One-Bit Soft Forwarding for Network Coded Uplink Channels with Multiple Sources

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***Abstract*—In this paper, we propose a threshold-based one-bit soft forwarding (TOB-SF) protocol for a multi-source relaying uplink system with network coding. In the TOB-SF protocol, the relay calculates the log-likelihood ratio (LLR) value of each network coded symbol, compares this LLR value with a pre-optimized threshold, and determines whether to transmit or keep silent. We first derive the bit error rate (BER) expression at the destination, based on which, we optimize the threshold to minimize the BER. Then we theoretically prove that the system can achieve the full diversity gain by using this threshold. Further, we optimize the power allocation at the relay to achieve a higher coding gain. Simulation results show that the proposed TOB-SF protocol outperforms other conventional relaying protocols in terms of error performance.**

I. Introduction

Two classical relay protocols, namely, amplify-and-forward (AF) and decode-and-forward (DF), have been widely studied in wireless relaying systems [1]. However, the AF protocol suffers from noise amplification, and the DF protocol propagates the erroneous decisions to the destination. Consequently, a new relaying concept, namely, soft information forwarding (SIF) has been proposed to achieve a better error performance by forwarding the intermediate soft decisions at the relay [2–5]. The soft information can be in the form of log-likelihood ratio (LLR) [2], soft symbol [3], or soft mutual information [4, 5]. The SIF based protocols are shown to achieve a better error performance than the AF and DF. However, these SIF protocols require the relay to forward real values via wireless channels, which is impractical.

More practically, LLR-threshold based one-bit soft information forwarding (TOB-SF) protocol is proposed [6–8], where the relay calculates the LLR value of each received symbol transmitted by the source, and compares the LLR value with a pre-determined threshold. The relay forwards the hard decision of the corresponding symbol if the LLR value is larger than the threshold, and keeps silent otherwise. The TOB-SF protocol requires the same bandwidth as the DF protocol, while it achieves a better performance. We note that the TOB-SF protocol is mainly investigated in one-way and two-way relay channels [6–8]. However, these results are not directly applicable for general multi-source relay channels.

This work is supported by Australian Research Council $LP110100110$ with ZTE for financial support, and National 973 Project $2012CB316106$.

Multi-source relay channels are common and basic building blocks in wireless networks, especially in cellular networks. Therefore, investigating efficient relaying protocols to control error propagations in such channels is an important issue [9, 10]. In [9], the LLR values of network coded symbols are forwarded by the relay. As discussed above, transmitting LLR values over bandwidth-limited wireless channels is impractical. Also, the sub-optimal combining of the signals at the destination will lead to a performance loss. In [10], the authors extend the power scaling schemes for one-way relay channels [11] to multi-source relay channels. Although the full diversity gain is achieved, the error performance is not fully optimized, i.e., the coding gain can be further improved.

In this paper, we design a network coded TOB-SF protocol in a two-source relaying system over fading channels. We first derive the bit error rate (BER) expressions at the destination and optimize the threshold by minimizing the BER. Then we theoretically prove that the system can achieve the full diversity gain by using the proposed threshold. Furthermore, we optimize the power allocation at the relay to achieve a higher coding gain. In the simulations, we use the DF, EF, and the power scaling scheme from [10] as benchmarks. Simulation results show that the proposed TOB-SF protocol outperforms the benchmarks in terms of the error performance.

II. System Model

Consider an uplink relay channel with two sources, one relay and one destination as shown in Fig. 1, where the two sources $\mathcal{S}_1$ and $\mathcal{S}_2$ broadcast their messages to the common destination $\mathcal{D}$ with the help of a half-duplex relay $\mathcal{R}$. Each transmission period consists of three phases. In the first phase, $\mathcal{S}_1$ broadcasts its message, and in second phase, $\mathcal{S}_2$ broadcasts its message, to the relay and the destination. After the first two phases, the relay generates the network coded message based on the signals from the two sources, which is then forwarded to the destination during the third phase. At the end of each transmission period, the destination decodes the messages of the sources based on the signals from the sources and the relay.

We denote by $h_{i\mathcal{R}}$, $i=1,2$, $h_{i\mathcal{D}}$, and $h_{\mathcal{R}\mathcal{D}}$ the channel coefficients between $\mathcal{S}_i$ and $\mathcal{R}$, between $\mathcal{S}_i$ and $\mathcal{D}$, and between $\mathcal{R}$ and $\mathcal{D}$, respectively, as shown in Fig. 1, and denote by $d_{i\mathcal{R}}$, $d_{i\mathcal{D}}$, and $d_{\mathcal{R}\mathcal{D}}$ the distances between $\mathcal{S}_i$ and $\mathcal{R}$, between

978-1-4799-2003-7/14/$31.00

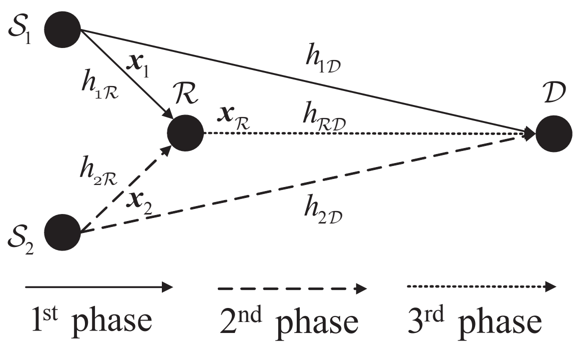

Fig. 1. The orthogonal uplink relay channel with two sources, one relay, and one destination.

$\mathcal{S}_i$ and $\mathcal{D}$, and between $\mathcal{R}$ and $\mathcal{D}$, respectively. We assume that $h_{i\mathcal{R}}$, $h_{i\mathcal{D}}$, and $h_{\mathcal{RD}}$ are independent and identically Rayleigh distributed with the channel gains as $\lambda_{i\mathcal{R}}$, $\lambda_{i\mathcal{D}}$, and $\lambda_{\mathcal{RD}}$, respectively. These channel gains are related to the corresponding distances with the attenuation exponent γ, i.e., $\lambda_{i\mathcal{R}} = 1/(d_{i\mathcal{R}})^\gamma$, $\lambda_{i\mathcal{D}} = 1/(d_{i\mathcal{D}})^\gamma$, and $\lambda_{\mathcal{RD}} = 1/(d_{\mathcal{RD}})^\gamma$. We consider quasi-static fading channels, i.e., the channel coefficients are constant during one transmission period, and change independently from one period to another.

Let us assume that each phase in a transmission period consists of l time slots, and thus each transmission period lasts $3l$ time slots. Each source $\mathcal{S}_i$ transmits the binary phase-shift keying (BPSK) symbol vector $\boldsymbol{x}_i = (x_i^1, \cdots, x_i^l)^T$, $x_i^j \in \{\pm 1\}$ and $j \in \{1, \cdots, l\}$, with the power E_i. The received signals at the relay and at the destination from $\mathcal{S}_i$ can be expressed as

$$\boldsymbol{y}_{i\mathcal{R}} = h_{i\mathcal{R}}\sqrt{E_i}\boldsymbol{x}_i + \boldsymbol{n}_{i\mathcal{R}}, \boldsymbol{y}_{i\mathcal{D}} = h_{i\mathcal{D}}\sqrt{E_i}\boldsymbol{x}_i + \boldsymbol{n}_{i\mathcal{D}}, \quad (1)$$

respectively, where the vectors $\boldsymbol{y}_{i\mathcal{R}}$ and $\boldsymbol{y}_{i\mathcal{D}}$ consist of l received signals, i.e., $\boldsymbol{y}_{i\mathcal{R}} = (y_{i\mathcal{R}}^1, \cdots, y_{i\mathcal{R}}^l)^T$ and $\boldsymbol{y}_{i\mathcal{D}} = (y_{i\mathcal{D}}^1, \cdots, y_{i\mathcal{D}}^l)^T$, the vector $\boldsymbol{n}_{i\mathcal{R}} = (n_{i\mathcal{R}}^1, \cdots, n_{i\mathcal{R}}^l)^T$ consists of l additive white Gaussian noise (AWGN) samples at the relay, and the vector, $\boldsymbol{n}_{i\mathcal{D}} = (n_{i\mathcal{D}}^1, \cdots, n_{i\mathcal{D}}^l)^T$, consists of l AWGN samples at the destination. We assume that all the noise samples at the relay and the destination are with a mean zero and the same variance σ^2, and the sources' power satisfies $E_1 = E_2 = 1$. We define the SNR as $\rho \triangleq 1/\sigma^2$.

After receiving $\boldsymbol{y}_{i\mathcal{R}}$, the relay detects/decodes $\boldsymbol{x}_i$ and generates the network coded message as $\boldsymbol{x}_{\mathcal{R}} = (x_{\mathcal{R}}^1, \cdots, x_{\mathcal{R}}^l)^T$, $x_{\mathcal{R}}^j \in \{\pm 1\}$, based on the hard decisions of $\boldsymbol{x}_i$, denoted by $\hat{\boldsymbol{x}}_i$. Since the network coding operation, i.e., XOR, between two bits is equivalent to the multiplication of their BPSK symbols, a network coded symbol $x_{\mathcal{R}}^j$ can be obtained by $x_{\mathcal{R}}^j = \hat{x}_1^j \hat{x}_2^j$, where $\hat{x}_i^j$ is the hard decision of x_i^j at the relay. Hence, the network coded message $\boldsymbol{x}_{\mathcal{R}}$ is obtained by implementing inner product (vector-wise multiplication) on $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. Then we have $\boldsymbol{x}_{\mathcal{R}} = \hat{\boldsymbol{x}}_1 \cdot \hat{\boldsymbol{x}}_2 = (\hat{x}_1^1\hat{x}_2^1, \cdots, \hat{x}_1^l\hat{x}_2^l)^T$. In our TOB-SF protocol, the relay only forwards those network coded symbols in $\boldsymbol{x}_{\mathcal{R}}$ whose absolute LLR values are larger than a preset threshold, and keeps silent otherwise. Then the received signal at the destination from the relay can be written as

$$\boldsymbol{y}_{\mathcal{RD}} = h_{\mathcal{RD}}\sqrt{\alpha}\boldsymbol{v} \cdot \boldsymbol{x}_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{RD}}, \quad (2)$$

where $0 < \alpha \leq 1$ is the power allocation at the relay, and $\boldsymbol{v} = (v_1, \cdots, v_l)^T$ is an indicator vector, with $v_j = 1$ representing the transmission of $x_{\mathcal{R}}^j$, and $v_j = 0$ representing being silent of the relay. Also in (2), $\boldsymbol{y}_{\mathcal{RD}} = (y_{\mathcal{RD}}^1, \cdots, y_{\mathcal{RD}}^l)^T$ is the received signal vector, and $\boldsymbol{n}_{\mathcal{RD}} = (n_{\mathcal{RD}}^1, \cdots, n_{\mathcal{RD}}^l)^T$ is the AWGN vector at the destination.

III. Threshold-based One-Bit Soft Forwarding Protocol

Without loss of generality, we focus on the j-th symbol in $\boldsymbol{x}_i$, i.e., x_i^j. Based on the detection/decoding results of x_1^j and x_2^j, the relay calculates the LLR value for each network coded symbol $x_{\mathcal{R}}^j$ in $\boldsymbol{x}_{\mathcal{R}}$. We denote by $L_{x_i^j,\mathcal{R}}$ the LLR value of x_i^j after detection/decoding at the relay, and denote by $L_{x_{\mathcal{R}}^j,\mathcal{R}}$ the LLR value of $x_{\mathcal{R}}^j$. Then we have

$$L_{x_{\mathcal{R}}^j,\mathcal{R}} = 2\tanh^{-1}\left(\tanh\left(\frac{L_{x_1^j,\mathcal{R}}}{2}\right)\tanh\left(\frac{L_{x_2^j,\mathcal{R}}}{2}\right)\right), \quad (3)$$

where $\tanh(z) = \frac{\exp(2z)-1}{\exp(2z)+1}$ is the hyperbolic function. The LLR $L_{x_i^j,\mathcal{R}}$ is calculated based on the conditional probability density function (PDF) of $y_{i\mathcal{R}}^j$, i.e.,

$$L_{x_i^j,\mathcal{R}} = \ln\frac{p(y_{i\mathcal{R}}^j | x_i^j = 1, h_{i\mathcal{R}})}{p(y_{i\mathcal{R}}^j | x_i^j = -1, h_{i\mathcal{R}})} = \frac{2h_{i\mathcal{R}}}{\sigma^2}y_{i\mathcal{R}}^j. \quad (4)$$

After obtaining $L_{x_{\mathcal{R}}^j,\mathcal{R}}$ based on (4), the relay compares $|L_{x_{\mathcal{R}}^j,\mathcal{R}}|$ with a preset threshold L_T ($L_T \geq 0$). If $|L_{x_{\mathcal{R}}^j,\mathcal{R}}|$ is larger than L_T, the j-th indicator v_j in the vector $\boldsymbol{v}$ is set to one, otherwise it is set to zero. By applying the vector $\boldsymbol{v}$, the relay either transmits BPSK symbols or keeps silent.

In the following, we will discuss the optimizations of the threshold L_T and the power allocation α at the relay to enhance the BER performance at the destination. Practically, we assume that the instantaneous channel state information (CSI) of $h_{1\mathcal{R}}$ and $h_{2\mathcal{R}}$, and the statistical CSI of $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$ are available at the relay during the optimizations.

IV. Optimizations and Performance Analysis

A. Threshold Optimization

We denote by $\mathcal{E}_c$ the event that $|L_{x_{\mathcal{R}}^j,\mathcal{R}}|$ is larger than L_T and $x_{\mathcal{R}}^j$ is correct, by $\mathcal{E}_e$ the event that $|L_{x_{\mathcal{R}}^j,\mathcal{R}}|$ is larger than L_T and $x_{\mathcal{R}}^j$ is in error, and by $\mathcal{E}_s$ the event that $|L_{x_{\mathcal{R}}^j,\mathcal{R}}|$ is no larger than L_T, i.e.,

$$\begin{aligned} \mathcal{E}_c: &\quad |L_{x_{\mathcal{R}}^j,\mathcal{R}}| > L_T, \text{ and } x_{\mathcal{R}}^j = x_1^j x_2^j, \\ \mathcal{E}_e: &\quad |L_{x_{\mathcal{R}}^j,\mathcal{R}}| > L_T, \text{ and } x_{\mathcal{R}}^j \neq x_1^j x_2^j, \\ \mathcal{E}_s: &\quad |L_{x_{\mathcal{R}}^j,\mathcal{R}}| \leq L_T. \end{aligned} \quad (5)$$

We denote by P_e the average BER of the two sources at the destination. We have $P_e = \frac{1}{2}(P_{e,1} + P_{e,2})$, where $P_{e,i}$ is the BER of $\mathcal{S}_i$ at the destination. The value of $P_{e,i}$ can be calculated as $P_{e,i} = P_{i,mrc}\Pr(\mathcal{E}_c) + P_{i,neg}\Pr(\mathcal{E}_e) + P_{i,dir}\Pr(\mathcal{E}_s)$, where $P_{i,mrc}$, $P_{i,neg}$, and $P_{i,dir}$ are the error probabilities of the symbol x_i^j at the destination given that the relay forwards the correct $x_{\mathcal{R}}^j$ to the destination, forwards the incorrect $x_{\mathcal{R}}^j$ to the destination, and keeps silent, respectively.

We focus on the calculations of $\Pr(\mathcal{E}_c)$, $\Pr(\mathcal{E}_e)$, and $\Pr(\mathcal{E}_s)$. First, we investigate the PDF of the LLR value $L_{x_{\mathcal{R}}^j,\mathcal{R}}$ at the relay. For a given channel realization, $L_{x_{\mathcal{R}}^j,\mathcal{R}}$ can be approximated as a Gaussian variable, whose variance is twice the absolute value of its mean [12]. Therefore, the PDF of $L_{x_{\mathcal{R}}^j,\mathcal{R}}$ can be written as

$$p_{L_{x_{\mathcal{R}}^j,\mathcal{R}}}(L) = \frac{1}{\sqrt{2\pi}\sigma_{L\oplus}} \exp\left(-\frac{(L-m_{L\oplus})^2}{2\sigma_{L\oplus}^2}\right), \quad (6)$$

where $m_{L\oplus}$ is the mean value, $\sigma_{L\oplus}^2$ is the variance of the $L_{x_{\mathcal{R}}^j,\mathcal{R}}$, and we have $\sigma_{L\oplus}^2 = 2|m_{L\oplus}|$.

Then we determine $\Pr(\mathcal{E}_c)$, $\Pr(\mathcal{E}_e)$, and $\Pr(\mathcal{E}_s)$, which depend on the PDF $p_{L_{x_{\mathcal{R}}^j,\mathcal{R}}}(L)$ and the threshold L_T. Without loss of generality, we consider $x_1^j x_2^j = 1$, and we have $\Pr(\mathcal{E}_c) = \int_{L_T}^{+\infty} p_{L_{x_{\mathcal{R}}^j,\mathcal{R}}}(L)\mathrm{d}L$, $\Pr(\mathcal{E}_e) = \int_{-\infty}^{-L_T} p_{L_{x_{\mathcal{R}}^j,\mathcal{R}}}(L)\mathrm{d}L$, and $\Pr(\mathcal{E}_s) = 1 - \Pr(\mathcal{E}_c) - \Pr(\mathcal{E}_e)$. At the destination, the average BER P_e can be minimized by optimizing the threshold L_T. The following theorem offers the solution of this optimization problem.

Theorem 1: The optimal L_T that minimizes the BER at the destination is expressed as

$$L_T^\star = \ln \frac{P_{neg} - P_{dir}}{P_{dir} - P_{mrc}}, \quad (7)$$

where $P_{mrc} = \frac{1}{2}(P_{1,mrc} + P_{2,mrc})$, $P_{neg} = \frac{1}{2}(P_{1,neg} + P_{2,neg})$, and $P_{dir} = \frac{1}{2}(P_{1,dir} + P_{2,dir})$. ■

Proof: We omit all of our proofs due to space considerations. Full proofs will appear in an extended version of this paper. ■

B. Error Probabilities Analysis

We can see from (7) that the optimal threshold $L_T^\star$ is based on $P_{i,mrc}$, $P_{i,neg}$, and $P_{i,dir}$ at the destination, which are related to the channels $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$. Now, we will determine these three probabilities when the statistical CSI of $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$ is available at the relay. We assume that LLR combining is applied at the destination to detect the symbol x_i^j. We denote by $L_{x_i^j,\mathcal{D}}$ and $L_{x_{\mathcal{R}}^j,\mathcal{D}}$ the received LLR values at the destination for x_i^j and $x_{\mathcal{R}}^j$, respectively, and denote by $L_{x_i^j,\mathcal{D},ext}$ the extrinsic LLR for x_i^j from the network coding between $x_{\bar{i}}^j$ and $x_{\mathcal{R}}^j$, where $\bar{i} = 1, 2$, $\bar{i} \neq i$. The combined LLR, denoted by $L_{x_i^j,\mathcal{D},comb}$, is calculated as

$$\begin{aligned} L_{x_i^j,\mathcal{D},comb} &= L_{x_i^j,\mathcal{D}} + L_{x_i^j,\mathcal{D},ext} \\ &= L_{x_i^j,\mathcal{D}} + 2\tanh^{-1}\left(\tanh\left(\frac{L_{x_{\bar{i}}^j,\mathcal{D}}}{2}\right)\tanh\left(\frac{L_{x_{\mathcal{R}}^j,\mathcal{D}}}{2}\right)\right). \end{aligned} \quad (8)$$

In (8), given $h_{i\mathcal{D}}$ and $h_{\mathcal{RD}}$, the received LLRs at the destination, i.e., $L_{x_i^j,\mathcal{D}}$ and $L_{x_{\mathcal{R}}^j,\mathcal{D}}$, are Gaussian distributed with their variances being twice the absolute values of their means. In addition, the LLR $L_{x_i^j,\mathcal{D},ext}$ from the network coding process can be approximated as a Gaussian variable with its variance being twice the absolute value of its mean [12].

Denote by $m_{L_{i\mathcal{D}}} \triangleq \frac{2h_{i\mathcal{D}}^2 x_i^j}{\sigma^2}$ the mean value of $L_{x_i^j,\mathcal{D}}$, and by $m_{L_{\mathcal{RD}}} \triangleq \frac{2h_{\mathcal{RD}}^2 \alpha v_j^2 x_{\mathcal{R}}^j}{\sigma^2}$ the mean value of $L_{x_{\mathcal{R}}^j,\mathcal{D}}$. Then the mean value of $L_{x_i^j,\mathcal{D},ext}$, denoted by $m_{L_{i\mathcal{D}},ext}$, can be calculated by utilizing the function $\phi(\cdot)$ defined in [12], i.e.,

$$\begin{aligned} m_{L_{i\mathcal{D}},ext} = x_{\bar{i}}^j x_{\mathcal{R}}^j \phi^{-1}(\phi(|m_{L_{\bar{i}\mathcal{D}}}|) + \\ \phi(|m_{L_{\mathcal{RD}}}|) - \phi(|m_{L_{\bar{i}\mathcal{D}}}|)\phi(|m_{L_{\mathcal{RD}}}|)), \end{aligned} \quad (9)$$

where the function $\phi(\cdot)$ is expressed as

$$\phi(z) = \begin{cases} 1 - \frac{1}{\sqrt{4\pi z}} \int_{-\infty}^{\infty} \tanh\left(\frac{u}{2}\right) \exp\left(-\frac{(u-z)^2}{4z}\right) \mathrm{d}u, & z > 0, \\ 1, & z = 0. \end{cases} \quad (10)$$

When $z > 0$, the function $\phi(\cdot)$ is bounded as [12]

$$\sqrt{\frac{\pi}{z}} \exp\left(-\frac{z}{4}\right)\left(1 - \frac{3}{z}\right) < \phi(z) < \sqrt{\frac{\pi}{z}} \exp\left(-\frac{z}{4}\right)\left(1 + \frac{1}{7z}\right). \quad (11)$$

We can see from (11) that when z is large enough, the two bounds converge to $\sqrt{\frac{\pi}{z}} \exp\left(-\frac{z}{4}\right)$. Therefore, given z_1 and z_2, when both of them are large enough, we have

$$\phi(z_1) + \phi(z_2) - \phi(z_1)\phi(z_2) \approx \max(\phi(z_1), \phi(z_2)). \quad (12)$$

Similarly, we can approximate $m_{L_{i\mathcal{D}},ext}$ in (9) in the high SNR region as

$$\begin{aligned} m_{L_{i\mathcal{D}},ext} &\approx x_{\bar{i}}^j x_{\mathcal{R}}^j \phi^{-1}(\max(\phi(|m_{L_{\bar{i}\mathcal{D}}}|), \phi(|m_{L_{\mathcal{RD}}}|))) \\ &= \frac{2x_{\bar{i}}^j x_{\mathcal{R}}^j}{\sigma^2} \min(h_{\bar{i}\mathcal{D}}^2, h_{\mathcal{RD}}^2 \alpha v_j^2). \end{aligned} \quad (13)$$

As we have $L_{x_i^j,\mathcal{D},comb} = L_{x_i^j,\mathcal{D}} + L_{x_i^j,\mathcal{D},ext}$, the combined LLR $L_{x_i^j,\mathcal{D},comb}$ can be viewed as a Gaussian variable with the mean $m_{L_{i\mathcal{D}}} + m_{L_{i\mathcal{D}},ext}$ and variance $2(|m_{L_{i\mathcal{D}}}| + |m_{L_{i\mathcal{D}},ext}|)$. Given the channel coefficients $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$, the instantaneous error probabilities can be obtained by utilizing $Q(\cdot)$ function, where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp\left(-\frac{u^2}{2}\right) \mathrm{d}u$. Specifically, when $\mathcal{E}_c$ happens to the symbol $x_{\mathcal{R}}^j$, we have $x_i^j = x_{\bar{i}}^j x_{\mathcal{R}}^j$. In this case, $m_{L_{i\mathcal{D}}}$ and $m_{L_{i\mathcal{D}},ext}$ have the same sign, and thus $|m_{L_{i\mathcal{D}}} + m_{L_{i\mathcal{D}},ext}| = |m_{L_{i\mathcal{D}}}| + |m_{L_{i\mathcal{D}},ext}|$. Therefore, we obtain the error probability of x_i^j as

$$P_{i,mrc} = Q\left(\sqrt{\frac{|m_{L_{i\mathcal{D}}}| + |m_{L_{i\mathcal{D}},ext}|}{2}}\right). \quad (14)$$

When $\mathcal{E}_e$ happens, we have $x_i^j = -x_{\bar{i}}^j x_{\mathcal{R}}^j$. In this case, $m_{L_{i\mathcal{D}}}$ and $m_{L_{i\mathcal{D}},ext}$ have the opposite signs, and we have $|m_{L_{i\mathcal{D}}} + m_{L_{i\mathcal{D}},ext}| = ||m_{L_{i\mathcal{D}}}| - |m_{L_{i\mathcal{D}},ext}||$. The calculation of $P_{i,neg}$ depends on the relation between $|m_{L_{i\mathcal{D}}}|$ and $|m_{L_{i\mathcal{D}},ext}|$, i.e.,

$$P_{i,neg} = \begin{cases} Q\left(\sqrt{\frac{(|m_{L_{i\mathcal{D}}}| - |m_{L_{i\mathcal{D}},ext}|)^2}{2(|m_{L_{i\mathcal{D}}}| + |m_{L_{i\mathcal{D}},ext}|)}}\right), & |m_{L_{i\mathcal{D}}}| > |m_{L_{i\mathcal{D}},ext}|, \\ 1 - Q\left(\sqrt{\frac{(|m_{L_{i\mathcal{D}}}| - |m_{L_{i\mathcal{D}},ext}|)^2}{2(|m_{L_{i\mathcal{D}}}| + |m_{L_{i\mathcal{D}},ext}|)}}\right), & |m_{L_{i\mathcal{D}}}| \le |m_{L_{i\mathcal{D}},ext}|. \end{cases} \quad (15)$$

When $\mathcal{E}_s$ happens, we have $P_{i,dir} = Q\left(\sqrt{|m_{L_{i\mathcal{D}}}|/2}\right)$.

The statistical CSI based error probabilities can be obtained by averaging $P_{i,mrc}$, $P_{i,neg}$, and $P_{i,dir}$ over $h_{1\mathcal{D}}^2$, $h_{2\mathcal{D}}^2$, and $h_{\mathcal{RD}}^2$, where the PDFs of $h_{i\mathcal{D}}^2$ and $h_{\mathcal{RD}}^2$ are $p_{h_{i\mathcal{D}}^2}(h) = \frac{1}{\lambda_{i\mathcal{D}}}\exp\left(-\frac{h}{\lambda_{i\mathcal{D}}}\right)$ and $p_{h_{\mathcal{RD}}^2}(h) = \frac{1}{\lambda_{\mathcal{RD}}}\exp\left(-\frac{h}{\lambda_{\mathcal{RD}}}\right)$, respectively. Alternatively, by defining $m_{L_{i\mathcal{D}},mrc} \triangleq |m_{L_{i\mathcal{D}}}| + |m_{L_{i\mathcal{D}},ext}|$, by defining $m_{L_{i\mathcal{D}},neg} \triangleq \frac{\left(|m_{L_{i\mathcal{D}}}|-|m_{L_{i\mathcal{D}},ext}|\right)^2}{|m_{L_{i\mathcal{D}}}|+|m_{L_{i\mathcal{D}},ext}|}$, and by defining $m_{L_{i\mathcal{D}},dir} \triangleq |m_{L_{i\mathcal{D}}}|$, we average $P_{i,mrc}$, $P_{i,neg}$, and $P_{i,dir}$ over the variables $m_{L_{i\mathcal{D}},mrc}$, $m_{L_{i\mathcal{D}},neg}$, and $m_{L_{i\mathcal{D}},dir}$, respectively. The PDFs of these three variables are given in the following lemma.

Lemma 1: The PDF of $m_{L_{i\mathcal{D}},mrc}$ can be expressed as $p_{m_{L_{i\mathcal{D}},mrc}}(m) = \frac{\lambda_i\bar{\lambda}_i}{\lambda_i-\bar{\lambda}_i}\left(\exp\left(-\bar{\lambda}_i m\right) - \exp\left(-\lambda_i m\right)\right)$, where $\lambda_i = \frac{1}{\frac{2}{\sigma^2}\lambda_{i\mathcal{D}}}$ and $\bar{\lambda}_i = \frac{1}{\frac{2}{\sigma^2}\lambda_{\bar{i}\mathcal{D}}} + \frac{1}{\frac{2\alpha}{\sigma^2}\lambda_{\mathcal{RD}}}$. The PDF of $m_{L_{i\mathcal{D}},neg}$ can be expressed as $p_{m_{L_{i\mathcal{D}},neg}}(m) = \frac{\lambda_i^2\bar{\lambda}_i^2}{\lambda_i^2-\bar{\lambda}_i^2}\int_0^\infty \frac{u^2}{\sqrt{m}}(\exp\left(-\bar{\lambda}_i u^2 - \lambda_i\sqrt{m}u\right) - \exp\left(-\lambda_i u^2 - \lambda_i\sqrt{m}u\right))\mathrm{d}u$. The PDF of $m_{L_{i\mathcal{D}},dir}$ can be expressed as $p_{m_{L_{i\mathcal{D}},dir}}(m) = \lambda_i\exp\left(-\lambda_i m\right)$. ■

Based on *Lemma 1*, we can derive the statistical CSI based error probabilities, i.e., the expectations of $P_{i,mrc}$, $P_{i,neg}$, and $P_{i,dir}$. Specifically, the expectation of $P_{i,mrc}$ is calculated as

$$\mathbb{E}(P_{i,mrc}) = \frac{\lambda_i}{\lambda_i-\bar{\lambda}_i}\int_0^\infty Q\left(\sqrt{\frac{m}{2}}\right)\bar{\lambda}_i\exp\left(-\bar{\lambda}_i m\right)\mathrm{d}m - \frac{\bar{\lambda}_i}{\lambda_i-\bar{\lambda}_i}\int_0^\infty Q\left(\sqrt{\frac{m}{2}}\right)\lambda_i\exp\left(-\lambda_i m\right)\mathrm{d}m = \frac{\frac{1}{2}\lambda_i}{(\lambda_i-\bar{\lambda}_i)}\left(1-\sqrt{\frac{1}{1+4\bar{\lambda}_i}}\right) - \frac{\frac{1}{2}\bar{\lambda}_i}{(\lambda_i-\bar{\lambda}_i)}\left(1-\sqrt{\frac{1}{1+4\lambda_i}}\right). \tag{16}$$

The expectation of $P_{i,neg}$ is calculated as follows. First, we calculate the probability that $|m_{L_{i\mathcal{D}}}| > |m_{L_{i\mathcal{D}},ext}|$, i.e., $\Pr(|m_{L_{i\mathcal{D}}}| > |m_{L_{i\mathcal{D}},ext}|)$. We have

$$\Pr(|m_{L_{i\mathcal{D}}}| > |m_{L_{i\mathcal{D}},ext}|) = \frac{\bar{\lambda}_i}{\lambda_i+\bar{\lambda}_i}. \tag{17}$$

Then we obtain the expectation of $P_{i,neg}$ as [6]

$$\begin{aligned}\mathbb{E}(P_{i,neg}) = &\int_0^\infty Q\left(\sqrt{\frac{m}{2}}\right)p_{m_{L_{i\mathcal{D}},neg}}(m)\mathrm{d}m\,\Pr(|m_{L_{i\mathcal{D}}}| > |m_{L_{i\mathcal{D}},ext}|)\\ &+\left(1-\int_0^\infty Q\left(\sqrt{\frac{m}{2}}\right)p_{m_{L_{i\mathcal{D}},neg}}(m)\mathrm{d}m\right)\cdot\\ &\Pr(|m_{L_{i\mathcal{D}}}| \le |m_{L_{i\mathcal{D}},ext}|)\\ =&\frac{\lambda_i}{\lambda_i+\bar{\lambda}_i} - \frac{\lambda_i-\bar{\lambda}_i}{\lambda_i+\bar{\lambda}_i}\int_0^\infty Q\left(\sqrt{\frac{m}{2}}\right)p_{m_{L_{i\mathcal{D}},neg}}(m)\mathrm{d}m\\ =&\frac{\lambda_i}{\lambda_i+\bar{\lambda}_i} - \frac{\lambda_i^2\bar{\lambda}_i^2}{(\lambda_i+\bar{\lambda}_i)^2}\left(f\left(\frac{1}{4},\bar{\lambda}_i,\lambda_i\right) - f\left(\frac{1}{4},\lambda_i,\lambda_i\right)\right),\end{aligned} \tag{18}$$

where the function $f(\cdot)$ is defined in Equation (A·4) in [6]. The expectation of $P_{i,dir}$ is calculated as

$$\begin{aligned}\mathbb{E}(P_{i,dir}) &= \int_0^\infty Q\left(\sqrt{\frac{m}{2}}\right)\lambda_i\exp\left(-\lambda_i m\right)\mathrm{d}m\\ &= \frac{1}{2}\left(1-\sqrt{\frac{1}{1+4\lambda_i}}\right).\end{aligned} \tag{19}$$

Hence, when the statistical CSI of $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$ is available at the relay, we can obtain the optimal threshold $L_T^\star$ based on (7) with $\mathbb{E}(P_{i,mrc})$, $\mathbb{E}(P_{i,neg})$, and $\mathbb{E}(P_{i,dir})$.

C. Diversity Gain

We now investigate the diversity order of the system with our TOB-SF protocol. When the SNR $\rho = \frac{1}{\sigma^2}$ is large enough, we make approximations on $\mathbb{E}(P_{i,mrc})$, $\mathbb{E}(P_{i,neg})$, and $\mathbb{E}(P_{i,dir})$. First, since the function $f(\cdot)$ in (18) is proportional to ρ^{-1} [6], it is straightforward to obtain the approximation of $\mathbb{E}(P_{i,neg})$ from (18), i.e.,

$$\mathbb{E}(P_{i,neg}) \approx \frac{\lambda_i}{\lambda_i+\bar{\lambda}_i} = \frac{\lambda_{\bar{i}\mathcal{D}}\lambda_{\mathcal{RD}}}{\lambda_{\bar{i}\mathcal{D}}\lambda_{\mathcal{RD}} + \lambda_{i\mathcal{D}}\lambda_{\mathcal{RD}} + \frac{1}{\alpha}\lambda_{i\mathcal{D}}\lambda_{\bar{i}\mathcal{D}}}. \tag{20}$$

By utilizing the property of the function $Q(\cdot)$, we have

$$\mathbb{E}(P_{i,mrc}) \approx \frac{3}{4\lambda_{i\mathcal{D}}\frac{\alpha\lambda_{\bar{i}\mathcal{D}}\lambda_{\mathcal{RD}}}{\lambda_{\bar{i}\mathcal{D}}+\alpha\lambda_{\mathcal{RD}}}}\rho^{-2}, \mathbb{E}(P_{i,dir}) \approx \frac{1}{2\lambda_{i\mathcal{D}}}\rho^{-1}. \tag{21}$$

Based on (20) and (21), when ρ is large enough, we can approximate the statistical CSI based optimal threshold as

$$\begin{aligned}L_T^\star &\approx \ln\frac{\mathbb{E}(P_{1,neg})+\mathbb{E}(P_{2,neg})}{\mathbb{E}(P_{1,dir})+\mathbb{E}(P_{2,dir})}\\ &\approx \ln\rho + \ln\frac{2\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}}{\lambda_{1\mathcal{D}}\lambda_{\mathcal{RD}} + \lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}} + \frac{1}{\alpha}\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}}.\end{aligned} \tag{22}$$

By defining $c \triangleq \ln\frac{2\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}}{\lambda_{1\mathcal{D}}\lambda_{\mathcal{RD}}+\lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}+\frac{1}{\alpha}\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}}$, we can see that c does not depend on ρ, but depends on the channel gains and the power allocation α. Hence, we have $\lim_{\rho\to\infty} L_T^\star = \ln\rho$.

We now focus on the diversity gain of each source with the optimal threshold $L_T^\star$. Since we have $\lim_{\rho\to\infty} L_T^\star = \ln\rho$, we only need to investigate the diversity gain with the threshold $\ln\rho$.

Theorem 2: By utilizing the TOB-SF protocol with the threshold $\ln\rho$, each source can achieve the full diversity gain of the system, i.e., a diversity of two. ■

Remark 1: The optimal threshold can be expressed in the form of $\ln\rho + c$*, where* $\ln\rho$ *and* c *are the keys to guaranteeing the full diversity gain and optimal coding gain, respectively.*

D. Power Allocation

In this subsection, we will further enhance the error performance by optimizing the power allocation α at the relay. Specifically, by averaging $P_{e,i}$ over the channels $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$, we have $\mathbb{E}(P_{e,i}) = \mathbb{E}(P_{i,mrc})\Pr(\mathcal{E}_c) + \mathbb{E}(P_{i,neg})\Pr(\mathcal{E}_e) + \mathbb{E}(P_{i,dir})\Pr(\mathcal{E}_s)$. We minimize the average BER, i.e., $\mathbb{E}(P_e) = \frac{1}{2}(\mathbb{E}(P_{e,1}) + \mathbb{E}(P_{e,2}))$, by optimizing α, and obtain the optimal α by solving $\frac{\partial\mathbb{E}(P_e)}{\partial\alpha} = 0$ with the constraint $0 < \alpha \le 1$.

However, from the expression of the optimal threshold $L_T^\star$ in (22), we can see that $L_T^\star$ is a function of α. If $L_T^\star$ is used as the threshold, it is difficult to obtain the closed form of the optimal α when solving $\frac{\partial \mathbb{E}(P_e)}{\partial \alpha} = 0$. Fortunately, from the previous subsection, we know that $\ln \rho$ can serve as a sub-optimal threshold to achieve the full diversity gain. Since $\ln \rho$ is independent of α, the items $\Pr(\mathcal{E}_c)$, $\Pr(\mathcal{E}_e)$, and $\Pr(\mathcal{E}_s)$ are constants relative to α if we use $\ln \rho$ as the threshold. Hence, the derivation of the optimal α can be simplified and a closed form of the optimal α is obtained. In the sequel, we optimize α by assuming that $\ln \rho$ is applied as the threshold.

With the power constraint $0 < \alpha \leq 1$, by utilizing the approximations in (20) and (21), we derive the optimal power allocation, denoted by $\alpha^\star$, at the relay as follows. First, we relax the constraint of α, and let $\frac{\partial \mathbb{E}(P_e)}{\partial \alpha} = 0$, which lead to two solutions for α, i.e.,

$$\begin{aligned} \alpha_1 &= \frac{\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}}{\sqrt{\frac{4\Pr(\mathcal{E}_e)}{3\Pr(\mathcal{E}_c)}\rho\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}} - \lambda_{1\mathcal{D}}\lambda_{\mathcal{RD}} - \lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}}, \\ \alpha_2 &= \frac{\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}}{-\sqrt{\frac{4\Pr(\mathcal{E}_e)}{3\Pr(\mathcal{E}_c)}\rho\lambda_{1\mathcal{D}}\lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}} - \lambda_{1\mathcal{D}}\lambda_{\mathcal{RD}} - \lambda_{2\mathcal{D}}\lambda_{\mathcal{RD}}}. \end{aligned} \tag{23}$$

Note that from (23), we have $\alpha_2 < 0$ and $\alpha_2 < \alpha_1$. Also, α_1 can be a negative value or a positive value, depending on $\Pr(\mathcal{E}_c)$, $\Pr(\mathcal{E}_e)$, ρ, and the channel gains.

Then we investigate the optimal power allocation $\alpha^\star$ based on the two value ranges of α_1, i.e., $\alpha_1 \leq 0$ and $\alpha_1 > 0$. When $\alpha_1 \leq 0$, we first relax the constraint of α and have the following two facts. (i) $\mathbb{E}(P_e)$ is a monotonically increasing function (MIF) of α if $\alpha_2 < \alpha \leq \alpha_1$, and (ii) $\mathbb{E}(P_e)$ is a monotonically decreasing function (MDF) of α if $\alpha \leq \alpha_2$ or $\alpha > \alpha_1$. Then we consider the optimal α with the constraint $0 < \alpha \leq 1$. We can see that within the range $0 < \alpha \leq 1$, $\mathbb{E}(P_e)$ is an MDF of α. Therefore, the optimal power allocation can be obtained as $\alpha^\star = 1$ when $\alpha_1 \leq 0$.

When $\alpha_1 > 0$, we first relax the constraint of α and have the following two facts. (i) $\mathbb{E}(P_e)$ is an MDF of α if $\alpha_2 < \alpha \leq \alpha_1$, and (ii) $\mathbb{E}(P_e)$ is an MIF of α if $\alpha \leq \alpha_2$ or $\alpha > \alpha_1$. Then we optimize α with the constraint $0 < \alpha \leq 1$. We can see that if $\alpha_1 > 1$, $\mathbb{E}(P_e)$ is an MDF of α in the range $0 < \alpha \leq 1$. Thus the optimal power allocation can be obtained as $\alpha^\star = 1$ when $\alpha_1 > 1$. Also, if $0 < \alpha_1 \leq 1$, $\mathbb{E}(P_e)$ is an MDF of α when $0 < \alpha \leq \alpha_1$, and is an MIF of α when $\alpha_1 < \alpha \leq 1$. Thus, the optimal power allocation can be obtained as $\alpha^\star = \alpha_1$ when $0 < \alpha_1 \leq 1$. With above discussions, the optimal power allocation $\alpha^\star$ at the relay is given as

$$\alpha^\star = \begin{cases} 1, & \alpha_1 \leq 0 \text{ or } \alpha_1 > 1, \\ \alpha_1, & 0 < \alpha_1 \leq 1. \end{cases} \tag{24}$$

Intuitively, from (23), when the source-to-relay channels are good which means $\frac{4\Pr(\mathcal{E}_e)}{3\Pr(\mathcal{E}_c)}$ is very small, we have $\alpha_1 < 0$. Thus, the relay should transmit with its full power. Once the source-to-relay channels are poor enough such that $0 < \alpha_1 \leq 1$, the relay should transmit with less power, i.e., $\alpha^\star = \alpha_1$.

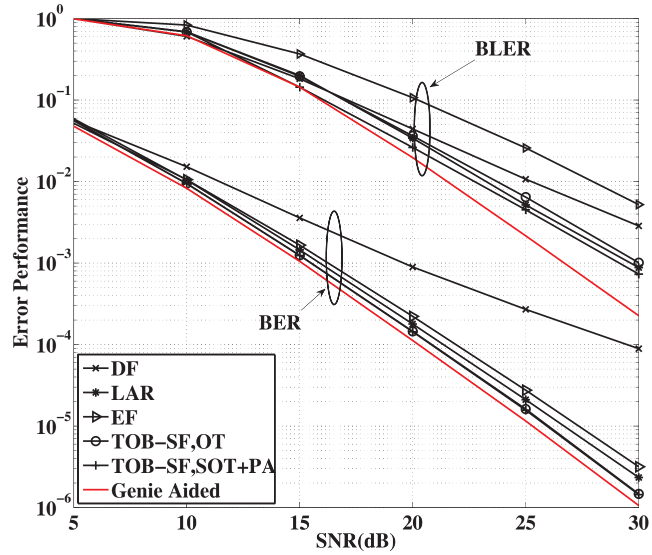

Fig. 2. Error performance for Case 1, i.e., the strong source-to-relay link scenario.

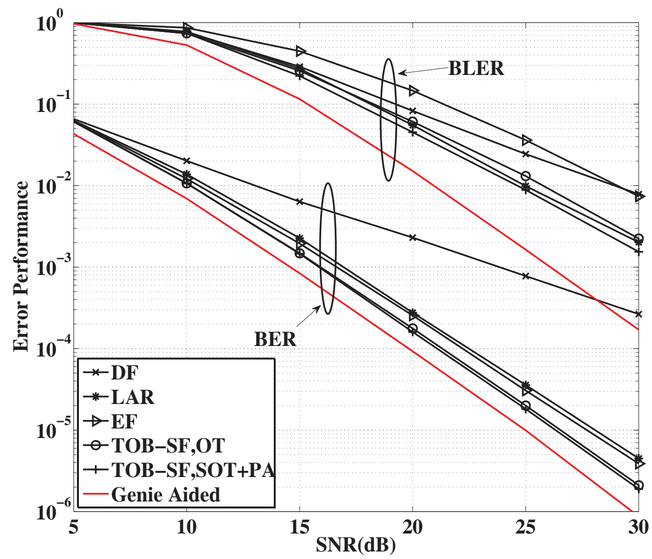

Fig. 3. Error performance for Case 2, i.e., the symmetric channel scenario.

V. Simulations

In the simulations, we set the frame (or codeword) length as $l = 10,000$. Hence, each transmission period is of length $30,000$. As we consider quasi-static fading channels, the channel coefficients $h_{1\mathcal{R}}$, $h_{2\mathcal{R}}$, $h_{1\mathcal{D}}$, $h_{2\mathcal{D}}$, and $h_{\mathcal{RD}}$ are constant in each transmission period, and change independently from one period to another.

We focus on a symmetric scenario where (i) the two sources have the same distance to the relay and the same distance to the destination, and (ii) the two sources, the relay, and the destination are aligned on the same horizontal line. The dis-

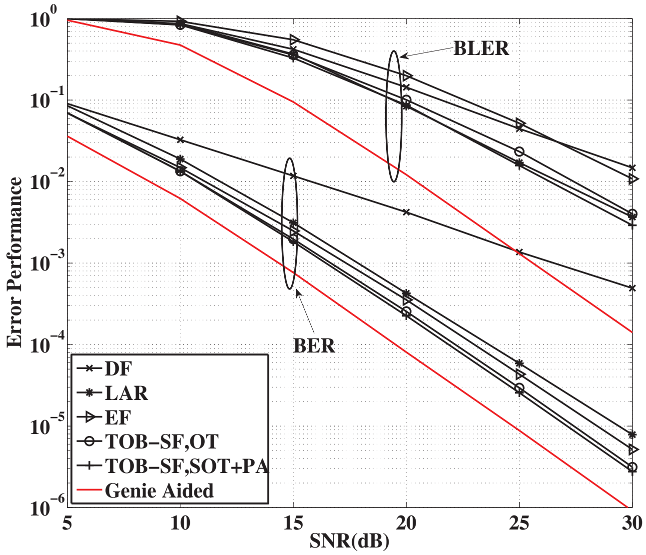

Fig. 4. Error performance for Case 3, i.e., the strong relay-to-destination link scenario.

tances between the sources and the destination are normalized as one, i.e., $d_{1\mathcal{D}} = d_{2\mathcal{D}} = 1$. The relay is located between the sources and the destination. In our simulations, we consider three cases, namely, Case 1, Case 2, and Case 3, according to the three different locations of the relay. Specifically, In Case 1, we consider a strong source-to-relay link scenario, where $d_{1\mathcal{R}} = d_{2\mathcal{R}} = 0.3$, and $d_{\mathcal{RD}} = 0.7$; In Case 2, we consider the symmetric scenario, where $d_{1\mathcal{R}} = d_{2\mathcal{R}} = 0.5$, and $d_{\mathcal{RD}} = 0.5$; In Case 3, we consider a strong relay-to-destination scenario, where $d_{1\mathcal{R}} = d_{2\mathcal{R}} = 0.7$, and thus $d_{\mathcal{RD}} = 0.3$. Also, we set the attenuation exponent $\gamma = 2$. Hence, we can obtain the channel gains $\lambda_{1\mathcal{R}}$, $\lambda_{2\mathcal{R}}$, $\lambda_{1\mathcal{D}}$, $\lambda_{2\mathcal{D}}$, and $\lambda_{\mathcal{RD}}$ based on γ and the corresponding distances.

We now evaluate the BER performance in the uncoded system. Besides the BER, we also investigate the block error rates (BLER) in the uncoded system, where a block consist two frames from the two sources. Fig. 2 shows the error performance for Case 1. In Fig. 2, the proposed TOB-SF protocol with the optimal threshold (OT) $L_T^\star$ is denoted by 'TOB-SF, OT', and the proposed TOB-SF protocol with the sub-optimal threshold (SOT) $\ln \rho$ plus the optimal power allocation $\alpha^\star$ (shown in (24)) is denoted by 'TOB-SF, SOT+PA'. We use the conventional DF and EF, and the link-adaptive regeneration (LAR) protocol (i.e., the power scaling scheme in [10]) as benchmarks, denoted by 'DF', 'EF', and 'LAR', respectively. Note that 'Genie Aided' protocol in Fig. 2 is a desired protocol, where the source-to-relay channels are always error free. From Fig. 2 we can see that 'DF' can only achieve a diversity of one, while all the other protocols can achieve the full diversity gain of the system, i.e., a diversity of two. Our 'TOB-SF, OT' and 'TOB-SF, SOT+PA' outperform 'DF', 'EF', and 'LAR' in terms of BER performance, and are close to the 'Genie Aided'. Also, 'TOB-SF, SOT+PA' is slightly better than 'TOB-SF, OT'. For the BLER performance, our 'TOB-SF, SOT+PA' outperforms 'DF', 'EF', 'LAR', and 'TOB-SF, OT'. 'LAR' is better than 'TOB-SF, OT'. And 'EF' performs the worst.

Fig. 3 and Fig. 4 show the error performance of Case 2 and Case 3, respectively. In both cases, our 'TOB-SF, OT' and 'TOB-SF, SOT+PA' can achieve the full diversity gain, and outperform the 'DF', 'EF' and 'LAR' in terms of BER performance. Also, 'TOB-SF, SOT+PA' has a better BER performance than 'TOB-SF, OT'. For the BLER performance in the two cases, the 'TOB-SF, SOT+PA' is always better than the 'LAR', while the 'LAR' is always better than 'TOB-SF, OT'. And the 'DF' and 'EF' always perform the worst.

VI. Conclusion

In this paper, we propose and optimize a TOB-SF protocol for a multi-source relay system with network coding. We first derive the BER expressions at the destination, and develop the optimal threshold that can minimize the BER performance. Then we prove theoretically that the TOB-SF protocol can achieve the full diversity gain by using the optimal threshold. Furthermore, we optimize the power allocation at the relay, by which, the system can achieve a better BER and BLER performance. Simulation results show that the proposed TOB-SF protocol with power allocation outperforms other conventional relaying protocols in terms of error performance.

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