# Compressed Channel Estimation for High-Mobility OFDM Systems: Pilot Symbol and Pilot Pattern Design

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Abstract—Orthogonal frequency-division multiplexing (OFD-M) has been widely adopted for broadband wireless communications due to its high spectral efficiency. However, it is sensitive to the time selectivity caused by the high-mobility, which largely degrades the accurate of estimating the channel state information (CSI). Therefore, the channel estimation in high-mobility OFDM systems has been a long-standing challenge. Recently, numerous experimental studies have shown that high-mobility broadband wireless channels tend to have some inherent sparsity. In this paper, we introduce the compressed sensing (CS) to utilize the inherent channel sparsity and estimate the high-mobility channel. Based on the CS minimization criterion, we propose two off-line pilot design algorithms to improve the estimation performance. One is to design the pilot symbol only and the other is to jointly design the pilot symbol and the pilot pattern. Simulation results show that the proposed methods achieve better estimation performances than conventional linear methods in high-mobility

*Index Terms*—Channel estimation, High-mobility, Compressed sensing, OFDM, Pilot design.

# I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted widely for broadband wireless communication systems due to the high spectral efficiency. However, the theoretical benefits of OFDM systems may not be fully achieved in broadband high-mobility environments since the channels are both rapidly time-varying and frequency-selective. In OFDM systems, each subcarrier has a narrow bandwidth to ensure the signal robust against the frequency selectivity caused by the multipath delay spread. In high-mobility environments, the channel may vary significantly during one OFDM symbol duration, which breaks down the orthogonality between subcarriers and introduces inter-carrier interference (ICI). Channel estimation over high-mobility environments has been considered in a number of recent papers [1]-[3]. In work [1], the Doppler information is used to design the basis for the basis expansion models (BEM). In work [2], the Doppler spread information is utilized for computing the channel correlations.

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In work [3], additional pilots with Doppler shift is designed for high-mobility users to improve the channel estimation. However, these methods are based on the implicit assumption of a rich underlying multipath environment.

Recently, numerous experimental studies show that the channels in broadband wireless communication systems tend to exhibit a sparse structure, and can be characterized with few parameters. To utilize the inherent sparsity of the highmobility channel, many researchers have studied the application of compressed sensing (CS) methods in [4]-[6]. The work [4] proposed a CS-based sparse channel estimator with a designed threshold for OFDM systems. The work [5] proposed a pilot design scheme based on the cross-entropy optimization method for OFDM systems. The work [6] gave a CS-based channel estimation method in ultra wide-band communication systems. However, these works seldom considered the coherence of CS, which influences the CS reconstruction performance directly. The fundamental work [7] concluded that a system with low CS coherence leads to a good performance. In our previous work [8], we designed the pilot symbol to minimize the CS coherence for MIMO-OFDM systems over high-mobility channels.

In this paper, we propose two off-line low coherence pilot design algorithms for the compressed channel estimation in the high-mobility OFDM systems. Based on the CS coherence minimization criterion, we design the pilot symbol and jointly design the pilot pattern and the pilot symbol with two algorithms to improve the estimation performance. Simulation results show that the both of the proposed algorithms achieve better performances than the conventional pilot in high-mobility environments without needing extra complexity.

# II. HIGH MOBILITY CHANNEL MODEL

## A. Physical Model

In the high-mobility environment, channels change rapidly and cause large Doppler frequency shift, which means that the Doppler spread  $v_{\rm max}$  is very large and cause time selective fading. Frequency selective fading is also unavoidable in high-mobility system, for the delay spread  $\tau_{\rm max}$  causes multipath effect. Therefore, we can consider the high-mobility channel as a time and frequency doubly-selective channel [9].

A typical physical multipath wireless channel model of the time-selective and frequency-selective channel can be accurately modeled as [9]:

$$\mathbf{H}(n,f) = \sum_{p=1}^{N_P} \beta_p \mathbf{a}_R(\theta_{R,p}) \, \mathbf{a}_T^H(\theta_{T,p}) \, e^{j2\pi v_p t} e^{-j2\pi \tau_p f}, \quad (1)$$

which represents signal propagation over  $N_P$  paths, here  $\beta_p$  denotes the complex path gain,  $\theta_{R,p}$  is the angle of arrival (AoA) at the receiver,  $\theta_{T,p}$  is the angle of departure (AoD) at the transmitter,  $\tau_p$  is the (relative) delay, and  $v_p$  is the Doppler shift associated with the pth path. The  $N_T \times 1$  vector  $\mathbf{a}_T(\theta_T)$  and the  $N_R \times 1$  vector  $\mathbf{a}_R(\theta_R)$  denote the array steering and response vectors, respectively. We assume that  $\tau_p \in [0, \tau_{\max}]$  and  $v_p \in \left[-\frac{v_{\max}}{2}, \frac{v_{\max}}{2}\right]$ , where  $\tau_{\max}$  denotes the maximum delay spread and  $v_{\max}$  the maximum Doppler spread of the channel.

#### B. Parameter Model

Let  $T_d$  be the packet duration, W be the bandwidth,  $T_0$  be the OFDM symbol duration,  $W_0$  be the bandwidth of each subcarrier,  $N_t = T_d/T_0$  and  $N_f = W/W_0$  be the number of the symbol and the subcarrier, respectively. The high-mobility channel between the transmitter and the receiver in the delay-Doppler domain can be modeled as

$$H(n,f) = \sum_{l=0}^{L-1} \sum_{m=-M}^{M} \beta_{l,m} e^{j2\pi \frac{m}{N_t} n} e^{-j2\pi \frac{l}{N_f} f}, \qquad (2)$$

where  $L = \lceil W\tau_{\max} \rceil + 1$  represents the maximum number of the resolvable delays and  $M = \lceil T_d v_{\max} \rceil$  represents the maximum number of the resolvable Doppler spreads of the high-mobility channel, f is the subcarrier frequency and n is the time slot. For the sake of writing convenience, we define two vectors  $\mathbf{u}_f = \left[1, e^{-j2\pi\frac{1}{N_f}f}, ..., e^{-j2\pi\frac{(L-1)}{N_f}f}\right]$  and  $\mathbf{u}_n = \left[e^{j2\pi\frac{-M}{N_t}n}, e^{j2\pi\frac{(-M+1)}{N_t}n}, ..., e^{j2\pi\frac{M}{N_t}n}\right]$ . Then the channel model can be rewritten as a matrix form:

$$H(n, f) = \mathbf{u}_f \mathbf{B} \mathbf{u}_n^T = (\mathbf{u}_n \otimes \mathbf{u}_f) \mathbf{b}, \tag{3}$$

where  ${\bf B}$  is an  $L \times (2M+1)$  channel coefficient matrix in the delay-Doppler domain of the high-mobility channel, i.e.,

$$\mathbf{B} = \begin{bmatrix} \beta_{0,-M} & \cdots & \beta_{0,M} \\ \vdots & \ddots & \vdots \\ \beta_{L-1,-M} & \cdots & \beta_{L-1,M} \end{bmatrix}. \tag{4}$$

Define  $\mathbf{b} \triangleq vec(\mathbf{B})$  is the stacking vector of the channel coefficient matrix, i.e.,

$$\mathbf{b} = [\beta_{0,-M}, \dots, \beta_{L-1,-M}, \dots, \beta_{0,M}, \dots, \beta_{L-1,M}]^T, \quad (5)$$

where each coefficient  $\beta_{l,m}$  is the sum of the complex gains of all physical paths lying in the unit sampling subspace in the delay-Doppler domain. The coefficients are considered constant in each OFDM symbol and different between two symbols. Thus, the total high-mobility channel model effectively captures the underlying multipath environment through D=L(2M+1) resolvable paths. Note that since our focus

is to discuss the performance of the channel estimator, the channel model error is omitted in this paper for convenience.

Here we define the dominant non-zero coefficients in **b** as those contributing significant channel coefficients, i.e.  $|\beta_{l,m}|^2 > \gamma$ , where  $\gamma$  is an appropriately chosen threshold whose value depends upon the design accuracy. For an appropriately chosen threshold  $\gamma \geq 0$ , the channel is said to be S-sparse, if  $|\beta_{l,m}|^2 > \gamma$  and  $\|\mathbf{b}\|_{\ell_0} = S \ll L(2M+1)$ . The works [9] and [10] have shown that the doubly-selective channels can be modeled accurately with sparse **b** in the delay-Doppler domain. In this way, CS is introduced in the following section to utilize the sparsity of high-mobility channels.

#### III. SYSTEM MODEL

Let us consider an OFDM system with K subcarriers in a high-mobility environment. In the nth OFDM symbol, the information signals  $X^n(k)$  are input in the frequency domain at K subcarriers, in which n=0,...,N-1 and k=0,...,K-1. The OFDM modulation is then implemented at the transmit antenna by performing the inverse discrete Fourier transform (IDFT). After the IDFT module and parallel to serial module,  $X^n(k)$  are transformed from the frequency domain into the time domain. Cyclic prefix (CP) is inserted into transmit signals to avoid the intersymbol interference (ISI). Then, the emitted signals pass the high-mobility wireless channels and arrive the receive antenna.

To ensure the estimation accuracy, in this paper, we send pilot signals in the frequency domain. Assume that there are P pilots which are placed at subcarriers  $k_1, k_2, ..., k_P$ , and  $P \le K$ . The received pilot vector at the receiver can be represented as a matrix form

$$\mathbf{Y}^n = \mathbf{X}^n \mathbf{H}^n + \mathbf{H}_{ICI}^n \mathbf{X}_{vec}^n + \mathbf{W}^n, \tag{6}$$

$$= \mathbf{X}^n \mathbf{H}^n + \mathbf{N}^n, \tag{7}$$

where  $\mathbf{Y}^n = [Y^n(k_1), Y^n(k_2), ..., Y^n(k_P)]^T$  is the received pilot vector at the receiver over all pilots subcarriers,  $\mathbf{X}^n = diag([X^n(k_1), X^n(k_2), ..., X^n(k_P)]^T)$  is a diagonal matrix of the transmitted pilot matrix at pilot subcarriers,  $\mathbf{X}^n_{vec}$  denotes the vector obtained by stacking the diagonal of  $\mathbf{X}^n$ ,  $\mathbf{W}^n = [W^n(k_1), W^n(k_2), ..., W^n(k_P)]^T$  is the noise vector,  $\mathbf{H}^n = [H^n(k_1), H^n(k_2), ..., H^n(k_P)]^T$  is the ICI-free highmobility channel matrix in the frequency domain, and  $\mathbf{H}^n_{ICI}$  is the channel matrix with zero diagonal entries and whose off-diagonal entries represent the ICI caused by time-variant channels. In this paper, we focus on the channel estimation and define  $\mathbf{N}^n = \mathbf{H}^n_{ICI}\mathbf{X}^n_{vec} + \mathbf{W}^n$ .

Substituting (3) into (7), and then we can get the matrix form:

$$\mathbf{Y}^n = \mathbf{X}^n \mathbf{U}^n \mathbf{b} + \mathbf{N}^n. \tag{8}$$

where  $\mathbf{U}^n = [\mathbf{u}_n \otimes \mathbf{u}_{k_1}, \mathbf{u}_n \otimes \mathbf{u}_{k_2}, \cdots, \mathbf{u}_n \otimes \mathbf{u}_{k_P}]^T$  is a  $P \times L(2M+1)$  channel model dictionary matrix of the highmobility channel of the nth symbol, and  $\mathbf{u}_{k_p} = \mathbf{u}_{f|f=k_p}$ .

For an appropriate threshold  $\gamma > 0$ , the channel is S-sparse if  $\|\mathbf{b}\|_{\ell_0} = S \ll L(2M+1)$ . In this way, we convert the task of estimating the high-mobility channel  $\mathbf{H}^n$  in the frequency domain to estimating the channel coefficients  $\mathbf{b}$  in the delay-Doppler domain, which fortunately are sparse in practice [10].

# IV. LOW COHERENCE COMPRESSED CHANNEL ESTIMATION

CS is an innovative and revolutionary idea that can utilize the inherent sparsity of the wireless channel, which is known as the compressed channel estimation [9]. Now we briefly introduces CS theory for better explanation. Let signal  $\mathbf{x} \in \mathbb{R}^m$  be an  $m \times 1$  vector and has the sparsity of S under the dictionary basis  $\mathbf{D} \in \mathbb{R}^{m \times U}$  (m < U), which means  $\mathbf{x} = \mathbf{D}\mathbf{a}$  and only S elements in vector  $\mathbf{a}$  are non-zero. Then  $\mathbf{x}$  can projects to  $\mathbf{y}$  by  $\mathbf{y} = \mathbf{P}\mathbf{x} = \mathbf{P}\mathbf{D}\mathbf{a}$  with a measurement matrix  $\mathbf{P} \in \mathbb{R}^{p \times m}$ , which is not related to the dictionary basis  $\mathbf{D}$ . If  $\mathbf{P}\mathbf{D}$  satisfies the restricted isometric property (RIP) [7], then CS reconstruction methods such as basis pursuit (BP) [11] can reconstruct  $\mathbf{x}$  from  $\mathbf{y}$ .

#### A. Coherence of CS

In this subsection, we review the definition of the coherence [7], which is a fundamental concept of CS, and then give an useful theorem.

Definition 1 (Coherence [12]): For a matrix M with the *i*th column of  $d_i$ , its coherence is defined as the largest absolute value of the normalized inner product between different columns in M and can be written as follows:

$$\mu\{\mathbf{M}\} = \max_{i \neq j} \frac{\left|\mathbf{d}_{i}^{H} \mathbf{d}_{j}\right|}{\left\|\mathbf{d}_{i}\right\| \cdot \left\|\mathbf{d}_{j}\right\|}.$$
 (9)

Previous works [12] and [13] established that both BP and orthogonal greedy algorithms (OGA) [14] are valid if the following theorem is satisfied.

Theorem 1: For a dictionary matrix  $\mathbf{D}$  and measurement matrix  $\mathbf{P}$ , assume that  $\mathbf{P}\mathbf{D}$  satisfies the RIP, if the representation  $\mathbf{y} = \mathbf{P}\mathbf{x} = \mathbf{P}\mathbf{D}\mathbf{a}$  satisfies the requirement

$$S = \|\mathbf{a}\|_{\ell_0} < \frac{1}{2} \left( 1 + \frac{1}{\mu \{ \mathbf{PD} \}} \right),$$
 (10)

then a)  $\bf a$  is the unique sparsest representation of  $\bf x$ ; b) the deviation of the reconstructed  $\hat{\bf a}$  from  $\bf a$  by BP or OGA can be bounded by

$$\|\hat{\mathbf{a}} - \mathbf{a}\|_{\ell_2}^2 \le \frac{\epsilon^2}{1 - \mu\{\mathbf{PD}\}(2S - 1)},$$
 (11)

for some constant  $\epsilon > 0$ .

From Theorem 1, we can find that once  ${\bf P}$  is designed with a fixed  ${\bf D}$  such that  $\mu\{{\bf PD}\}$  is as small as possible, then a large number of candidate signals are able to reside under the umbrella of successful CS behavior which leads to better CS performance.

#### B. Problem Statement

As we have already known that a lower  $\mu$  leads to a better CS performance, we are going to reduce the coherence in our system to improve the estimation performance. In this paper, both of the pilot symbol and the pilot pattern are discussed. Since we only consider the estimation process in one OFDM symbol, the superscripts n in the rest of the paper are omitted for compactness. Therefore, our objective is to

# Algorithm 1 : Pilot Symbol Design

- 1: Set  $\mathbf{X} \in \mathbb{R}^{P \times P}$ , the shrink factor  $\lambda$  and the maximum iteration time Iter.
- 2: Normalize the columns in the matrix  $\mathbf{X}\mathbf{U}$  and obtain the effective dictionary  $\hat{\mathbf{D}}$ .
- 3: Compute Gram Matrix:  $\mathbf{G} = \hat{\mathbf{D}}^T \hat{\mathbf{D}}$ .
- 4: Update the Gram matrix and obtain  $\hat{\mathbf{G}}$  with

$$g_{ij} = \begin{cases} \lambda g_{ij}, & |g_{ij}| \ge \delta, \\ \lambda \delta \cdot sign(g_{ij}), & \delta \ge |g_{ij}| \ge \lambda \delta, \\ g_{ij}, & \lambda \delta \ge |g_{ij}|, \end{cases}$$

where 
$$sign(x) = \begin{cases} 1, & x \ge 0 \\ -1, & x < 0 \end{cases}$$

- 5: Apply SVD and reduce the rank of  $\hat{\mathbf{G}}$  to P by only keeping P elements of the diagonal matrix.
- 6: Build the squared-root of  $\hat{\mathbf{G}}$ ,  $\mathbf{S}^T\mathbf{S} = \hat{\mathbf{G}}$ .
- 7: Find the new **X** that minimizes the error  $\|\mathbf{S} \mathbf{X}\mathbf{U}\|_F^2$  and goto  $Step\ 2$ .
- 8: Output the measurement matrix  $X^*$  after Iter iterations.

minimize the coherence  $\mu$  {XU}. This optimization problem can be formulated as

$$\min_{|\mathbf{X}|,\mathbf{p}} \mu \left\{ \mathbf{X}\mathbf{U} \right\},\tag{12}$$

where  $|\mathbf{X}|$  denotes the pilot symbols in  $\mathbf{X}$  and  $\mathbf{p}$  denotes the set of pilot subcarrier placement. Hence, the optimal pilot matrix is given as

$$(\mathbf{X}^*, \mathbf{p}^*) = \underset{|\mathbf{X}|, \mathbf{p}}{\arg \min} \, \mu \left\{ \mathbf{X} \mathbf{U} \right\}, \tag{20}$$

where  $X^*$  is the optimal pilot symbol matrix and  $p^*$  is the optimal pilot pattern vector.

# C. Pilot Symbol Design

In this subsection, follow the spirit of [8], we give a pilot symbol design algorithm to reduce the system coherence. The pilot pattern is considered as the equidistant pattern. We are going design |X| to reduce the coherence  $\mu\{XU\}$  with the fixed dictionary U. The details are given in Algorithm 1. Steps 2-4 address the objective of the process and the reduction of XU, and Steps 5-7 are responsible for the feasibility of the proposed new Gram matrix and the identity of the emerged projection matrix. Since the objective function in Step 4 can be evaluated after every iteration with almost no additional cost, this scheme could be used in practice easily. After enough iteration, the optimized measurement matrix X\* can be obtained and used to estimate the channel state information (CSI). In this way, the entries in pilot matrix X is optimized directly to reduce its coherence with the fixed channel dictionary matrix U. Furthermore, as X\* is optimized for a known channel model dictionary (with known system parameters) before transmission, this is an off-line algorithm which means its complexity can be ignored.

Algorithm 2: Joint Pilot Symbol and Pilot Pattern Design

```
Input: Random pilot matrix X_0
Output: Optimized pilot matrix X^*, optimized pattern p^*.
  1: Initialization: Set I[0] = \mathbf{0}_{N_x}, I[0,0] = 1, u = 0, v = 0 and
       \hat{\mathbf{p}}_0 = \mathbf{p}_0, set Iter.
  2: for n = 0, 1, ..., Iter - 1 do
             for k = 0, 1, ..., P - 1 do
  3:
  4:
                   m \Leftarrow n \times P + k;
                   generate \tilde{\mathbf{p}}_m with operation \tilde{\mathbf{p}}_m \leftarrow \mathbf{p}_m;
  5:
  6:
                   if \mu\{\mathbf{X}(\tilde{\mathbf{p}}_m)\mathbf{U}\} < \mu\{\mathbf{X}(\mathbf{p}_m)\mathbf{U}\} then
                          if \mu\{\mathbf{X}(\tilde{\mathbf{p}}_m^{E_1})\mathbf{U}\} < \mu\{\mathbf{X}(\tilde{\mathbf{p}}_m^{E_2})\mathbf{U}\} then
  7:
                                \mathbf{X}_{m+1} \Leftarrow \mathbf{X}(\tilde{\mathbf{p}}_m^{E_1});
  8:
  9:
                                \mathbf{X}_{m+1} \Leftarrow \mathbf{X}(\tilde{\mathbf{p}}_m^{E_2});
10:
                          end if
11:
12:
                          if \mu\{\mathbf{X}(\mathbf{p}_m^{E_1})\mathbf{U}\} < \mu\{\mathbf{X}(\mathbf{p}_m^{E_2})\mathbf{U}\} then
13:
                                 \mathbf{X}_{m+1} \Leftarrow \mathbf{X}(\mathbf{p}_m^{E_1});
14:
15:
                                \mathbf{X}_{m+1} \Leftarrow \mathbf{X}(\mathbf{p}_m^{E_2});
16:
                          end if
17.
18:
                   end if
19:
                   u \Leftarrow m + 1;
                   I[m+1] \Leftarrow I[m] + (D[m+1] - I[m])/(m+1);
20:
21:
                   if I[m+1, u] > I[m+1, v] then
22:
                          \mathbf{X}_{m+1} \Leftarrow \mathbf{X}_{m+1}; v \Leftarrow u;
23:
                   else
                         \hat{\mathbf{X}}_{m+1} \Leftarrow \hat{\mathbf{X}}_m;
24:
                   end if
25:
26:
             end for (k)
27: end for (n)
```

#### D. Joint Pilot Symbol and Pilot Pattern Design

In this subsection, follow the spirit of the discrete stochastic optimization [16], we propose a joint pilot symbol and pilot pattern design algorithm to reduce  $\mu\{XU\}$ , which jointly consider |X| and p. The key idea of this algorithm is to generate a sequence of pilot sets, where each new set is obtained from the previous one by taking a step towards the global optimum. In this paper, we assume that there are two pilot symbol power  $E_1$  and  $E_2$ . Define  $\mathbf{p}_m$ ,  $\tilde{\mathbf{p}}_m$  and  $\hat{\mathbf{p}}_m$ as different pilot placement sets at the mth iteration. Iter denotes the iteration times, and  $N_x$  denotes the number of total pilot sets. The pilot placement set occupation probability vector  $\mathbf{I}[m] = [I[m,1], I[m,2], ..., I[m,N_x]]^T$  indicates the occupation probability of each pilot placement set at the mth iteration, in which  $I[m,i] \in [0,1]$  and  $\sum_i I[m,i] = 1$ . The details are given in Algorithm 2. According to [15], this process can quickly converge to the global optimal solution. Furthermore, as U can be obtained before, Algorithm 2 is also off-line, thus its complexity can be ignored in the practical systems.

#### E. Practical Applicability

In practical high-mobility systems, the system parameters (such as  $\tau_{max}$ ,  $v_{max}$ , and etc.) can be estimated in advance. Thus, we can pre-calculate the channel model dictionary U, which reflects the properties of the high-mobility channel. In this way, the optimal pilot can be pre-designed by the given algorithms and pre-stored at the transmitter and the receiver, which is an off-line process. When the system runs,

the transmitter sends  $X^*$  to estimate the channel. After passing the high-mobility channel, at the receiver, CS reconstruction algorithms (such as BP and OGA) can reconstruct the estimated coefficients  $\tilde{\mathbf{b}}$  in the delay-Doppler domain. After that, the CSI is recovered by  $\tilde{\mathbf{H}} = U\tilde{\mathbf{b}}$ . Finally, the estimated CSI  $\tilde{\mathbf{H}}$  is used to recover the received symbol.

Moveover, except for the system parameters needed by conventional pilot-assisted channel estimation methods, the only necessarily priori information of the proposed scheme is the maximum Doppler frequency spread  $v_{max}$ . In practical,  $v_{max}$  is available in some systems. For example, in a high speed train (HST) communication system, as the HST moves along the railway and its maximum speed is known, it is easy to get  $v_{max}$ . Therefore, the proposed schemes are feasible to implement in the current OFDM communication systems.

#### V. SIMULATION RESULTS

In this section, in the high-mobility environment, we compare the MSE performance of the compressed channel estimation BP with the proposed pilot design algorithms and with the conventional equidistant pilot. The conventional least square (LS) and linear minimum mean square error (LMMSE) estimators are also included.

Here we consider an OFDM system in the high-mobility environment. Assumed that there are 512 subcarriers in OFD-M, and 120 are pilot subcarriers. The bandwidth is 5MHz, the packet duration is T=40ms, and carrier frequency is operated at  $f_c = 2.5 \text{GHz}$ . The additive noise is a Gaussian and white random process. The high-mobility channel is modeled as (3). We take the maximum delay spread as  $\tau_{max} = 50 \mu s$ and the maximum Doppler frequency is  $v_{max} = 1.389 \text{KHz}$ , which means the maximum velocity of the mobile user is 600km/h. The pilots and symbols are modulated by the star 16-QAM with two symbol powers. In our experiment, we assumed that there are only 10% of the channel coefficients are nonzero. The simulations setup corresponding to realizing the channel matrix by first randomly selecting the locations of 10\% non-zeros channel coefficients and then generating their values independently. Here we consider two designed pilot sets. One is with the equidistant pilot pattern and the pilot symbol designed by Algorithm 1. The other is considered with the pilot pattern and pilot symbol designed by Algorithm 2.

Fig. 1 presents the comparison of the MSE performances of different pilot sets with the BP, the LS, and the LMMSE estimators versus the SNR at 600km/h. Algorithm 1 and Algorithm 2 are set as Iter = 200. The LS and the LMMSE are both equipped with the "equidistant" pilot in [17] with random symbols, which is claimed as the optimal pilot placement to the doubly selective channels. It is seen that the CS channel estimators improve the MSE performances for taking use of the sparse feature. As can be seen, 120 pilots are not enough for linear estimators to get sufficient channel information and reconstruct the channel exactly. As expected, both BP with the proposed pilot design algorithms get better performances than the one with the equidistant pattern and random symbols. It means that, with the proposed algorithms, the coherence between the pilot and the high-mobility channel is effectively reduced and hence improve the estimation performance.

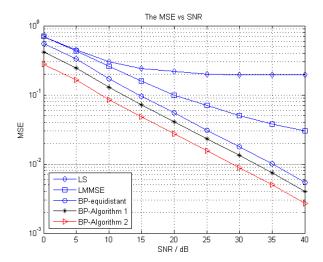


Fig. 1. MSE performances of different channel estimators in an OFDM system with 512 subcarriers at 600km/h, in which there are 120 pilot subcarriers.

Fig. 2 presents the comparison of the MSE performances of the system at 300km/h with 120 pilot subcarriers. As can be seen, comparing with Fig. 1, all estimators get better performances at lower speed for suffering less Doppler spread. However, the LS and LMMSE still need more pilots to get the accurate CSI, while the BP methods perform well with the same number of pilots. Simulation results show that the proposed algorithms can always improve the channel estimate performance in high-mobility environments. In addition, from Fig. 1 and Fig. 2, we find that Algorithm 2 is more effective than Algorithm 1 for also considering the pilot pattern. Moreover, as the proposed algorithms are off-line, the optimal pilots can be pre-designed in the practice systems, which makes the proposed schemes can be easily used in the current OFDM systems.

# VI. CONCLUSION

In this paper, two off-line low coherence pilot design algorithms are proposed for the compressed channel estimation in high-mobility OFDM systems, in which the pilot symbol and the pilot pattern are both studied. Simulation results demonstrated that the proposed algorithms efficiently improve the system performance in the high-mobility environments. Furthermore, except for the system parameters needed by conventional pilot-assisted channel estimation methods, the only necessarily priori information of the proposed schemes is the maximum Doppler frequency  $v_{max}$ . This makes the proposed methods are feasible for the implementation in the current wireless OFDM communication systems.

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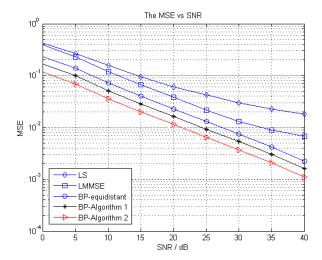


Fig. 2. MSE performances of different channel estimators in an OFDM system with 512 subcarriers at 300km/h, in which there are 120 pilot subcarriers.

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