Network Coded Power Adaptation Scheme in Non-orthogonal Multiple-Access Relay Channels

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***Abstract*—In this paper we propose a new power adaptive network coding (PANC) strategy for a non-orthogonal multiple-access relay channel (MARC), where two sources transmit their information simultaneously to the destination with the help of a relay. In contrast to the conventional XOR-based network coding (CXNC), the relay in PANC generates network coded symbols by considering the coefficients of the source-to-relay channels, and forwards each symbol with a pre-optimized power level. Next, we obtain the optimal power level by decomposing it as a multiplication of a power scaling factor and a power adaptation factor. We prove that with the power scaling factor at the relay, our PANC scheme can achieve a full diversity gain, i.e., an order of two diversity gain, while the CXNC can achieve only an order of one diversity gain. In addition, we optimize the power adaptation factor at the relay to minimize the SPER at the destination by considering of the relationship between SPER and minimum Euclidean distance of the received constellation, resulting in an improved coding gain. Simulation results show that the PANC scheme with power adaptation optimizations and power scaling factor design can achieve a full diversity, and obtain a much higher coding gain than other network coding schemes.**

I. INTRODUCTION

Network coding, originated from wire-line networks [1], [2], has been recently applied to wireless networks to enhance the network throughput [3]. Among the typical network models which are suitable for the applications of network coding, multiple access relay channel (MARC) has been recognized as a fundamental building block for cellular and wireless sensor networks. The MARC is a model for network topologies where multiple sources communicate with a single destination in the presence of a relay. A typical example is in a cellular systems where two mobile users communicate with a base station with the help of a relay. Different from the two-way relay channels (TWRC), the MARC only has imperfect information at the destination from the sources. Currently, there are a number of interesting research on network coding design in the MARC [4]–[6]. However, we note that most of these works consider orthogonal MARC, where multiple sources transmit their signals by using time-division or frequency-division multiple access. How to efficiently reduce the influence of error propagation without losing any source-relay information, e.g., dropping erroneous source-relay symbols, or stopping the communication when source-relay channel fails, is not considered in these works.

In this paper, we are interested in designing novel network coding schemes for a non-orthogonal MARC over fading channels to achieve a full diversity gain and a high coding gain. Although by dropping erroneous source-relay symbols, conventional XOR-based network coding (CXNC) has been shown to achieve a full diversity in both orthogonal and non-orthogonal MARC [7], we will show that CXNC without any error propagation mitigation process at relay cannot achieve full diversity gain in a non-orthogonal MARC due to multi-user interference.

There are two major concerns in our network coding design, namely diversity gain and coding gain. The first concern is how to achieve the full diversity gain of the MARC. We propose a novel power adaptation network coding (PANC) scheme to achieve the full diversity gain, in which the power at the relay has two levels. In contract to the CXNC, the relay in the proposed PANC generates network coded symbols by considering the coefficients of the source-to-relay channels, and forwards each symbol with one of the two given power levels. Based on the received signals, the relay decides which power level should be applied to each network coded symbol. We prove that the PANC scheme with the design of power scaling at the relay can achieve a full diversity gain, i.e., a diversity of two, while the CXNC scheme can achieve only a diversity of one. The second concern is how to achieve a high coding gain. We will minimize the SPER by optimizing the two power adaptation levels. Specifically, we propose a criteria based on the relationship between the Euclidean distance and the SPER, and formulate a convex optimization problem to develop the optimal power adaptation levels at the relay. Simulation results show that the PANC scheme with power level optimizations and power scaling factor design can achieve a full diversity and obtain a much higher coding gain than the PANC scheme with fixed chosen power levels. Also, the CXNC scheme cannot achieve a full diversity with or without the power scaling design.

The notations used in this paper are as follows. A line segment is denoted by AB, and $\overline{AB}$ denotes the length between points A and B. The one-dimensional Q-function is defined as $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta$. .

II. SYSTEM MODEL

Consider a two-source single-relay multiple-access relaying system, where two sources $\mathcal{S}_1$ and $\mathcal{S}_2$ transmit their information to the common destination $\mathcal{D}$ with the assistance of a half-duplex relay $\mathcal{R}$. Each transmission period is divided into two transmission phases. In a symbol time slot of the first transmission phase, the two sources simultaneously broadcast their symbols x_1 and x_2 to both the destination and the relay. In a symbol time slot of the second transmission phase, the two sources keep silent, while the relay processes the received signals and forwards the network coded symbol $x_{\mathcal{R}}$ to the destination. At the end of the second phase, the destination decodes the two sources' information based on the received signals.

We assume that all the transmitted signals are BPSK modulated with equal probability, i.e., $x_1, x_2, x_{\mathcal{R}} \in \{\pm 1\}$, and all the signals are transmitted in the same frequency band. The channel between any two given nodes j and k, $j \in \{1, 2, \mathcal{R}\}, k \in \{\mathcal{R}, \mathcal{D}\}$, and $j \neq k$, is denoted by h_{jk} with a subscript indicating the nodes under consideration. We assume that h_{jk} for all the j and k are Rayleigh distributed with a mean zero and variance $\bar{\gamma}_{jk}$. We consider slow fading channels in our system, i.e., the channel coefficients are constant during a transmission period, while they change independently from one transmission period to another.

Also, we implement the channel phase pre-equalization for both the source-to-destination multiple access channels (MAC) and the relay-to-destination channel before each transmission. Thus, the effective source-to-destination and relay-to-destination channel coefficients can be regarded as real-valued channels, i.e., real channel coefficients and real values of noise samples. The carrier phase synchronization is practical by adopting methods shown in [8] [9].

Based on the aforementioned system settings and assumptions, the received signals at the relay and destination in the first transmission phase can be written as

$$\begin{aligned} y_{\mathcal{R}} &= \sqrt{E_1} h_{1\mathcal{R}} x_1 + \sqrt{E_2} h_{2\mathcal{R}} x_2 + n_{\mathcal{R}}, \\ y_1 &= \sqrt{E_1} |h_{1\mathcal{D}}| x_1 + \sqrt{E_2} |h_{2\mathcal{D}}| x_2 + n_1, \end{aligned} \tag{1}$$

respectively, where E_1 and E_2 denote the transmission power of $\mathcal{S}_1$, $\mathcal{S}_2$, respectively, $n_{\mathcal{R}}$ is a complex additive white Gaussian noise (AWGN) sample at the relay with a zero mean and variance $\sigma^2/2$ per dimension, and n_1 is a real AWGN sample at the destination with zero mean and variance σ^2.

As we adopt the joint power scaling and adaptation scheme at the relay, the instantaneous power at the relay is optimized given the channel realization within each transmission period with the aim to minimize the SPER and achieve a full diversity at the destination. Specifically, in the power scaling, we have the scaling factor α $(0 \leq \alpha \leq 1)$ which is determined based on the channel conditions. In the power adaptation, we have two power levels, namely, $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$, related as $\tilde{\kappa}_1^2 + \tilde{\kappa}_2^2 \leq 2E_{\mathcal{R}}^{\text{ave}}$, where $E_{\mathcal{R}}^{\text{ave}}$ denotes the relay average transmission power. The calculations of $\alpha, \tilde{\kappa}_1, \tilde{\kappa}_2$ will be discussed later. Therefore, in the second transmission phase, the received signal at destination can be expressed as

$$y_2 = \sqrt{E_{\mathcal{R}}} |h_{\mathcal{R}\mathcal{D}}| x_{\mathcal{R}} + n_2, \tag{2}$$

where n_2 is a real AWGN sample at destination with a zero mean and variance σ^2, and $E_{\mathcal{R}} \in \{\kappa_1, \kappa_2\}$ represents the transmission power at relay with $\kappa_i = \alpha \tilde{\kappa}_i$ for $i \in \{1, 2\}$.

III. Network Coded Power Adaptation Scheme at the Relay

In the conventional network coding based MARC, XOR operations are implemented at the relay on the two sources' information. We will show later in Section IV that the system cannot achieve a full diversity with the conventional network coding. To achieve a full diversity, we propose the PANC scheme, i.e., based on the received signals, the relay transmits a network coded symbol multiplied with a perfected power level.

Firstly, the relay obtain the two sources' message symbols (x_1, x_2) from its received signal $y_{\mathcal{R}}$ by utilizing the maximum likelihood (ML) detection, i.e.,

$$(\hat{x}_1, \hat{x}_2) = \underset{\tilde{x}_1, \tilde{x}_2 \in \{\pm 1\}}{\arg\min} \left| y_{\mathcal{R}} - \sqrt{E_{1\mathcal{R}}} h_{1\mathcal{R}} \tilde{x}_1 - \sqrt{E_{2\mathcal{R}}} h_{2\mathcal{R}} \tilde{x}_2 \right|^2, \tag{3}$$

where $(\hat{\cdot})$ denotes the detected symbol, and $(\tilde{\cdot})$ denotes the trial symbol used in the hypothesis-detection problem. Then the relay performs a network coding operation on the two detected symbols. The network coded operation in our PANC [1] is denoted by $\boxplus$, which is different from the conventional XOR operation. That is, we calculate $x_{\mathcal{R}}$ by $x_{\mathcal{R}} = \hat{x}_1 \boxplus \hat{x}_2 = \text{sign}(|h_{1\mathcal{R}}|\hat{x}_1 + |h_{2\mathcal{R}}|\hat{x}_2)$. Next, the relay chooses the power level $E_{\mathcal{R}}$ based on the decoded symbols, i.e., if $(\hat{x}_1 = 1, \hat{x}_2 = 1)$ or $(\hat{x}_1 = -1, \hat{x}_2 = -1)$, power level is chosen as $\tilde{\kappa}_1$; else if $(\hat{x}_1 = 1, \hat{x}_2 = -1)$ or $(\hat{x}_1 = -1, \hat{x}_2 = 1)$, power level is chosen as $\tilde{\kappa}_2$. The reason for adopting the new proposed network coding operation and power level allocation method is that the received constellation at destination is a parallelogram, on which the $(\hat{x}_1 = 1, \hat{x}_2 = 1)$ corresponding constellation point lies in a diagonal with the $(\hat{x}_1 = -1, \hat{x}_2 = -1)$ corresponding constellation point. While for XOR operation, the received constellation is an irregular quadrilateral no matter what power level allocation result we implement. The values of power adaptation factors $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are optimized at the destination, and then feedbacks the values to the relay before the transmission period starts. Also, the power scaling factor is determined at relay. The relay will use the power level to transmit the network coded symbol $x_{\mathcal{R}}$. The details of power levels optimization will be introduced in Section IV.

Based on the observations y_1 and y_2, the destination jointly decode the two source symbols with the minimum Euclidean

[1] Although the proposed PANC strategy shares some similarities compared with DF strategy with a 4-PAM modulation, it is essentially different from such scheme since the power levels $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are not fixed in different system settings like 4-PAM modulation. They are optimized at destination according to instantaneous CSI, and feedback to relay to achieve a better error performance.

distance detection. Then we have

$$(\hat{x}_1, \hat{x}_2) = \underset{\tilde{x}_1, \tilde{x}_2 \in \{\pm 1\}}{\arg\min} \left(\left| y_1 - \sum_{j=1}^{2} |h_{j\mathcal{D}}| \tilde{x}_j \right|^2 + \left| y_2 - |h_{\mathcal{R}\mathcal{D}}| \sqrt{\tilde{E}_{\mathcal{R}}} \left(\tilde{x}_1 \boxplus \tilde{x}_2 \right) \right|^2 \right), \tag{4}$$

where $\tilde{E}_{\mathcal{R}} \in \{\kappa_1, \kappa_2\}$ is determined by $\tilde{x}_1$ and $\tilde{x}_2$.

IV. System Optimization

In this section, we first develop a practical method at the relay side to address the error propagation problem. With the designed methods, the system is proved to achieve a full diversity when the relay cannot detect all the received signals successfully. Specifically, we propose a power scaling scheme where the relay power adapts to the channel conditions. For such a link adaptive relaying (LAR) scheme, we model a complex MARC system as a degraded virtual one-source one-relay one destination model (triangle model), and show that the relay power should be chosen to balance the SNRs of source-relay channel and relay-destination channel. Moreover, we formulate a sub-optimal Euclidean distance optimization method to obtain the optimized system parameters $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ that minimize the end-to-end SPER.

A. The Design of Power Scaling Factor at the Relay

Before introducing the design of power scaling factor at the relay, we present the diversity performance of the proposed PANC and the CXNC scheme in the following Proposition. In the CXNC scheme, we do not consider any relay processes to identify or drop erroneous symbols and thus mitigate error propagation from source-relay channel to destination, e.g., outage event detection, CRC, and etc.

Theorem 1: Without power scaling factor at the relay, both the PANC scheme and the CXNC scheme can only achieve the diversity of one in MARC system for error propagation issue.

We leave the detailed proof for a longer version of this work. From *Theorem 1*, we can see that the error propagation from the source-relay hop degrades the performance of the system. In this case, we adopt a power scaling factor at the relay to mitigate the effect of error propagation by adjusting the relay transmission power to the channel conditions. Such a link adaptive ratio (LAR) was first introduced for the single-source decode-and-forward (DF) system in [10]. However, LAR cannot be directly applied to a multi-user power adaptive network coding system.

To extend the LAR concept, we first develop a virtual channel model for the source-relay-destination link, as shown in Fig. 4. In the first phase, the two sources transmit to the relay simultaneously. For such multiple-access channel, the upper bound for the instantaneous SPER is given by

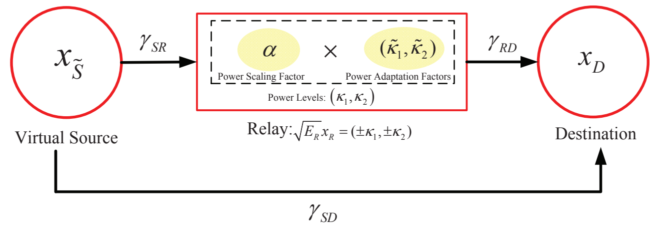

Fig. 1. Virtual channel model. In particular, the channel gains $\gamma_{\mathcal{S}\mathcal{R}}$ and $\gamma_{\mathcal{S}\mathcal{D}}$ with respect to virtual channels will be introduced in detail in the following paragraph. $x_{\tilde{\mathcal{S}}}$ is the virtual source message, which is generated from the original transmitted signal as $x_{\tilde{\mathcal{S}}} = x_1 \boxplus x_2$. $x_{\mathcal{R}}$ is the real transmitted signal from the relay to destination. And $x_{\mathcal{D}}$ is also equal to $x_1 \boxplus x_2$.

$$P_{\text{MAC}} \le P_{\text{MAC}}^{U} \triangleq Q\left(\sqrt{2E_1|h_{1\mathcal{R}}|^2/\sigma^2}\right) + Q\left(\sqrt{2E_2|h_{2\mathcal{R}}|^2/\sigma^2}\right) + Q\left(\sqrt{2|\sqrt{E_1}h_{1\mathcal{R}} + \sqrt{E_2}h_{2\mathcal{R}}|^2/\sigma^2}\right), \tag{5}$$

where the first term in the right side of (5) represents the error probability that the relay fails to decode x_1 and successfully decode x_2. Likewise, the second term is the error probability that the relay fails to decode x_2 while successfully decode x_1. Also, the third term is the error probability that the relay fails to decode both x_1 and x_2. The union bound $P_{\text{MAC}}^{\text{upper}}$ can be further approximated as (6), which is quite tight when $E_1|h_{1\mathcal{R}}|^2/\sigma^2$, $E_2|h_{2\mathcal{R}}|^2/\sigma^2$, $|\sqrt{E_1}h_{1\mathcal{R}} + \sqrt{E_2}h_{2\mathcal{R}}|^2/\sigma^2$ and their difference are reasonably large, as the Q-function $Q(x)$ decays fast as x grows. The advantage of such an approximation is that we can now model the multiple access source-relay channel as a single-input single-output channel with the input being the virtual source message $x_{\tilde{\mathcal{S}}} = x_1 \boxplus x_2$ and the instantaneous channel SNR being $\gamma_{\mathcal{S}\mathcal{R}} \triangleq \min(E_1|h_{1\mathcal{R}}|^2/\sigma^2, E_2|h_{2\mathcal{R}}|^2/\sigma^2, |\sqrt{E_1}h_{1\mathcal{R}} + \sqrt{E_2}h_{2\mathcal{R}}|^2/\sigma^2)$, which represents the SNR of the worse source-relay channel[2]. The idea of regarding a virtual source message as network coded sources' signals is that we implement network coding at the relay. Thus, the virtual transmitting information from source to destination via the aid of relay becomes the same, i.e., $x_{\tilde{\mathcal{S}}} = x_{\mathcal{R}}$. Similarly, we can model the multiple access source-destination channel as a point-to-point channel with the channel SNR, denoted by $\gamma_{\mathcal{S}\mathcal{D}}$, which is defined as

$$\gamma_{\mathcal{S}\mathcal{D}} \triangleq \min\left(\frac{E_1|h_{1\mathcal{D}}|^2}{\sigma^2}, \frac{E_2|h_{2\mathcal{D}}|^2}{\sigma^2}, \frac{|\sqrt{E_1}|h_{1\mathcal{D}}| + \sqrt{E_2}|h_{2\mathcal{D}}||^2}{\sigma^2}\right). \tag{7}$$

So far, we have successfully reduced the complex MARC system to a traditional triangle model. Based on the conclusion in [10], the power scaling factor α with instantaneous $\gamma_{\mathcal{S}\mathcal{R}}$ and

[2]Note that our approximation is different from the one shown in [11] [12], in which the authors consider an orthogonal MARC system.

$$P_{\text{MAC}}^{U} \approx Q\left(\sqrt{2\min\left[E_1|h_{1\mathcal{R}}|^2/\sigma^2, E_2|h_{2\mathcal{R}}|^2/\sigma^2, |\sqrt{E_1}h_{1\mathcal{R}}+\sqrt{E_2}h_{2\mathcal{R}}|^2/\sigma^2\right]}\right). \quad (6)$$

$\gamma_{\mathcal{RD}}$ is

$$\alpha = \min\left(\frac{\gamma_{\mathcal{SR}}}{\gamma_{\mathcal{RD}}}, 1\right). \quad (8)$$

Note that, the instantaneous channel SNR $\gamma_{\mathcal{RD}}$ can be replaced by statistical channel SNR $\bar{\gamma}_{\mathcal{RD}}$. The advantage of using $\bar{\gamma}_{\mathcal{RD}}$ to obtain α is that the relay does not need the feedback of relay-destination channel. Later, we will show that using both instantaneous and statistical relay-destination channel SNR can result in a full diversity in the proposed PANC scheme.

Theorem 2: Given the instantaneous source-relay channel SNR, and instantaneous (or statistical) relay-destination channel SNR, the power scaled PANC scheme can achieve a diversity of two, i.e., the full diversity, in the MARC system with two sources, while the power scaled CXNC scheme can only achieve the diversity of one even with the power scaling. We leave the detailed proof for a longer version of this work.

B. The Design of Power Adaptation Factors

Note that the error performance metric, e.g., SPER, is a function of power adaptation levels $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$. In particular, denote the sources' symbol pairs by $T_i \triangleq (x_1, x_2)$. We have $T_1 \triangleq (1,1), T_2 \triangleq (-1,1), T_3 \triangleq (1,-1)$, and $T_4 \triangleq (-1,-1)$. Assuming that symbols are transmitted with equal probability, then the general expression of the system SPER of the PANC scheme is

$$P_{e,inst} = \sum_{i=1}^{4} P(\mathcal{E}|T_i, \mathbf{h})P(T_i) = \frac{1}{4}\sum_{i=1}^{4} P(\mathcal{E}|T_i, \mathbf{h}), \quad (9)$$

where $\mathcal{E}$ denotes the symbol error event at the destination that a transmitted symbol pair from two sources is decoded to an erroneous pair, i.e., either x_1 or x_2 is wrongly detected or both x_1 and x_2 are wrongly detected, $P(\mathcal{E}|T_i, \mathbf{h})$ is the conditional SPER given T_i is transmitted and the channel realization vector $\mathbf{h} = [h_{1\mathcal{R}}, h_{2\mathcal{R}}, h_{1\mathcal{D}}, h_{2\mathcal{D}}, h_{\mathcal{RD}}]$, and $P(T_i) = \frac{1}{4}$ is the probability that T_i is sent by the two sources.

To minimize the system error performance requires an optimization of $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$. However, directly minimizing the SPER is very complex and leads to no closed form expressions for $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$. Here, we first transform the parallelogram-shaped constellation at destination to a rectangle-shaped constellation, which preserves the length of the IDC's sides. For more details of the coordinate transformation, please refer to a longer version of this work. Next, we propose a sub-optimal criterion for the instantaneous SPER minimization, i.e., maximizing the minimum Euclidean distances of the coordinate transformed IDC [13]. Since we consider slow fading channel, i.e., the channel coefficients are constant with a transmission period, the power adaptation factors $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are optimized for each transmission period. For the coordinate transformed IDC, we only consider the two edges of the rectangle, since the two edges are always less than the diagonals of the rectangle. The Euclidean distance optimization problem under the power constraint is formulated as

$$\begin{aligned}(\tilde{\kappa}_1^*, \tilde{\kappa}_2^*) = \arg\max_{\tilde{\kappa}_1,\tilde{\kappa}_2} \min_{j=2,3} \left\{||\bar{V}_1^{\mathcal{D}} - \bar{V}_j^{\mathcal{D}}||^2\right\} \\ \text{s. t. } \tilde{\kappa}_1^2 + \tilde{\kappa}_2^2 \le 2E_{\mathcal{R}}^{\text{ave}}, \ \ \tilde{\kappa}_1, \tilde{\kappa}_2 \in \mathbb{R},\end{aligned} \quad (10)$$

where the lengths of the rectangle's two edges are $\overline{\bar{V}_1^{\mathcal{D}}\bar{V}_2^{\mathcal{D}}} = \overline{V_1^{\mathcal{D}}V_2^{\mathcal{D}}} = ||V_1^{\mathcal{D}} - V_2^{\mathcal{D}}||^2 = 4E_1|h_{1\mathcal{D}}|^2 + \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 - \tilde{\kappa}_2)^2$ and $\overline{\bar{V}_1^{\mathcal{D}}\bar{V}_3^{\mathcal{D}}} = \overline{V_1^{\mathcal{D}}V_3^{\mathcal{D}}} = 4E_2|h_{2\mathcal{D}}|^2 + \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 + \tilde{\kappa}_2)^2$ because the coordinate transformation of IDC preserves the length of the geometry's sides. Defining $\mathcal{V}$ as the set of $\left\{\overline{\bar{V}_1^{\mathcal{D}}\bar{V}_2^{\mathcal{D}}}, \overline{\bar{V}_1^{\mathcal{D}}\bar{V}_3^{\mathcal{D}}}\right\}$, and introducing a variable $u \triangleq \min\{\mathcal{V}\}$, after some manipulations, the Euclidean distance optimization problem in (10) can be further described as a maximization problem

$$\begin{aligned}&\max u \\ &\text{s. t.} \ -(4E_1|h_{1\mathcal{D}}|^2 + \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 - \tilde{\kappa}_2)^2) \le -u, \\ &\quad\ \ -(4E_2|h_{2\mathcal{D}}|^2 + \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 + \tilde{\kappa}_2)^2) \le -u, \\ &\quad\ \ \tilde{\kappa}_1^2 + \tilde{\kappa}_2^2 \le 2E_{\mathcal{R}}^{\text{ave}}.\end{aligned} \quad (11)$$

Since the objective function of the new maximization problem is an affine function and the constraints are quadratic functions of $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ in (11), it is a convex optimization problem. We can adopt the Lagrange Multiplier method to obtain the solutions. The Lagrange equation is given by

$$\begin{aligned}&L(\tilde{\kappa}_1, \tilde{\kappa}_2, u, \mu_1, \mu_2, \mu_3) \\ &= u + \mu_1(u - 4E_1|h_{1\mathcal{D}}|^2 - \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 - \tilde{\kappa}_2)^2) \\ &+ \mu_2(u - 4E_2|h_{2\mathcal{D}}|^2 - \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 + \tilde{\kappa}_2)^2) \\ &+ \mu_3(\tilde{\kappa}_1^2 + \tilde{\kappa}_2^2 - E_{\mathcal{R}}^{\text{ave}}).\end{aligned} \quad (12)$$

Specifically, when $\mu_i = 0$, it represents that the ith constraint is not binding. Then we can ignore the ith constraint, and derive the optimal $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ combining the Lagrangian function and other KKT conditions. When $\mu_i \neq 0$, it represents that the ith constraint is binding. Then we can obtain an equality of parameters $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ with respect to the ith constraint. For instance, when $\mu_1 \neq 0$, we will have

$$u - 4E_1|h_{1\mathcal{D}}|^2 - \alpha|h_{\mathcal{RD}}|^2(\tilde{\kappa}_1 - \tilde{\kappa}_2)^2 = 0. \quad (13)$$

In this case, there are totally 8 solutions with respect to the value of μ_i, among which, we present positive real $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$

that correspond to the maximum u as follows.

$$\tilde{\kappa}_1^* = \frac{1}{2}\left(\sqrt{\frac{2\left(\alpha E_{\mathcal{R}}^{\text{ave}}|h_{\mathcal{R}\mathcal{D}}|^2 + E_2|h_{2\mathcal{D}}|^2 - E_1|h_{1\mathcal{D}}|^2\right)}{\alpha|h_{\mathcal{R}\mathcal{D}}|^2}} + \sqrt{\frac{2\left(\alpha E_{\mathcal{R}}^{\text{ave}}|h_{\mathcal{R}\mathcal{D}}|^2 + E_1|h_{1\mathcal{D}}|^2 - E_2|h_{2\mathcal{D}}|^2\right)}{\alpha|h_{\mathcal{R}\mathcal{D}}|^2}}\right),$$

$$\tilde{\kappa}_2^* = \frac{1}{2}\left(\sqrt{\frac{2\left(\alpha E_{\mathcal{R}}^{\text{ave}}|h_{\mathcal{R}\mathcal{D}}|^2 + E_1|h_{1\mathcal{D}}|^2 - E_2|h_{2\mathcal{D}}|^2\right)}{\alpha|h_{\mathcal{R}\mathcal{D}}|^2}} - \sqrt{\frac{2\left(\alpha E_{\mathcal{R}}^{\text{ave}}|h_{\mathcal{R}\mathcal{D}}|^2 + E_2|h_{2\mathcal{D}}|^2 - E_1|h_{1\mathcal{D}}|^2\right)}{\alpha|h_{\mathcal{R}\mathcal{D}}|^2}}\right). \quad (14)$$

V. Simulation Results

In this section, we evaluate the performance of the proposed PANC scheme by simulations. Consider a two-dimension cartesian coordinate system, where nodes $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{D}$ are located at $(0, \frac{\sqrt{3}}{3})$, $(0, -\frac{\sqrt{3}}{3})$, and $(1, 0)$, respectively. The relay is moving from the origin point $(0, 0)$ to $(1, 0)$ at X-axis. Throughout our simulations, we use the path loss model $\gamma_{ij} = d_{ij}^{-3}$, where γ_{ij} is the channel gain. The notation d_{ij} is the distance between two terminals, where $i \in \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{R}\}$ and $j \in \{\mathcal{R}, \mathcal{D}\}$. We assume that $E_1 = E_2 = E_{\mathcal{R}}^{ave} = 1$, and the SNR in the simulation is defined as $\rho = E_1/\sigma^2$. The average SNR range is $[0, 30]$ dB. To simplify the legends of simulation results figures, 'sim' stands for Monte-Carlo simulation result, 'thy' stands for the theoretical results.

In order to investigate the performance of our proposed scheme comprehensively, we consider the relay placed at different locations, resulting in different channel scenarios. Firstly, we consider the relay is located at $(0, 0)$, so the relay is close to the sources, i.e., forming an asymmetric network with strong source-relay channel, shown in Fig. 2. Then, we consider the relay is located at $(\frac{1}{3}, 0)$, so the distance between the source and relay is equal to the distance between the relay and destination, i.e., forming a symmetric network, shown in Fig. 3. Finally, we consider the relay is located at $(0.8, 0)$, so the distance between the sources and relay is larger than the distance between the relay to destination, i.e., forming an asymmetric network with a strong relay-destination channel, shown in Fig. 4.

In each realization of nodes locations, we evaluate the SPER performance of the proposed PANC scheme with optimized $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ given in (14), denoted by *PANC*, respectively. As the references, we simulate the following schemes: (a) the SPER performance of the CXNC scheme [14], [15] in which the relay transmits an XORed signal to the destination in the second transmission phase, denoted by *CXNC*; (b) the SPER performance of the CXNC scheme in which the relay transmits a power scaled XORed signal to the destination in the second transmission phase, denoted by *CXNC*$_\alpha$; (c) the SPER performance of the PANC scheme with randomly generated $\tilde{\kappa}_1 = 3\tilde{\kappa}_2 = 3/\sqrt{5}$, denoted by *Fixed*.

From the simulation results, we investigate that the CXNC without power scaling can only achieve a diversity of one, due

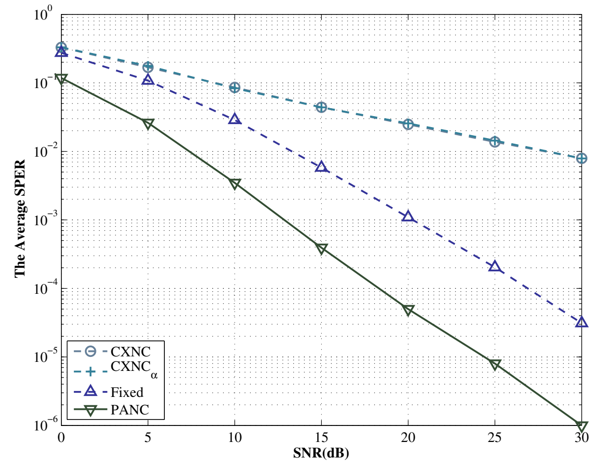

Fig. 2. Error performance with strong source-relay channel.

to the error propagation from the source-relay hop as *Theorem 1* indicates. The CXNC with power scaling still cannot achieve a full diversity due to the multi-user interference of non-orthogonal MARC as *Theorem 2* infers. The proposed PANC scheme with a power scaling factor can achieve a full diversity no matter what power levels it adopts at the relay, which verify the proof of *Theorem 2*. We can conclude from the simulation results that the PANC scheme with the proposed design of allocating different power levels and adopting a power scaling factor can achieve full diversity in a MARC system. Note that, allocating different power levels at the relay may vary the coding gain of the system. In particular, the SPER performance of optimized $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$, based on the Euclidean distance optimization method, has the best coding gain. The SPER performance for a fixed chosen $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ has a lower coding gain performance because it is not adaptive to the instantaneous CSI comparing with the optimized $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$.

VI. Conclusion

In this paper we propose a novel PANC scheme to achieve full diversity and a high coding gain for a non-orthogonal MARC. By applying the power scaling factor at the relay, the proposed PANC scheme can achieve a full diversity. In addition, we propose a Euclidean distance optimization criterion to obtain the optimal power adaptation factors at the relay. Simulation results show our PANC scheme with power adaptation factors optimizations and power scaling factor design can achieve a full diversity and a higher coding gain compared to other network coding schemes.

VII. Acknowledgment

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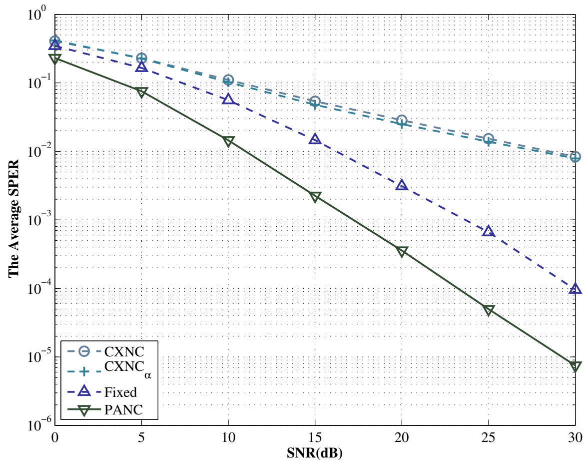

Fig. 3. Error performance in a symmetric network.

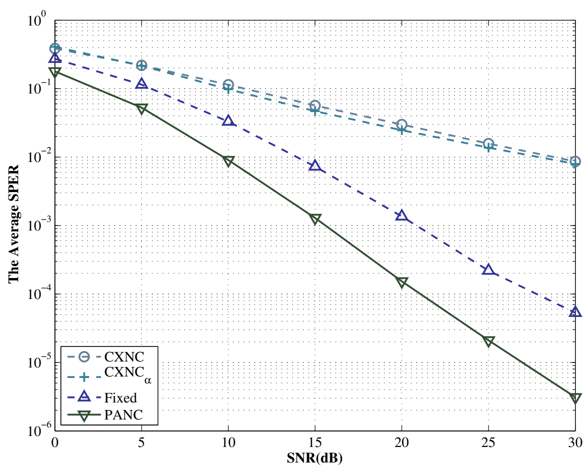

Fig. 4. Error performance with strong relay-destination channel.

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