \tag{33}$$

$$\mathbf{E}_r^k = \sum_{j=1}^R \left( vec(\mathbf{P}_r^T \bar{\mathbf{h}}_{rk} \bar{\mathbf{h}}_{jk}^H \mathbf{P}_j)^T \otimes ((\mathbf{P}_j^T \bar{\mathbf{h}}_{jk}^*)^T \otimes \mathbf{P}_r) \right) \mathbf{T}_{f^T}. \tag{34}$$

Next, we need to find the upper bound the relay power consumption,

$$\begin{aligned}
&\sum_{r=1}^R Tr \left[ \mathbf{F}_r \left( \sigma_s^2 \sum_{i=1}^{2K} \mathbf{h}_{ri} \mathbf{h}_{ri}^H + \sigma_r^2 \mathbf{I}_{M_r} \right) \mathbf{F}_r^H \right] \\
&\leq Tr(\mathbf{Q}_R \mathbf{f}^H) + 2\sigma_s^2 \sum_{r=1}^R \sum_{i=1}^{2K} \sqrt{\varepsilon_{ri}} \|\mathbf{Z}_{ri} vec(\mathbf{f}^H)\|, \tag{35}
\end{aligned}$$

where

$$\mathbf{Q}_R = \mathbf{T}^H \left( \sum_{r=1}^R \left( \left( \sum_{i=1}^{2K} \sigma_s^2 \left( (\mathbf{P}_r^T \bar{\mathbf{h}}_{ri})^T \otimes \mathbf{P}_r \right)^H \left( (\mathbf{P}_r^T \bar{\mathbf{h}}_{ri})^T \otimes \mathbf{P}_r \right) \right) + \sigma_r^2 \left( (\mathbf{P}_r^T \mathbf{P}_r)^T \otimes (\mathbf{P}_r^T \mathbf{P}_r) \right) \right) \mathbf{T}, \tag{36}$$

$$\mathbf{Z}_{ri} = \left( vec(\mathbf{P}_r^T \mathbf{P}_r)^T \otimes (\mathbf{P}_r \otimes \bar{\mathbf{h}}_{ri}^H \mathbf{P}_r) \right) \mathbf{T}_{f^H}. \tag{37}$$

Then, with the upper bound of relay power consumption,  $\underline{\psi}_{2k}$ ,  $\underline{\psi}_{3k}$  and  $\underline{\psi}_{4k}$ , the power minimization problem can be reformulated as

$$\begin{aligned}
\min_{\mathbf{f}} \quad &Tr(\mathbf{Q}_R \mathbf{f}^H) + 2\sigma_s^2 \sum_{r=1}^R \sum_{i=1}^{2K} \sqrt{\varepsilon_{ri}} \|\mathbf{Z}_{ri} vec(\mathbf{f}^H)\|, \\
\text{s.t.} \quad &Tr(\mathbf{Q}_k \mathbf{f}^H) \geq 2\eta_k \sigma_r^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{A}_r^k vec(\mathbf{f}^H)\| \\
&+ 2\eta_k \sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{B}_{ri}^k vec(\mathbf{f}^H)\| \\
&+ 2\eta_k \sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \sqrt{\varepsilon_{ri}} \|\mathbf{C}_{ri}^k vec(\mathbf{f}^H)\| \\
&+ 2\sigma_s^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{D}_r^k vec(\mathbf{f}^H)\| + 2\sigma_s^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{E}_r^k vec(\mathbf{f}^H)\| + \eta_k \sigma_r^2, \forall k \in 1, 2, \dots, 2K, \tag{38}
\end{aligned}$$

where

$$\mathbf{Q}_k = \mathbf{Q}_{4k} - \eta_k \mathbf{Q}_{2k} - \eta_k \mathbf{Q}_{3k}. \tag{39}$$

(32) By introducing  $\mathbf{W} = \mathbf{f}^H$  and  $\tau$ , and neglecting the rank constraint of  $\mathbf{W}$ , the optimization problem turns into

$$\begin{aligned}
\min_{\mathbf{W}, \tau} \quad &\tau, \\
\text{s.t.} \quad &\tau - Tr(\mathbf{Q}_R \mathbf{W}) \geq 2\sigma_s^2 \sum_{r=1}^R \sum_{i=1}^{2K} \sqrt{\varepsilon_{ri}} \|\mathbf{Z}_{ri} vec(\mathbf{W})\|, \\
&Tr(\mathbf{Q}_k \mathbf{W}) \geq 2\eta_k \sigma_r^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{A}_r^k vec(\mathbf{W})\| \\
&+ 2\eta_k \sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{B}_{ri}^k vec(\mathbf{W})\| \\
&+ 2\eta_k \sigma_s^2 \sum_{i=1, i \neq k, \bar{k}}^{2K} \sum_{r=1}^R \sqrt{\varepsilon_{ri}} \|\mathbf{C}_{ri}^k vec(\mathbf{W})\| \\
&+ 2\sigma_s^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{D}_r^k vec(\mathbf{W})\| + 2\sigma_s^2 \sum_{r=1}^R \sqrt{\varepsilon_{rk}} \|\mathbf{E}_r^k vec(\mathbf{W})\| + \eta_k \sigma_r^2, \forall k \in 1, 2, \dots, 2K, \tag{40}
\end{aligned}$$

which is a typical semidefinite programming (SDP) problem [14] and can be solved by CVX [15]. For the solutions that don't conform the rank constraint, we can use randomization technique to obtain the approximate vector solution [16].

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will provide simulation results to verify the effectiveness of our proposed robust precoding scheme.

A vector  $[K \ N \ R \ M]$  represents a network with  $K$  user pairs and  $R$  relays, each equipped with  $N$  and  $M$  antennas. For simplicity, we assume  $N_r = N$ ,  $\sigma_s^2 = \sigma_r^2 = 1$ ,  $\varepsilon_{rk} = \varepsilon$  and  $\eta_k = \eta$ ,  $\forall r = 1, 2, \dots, R, \forall k = 1, 2, \dots, 2K$ . We mainly investigate the SINRs achieved by our robust strategy and the non-robust one.  $\bar{\mathbf{h}}_{rk}$  is fixed and we generate 1000 times  $\Delta\mathbf{h}_{rk}$  and record their corresponding SINRs.

Firstly, in Fig. 2, we execute simulations for the case  $[1 \ 1 \ 2 \ 4]$  which means there are two four-antenna relays and a pair of single-antenna users. With fixed  $\varepsilon = 0.001$  and  $\eta = 2$ , we can see that our proposed robust algorithm can always achieve SINR values above the threshold  $\eta$ . The non-robust precoding scheme can be obtained by setting all  $\varepsilon_{rk}$  to null in (43). However, almost half of the non-robust scheme's SINRs fall below the threshold  $\eta$ . Therefore, it proves that our robust algorithm can get a better performance of outage probability.

In Fig. 3, a  $[2 \ 1 \ 2 \ 4]$  network has been investigated with fixed  $\varepsilon = 0.001$  and  $\eta = 1$ . The difference between Fig. 1 and Fig. 2 is the number of user pairs, we have single user pair in Fig. 1 while there are two in Fig. 2. We can observe two peaks in Fig. 2 for our proposed algorithm. Since we only generate  $\bar{\mathbf{h}}_{rk}$  and the channel qualities of distinct users vary largely. Even with the same  $\varepsilon$ , each user pair exhibits different performance. Besides, we can see that our proposed algorithm demonstrates better performance than the non-robust one.

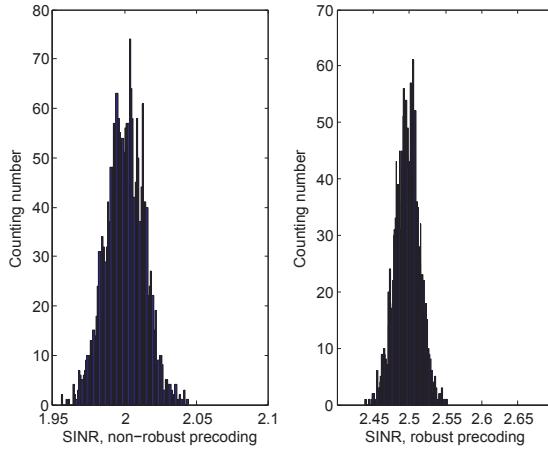


Fig. 2. Performance comparison between robust scheme and non-robust scheme,  $[1 \ 1 \ 2 \ 4]$ ,  $\eta = 2$ ,  $\varepsilon = 0.001$

#### V. CONCLUSION

This paper studies joint relay precoding optimization scheme under the bidirectional multi-user multi-relay scenario with norm-bounded channel estimation error. A robust precoding strategy is proposed and simulation results verify the effectiveness of this scheme.

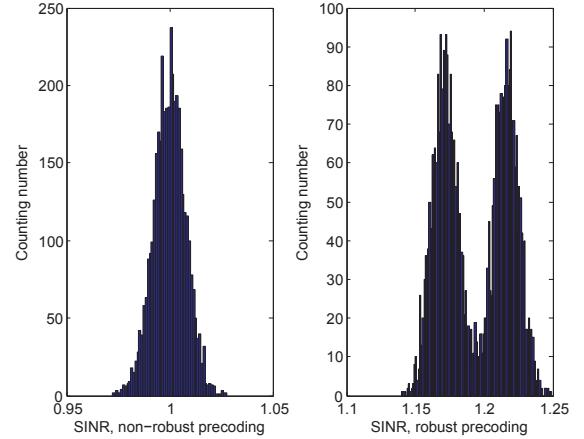


Fig. 3. Performance comparison between robust scheme and non-robust scheme,  $[2 \ 1 \ 2 \ 4]$ ,  $\eta = 1$ ,  $\varepsilon = 0.001$

#### REFERENCES

- [1] I. E. Telatar, "Capacity of Multi-Antenna Gaussian Channels", *Tech. Rep. Bell Labs*, Lucent Technologies, 1995.
- [2] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels", in *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073-1096, May, 2003.
- [3] H. Wan and W. Chen, "Joint Source and Relay Design for Multi-user MIMO Non-regenerative Relay Networks with Direct Links," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 6, pp. 2871-2876, 2012.
- [4] C. Shannon, "Two-way communication channels," *Proc. 4th Berkeley Symp. Mathematical Statistics Probability*, Berkeley, CA, pp. 611-644, 1961.
- [5] R. Zhang, Y. Liang, C. Chai and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding", in *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 5, pp. 699-712, Jun, 2009.
- [6] R. Wang and M. Tao, "Joint Source and Relay Precoding Designs for MIMO Two-Way Relaying Based on MSE Criterion", in *IEEE Transactions on Signal Processing*, vol. 60, no. 3, pp. 1352-1365, 2011.
- [7] J. Zou, W. Liu, M. Ding, H. Luo and H. Yu, "Transceiver Design for AF MIMO Two-Way Relay Systems with Imperfect Channel Estimation", in *GLOBECOM*, 1-5, Dec, 2011.
- [8] W. Huang and X. Wang, "Capacity Scaling of General Cognitive Networks," in *IEEE/ACM Transactions on Networking*, vol 20, no. 5, pp. 1501-1513, 2012.
- [9] X. Wang, W. Huang, S. Wang, J. Zhang and C. Hu, "Delay and Capacity Tradeoff Analysis for MotionCast," in *IEEE/ACM Transactions on Networking*, vol. 19, no. 5, pp. 1354-1367, 2011.
- [10] E. Gharavol and E. Larsson, "Robust joint optimization of MIMO two-way relay channels with imperfect CSI," *Communication, Control, and Computing (Allerton), 2011 49th Annual Allerton Conference on*, pp. 1657-1664, 28-30, Sep, 2011.
- [11] M. Tao, R. Wang, "Robust Relay Beamforming for Two-Way Relay Networks," *Communications Letters, IEEE*, vol. 16, no. 7, pp. 1052-1055, Jul, 2012.
- [12] B. Chalise and L. Vandendorpe, "MIMO Relay Design for Multipoint-to-Multipoint Communications With Imperfect Channel State Information," *Signal Processing, IEEE Transactions on*, vol. 57, no. 7, pp. 2785-2796, Jul. 2009.
- [13] R. A.Horn, and C. R. Johnson, *Topics in Matrix Analysis*, Cambridge University Press, New York, 1991.
- [14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [15] CVX Research, Inc. CVX: Matlab software for disciplined convex programming, version 2.0 beta. <http://cvxr.com/cvx>, September 2012.
- [16] Y. Huang and D. Palomar, "Rank-Constrained Separable Semidefinite Programming With Applications to Optimal Beamforming," *Signal Processing, IEEE Transactions on*, vol. 58, no. 2, pp. 664-678, Feb. 2010.