Joint Codebook Assignment and Power Allocation for SCMA Based on Capacity with Gaussian Input

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***Abstract*—Sparse Code Multiple Access (SCMA) is a non-orthogonal multi-dimensional spreading technique based on layered codebooks. In SCMA, incoming bits are directly mapped to multi-dimensional codewords of pre-defined codebooks. In comparison to simple repetition of QAM symbols in Low Density Signature (LDS), shaping gain of the multi-dimensional constellation is the main advantage of SCMA for performance improvement. Similar to LDS, SCMA can take the advantage of a near optimal Message Passing Algorithm (MPA) receiver with reasonable complexity. In this paper, the uplink SCMA sum capacity is derived and a specific codebook design method is proposed, which can cancel the effect of the dependency between the non-zero entries of codewords. Under this condition, we study the uplink sum-rate optimization problem. A joint codebook assignment and power allocation method is proposed to achieve a near optimal solution. Simulation results show the significant performance gain of the proposed algorithm.**

***Index Terms*—SCMA, capacity, shaping gain, codebook, power allocation**

I. Introduction

Future 5G wireless network are expected to support massive connectivity, high throughput, low latency and low controlling signal overhead. To meet the requirements of 5G, Sparse Code Multiple Access (SCMA) [1], a new non-orthogonal codebook-based multiple access method, is proposed for multiple user access.

The idea of sparse spreading, called Low Density Signature (LDS), was first proposed by Reza Hoshyar [2]. LDS is a special case of multi-carrier CDMA with a few non-zero elements in a long spreading signature. This characteristic allows us to use the multi-user detection called Message Passing Algorithm (MPA) with reasonable complexity [3] [4]. In LDS-Orthogonal Frequency Division Multiplexing (OFDM) system [5], data symbols are spreading over only a small subset of the subcarriers. In other words, each subcarrier will be allocated to a small subset of the users set. Therefore, the users transmitting over a specific subcarrier are interfered by a small amount of other users.

In LDS, incoming bits are mapped to a QAM symbol, and the repetitions of the QAM symbol are transmitted through the subcarriers according to the designed signature. Instead of simple repetition of QAM symbol in LDS, SCMA provides significant shaping gain with the multi-dimensional constellation design [6]. In SCMA, incoming bits are directly mapped to multi-dimensional complex codewords selected from predefined codebook [7]. In the SCMA codebook, the non-zero dimensions are corresponding to the subcarriers in use. Besides, SCMA can also utilize iterative MPA detection with near optimal performance in the MAP sense.

In SCMA system, each user is allocated with a specific predefined codebook, which determines the correspondence between users and subcarriers. Since the users experience independent channel fading, it is necessary to use dynamic codebook assignment and power allocation in order to utilize the multiuser diversity. The SCMA downlink power allocation to maximize weighted sum-rate has been studied in [8]. But it only considers two users without taking into account dynamic codebook assignment. The energy efficient communication for uplink SCMA system was investigated in [9] and uplink contention based SCMA in [10].

This paper first derive the uplink SCMA channel capacity based on Gaussian input. The mathematical form of SCMA channel capacity is similar to that of multi-user multiple-input and multiple-output (MU-MIMO) system, but with sparse channel matrix. We can hence utilize some techniques in deriving the capacity region of MU-MIMO by David Tse [11] and obtain the SCMA channel capacity. Based on the derived capacity, we then study the joint dynamic codebook assignment and power allocation to maximize the system sum-rate. We propose a codebook assignment algorithm, where the objective function becomes a concave function. Thus, convex optimization can be utilized to obtain the optimal power allocation.

In this paper, the sets of binary, natural, integer, real and complex numbers are denoted by $\mathbb{B}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$, respectively. x is a scalar, $\boldsymbol{x}$ means a vertical vector and $\boldsymbol{X}$ represents a matrix. The conjugate transpose of $\boldsymbol{x}$ is denoted as $\boldsymbol{x}^*$, and the transpose of $\boldsymbol{x}$ is denoted as $\boldsymbol{x}^T$. For $\boldsymbol{x} = (x_1, x_2, ..., x_N)^T$ $diag(\boldsymbol{x})$ is a diagonal matrix in which the nth diagonal entry is x_n. $\mathbf{0}_N$ is an all-zero $N \times N$ matrix, and $\boldsymbol{I}_N$ stands for an $N \times N$ identity matrix.

The rest of the paper is organized as follows. Section II introduces the SCMA transceiver scheme and derive the SCMA sum capacity. In section III, we formulate the optimization problem to maximize the sum-rate. We propose a joint codebook assignment and dynamic power allocation algorith-

m. In section IV, the simulation results show the significant improvement in comparison with the equal power allocation and/or random codebook assignment. The conclusion is made in section V.

II. System Model and Capacity

In this section, we will introduce the SCMA system model and derive the SCMA uplink capacity with Gaussian input.

A. Uplink SCMA Model

Consider a single cell uplink SCMA system as shown in Fig. 1. SCMA encoder can be defined as a mapping from $\log_2(M)$ bits to a K-dimensional codeword of size M selected from a predefined codebook. K dimensions is corresponding to K different orthogonal tones, such as OFDM subcarriers. The K-dimensional codeword is a vector with only $N < K$ non-zero entries. Users can't transmit data through the subcarriers represented by the other $N-K$ zero entries. Theoretically, each user can be allocated to more than one codebook, and each codebook can't be utilized by more than one user generally. But in the considered uplink SCMA system, there is an one-to-one correspondence, which means J codebooks corresponding to J users. In uplink channel, the received signal in the kth subcarrier can be written as

$$y_k = \sum_{j=1}^{J} h_{j,k} x_{j,k} + n_k, \tag{1}$$

where $h_{j,k}$ is the channel state information of the jth user in the kth subcarrier. $x_{j,k}$ is the element of the jth user's codeword on the kth subcarrier, and this element can be zero, which is determined by the codebook of the jth user. n_k represents the Gaussian noise in the kth subcarrier.

Fig. 1 shows an example of an SCMA encoder, with 6 layered codebooks and 4 subcarriers. Each row denotes a dimension, and each column means a 4-dimensional codeword. In each codebook, the constellation size is 4, which means there are 4 different codewords can be chosen. The white entries denote the zero elements and the colored entries denote the non-zero elements in the codebooks. For example, in Codebook 1, the entries in the first row is colored and the entries in the third row is white, which means that the first dimension is non-zero and third dimension is zero. In each codebook, there are 2 non-zero dimensions with colored lattice. In AWGN channel, the signal received in the base station is the superposition of the codewords selected from the codebooks.

Considering the received signal by the base station, we combine the signals in each subcarrier and rewrite the received signal in vector form as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{y} = \left(\; y_1, y_2, \ldots, y_K \; \right)^T$, $\boldsymbol{H} = \left(\; \boldsymbol{H}_1, \boldsymbol{H}_2, \ldots, \boldsymbol{H}_J \; \right)$

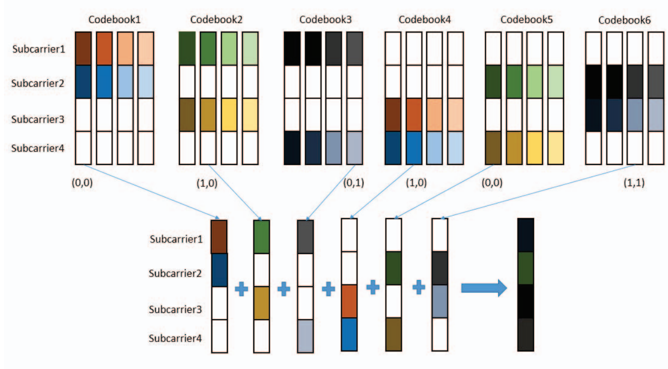

Fig. 1: SCMA encoder.

is the $K \times KJ$ channel matrix,

$$\boldsymbol{H}_j = \begin{pmatrix} h_{j,1} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & h_{j,K} \end{pmatrix}$$

is a $K \times K$ diagonal matrix; $\boldsymbol{x} = \left(\; \boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_J \; \right)^T$, and $\boldsymbol{x}_j = \left(\; x_{j,1}, x_{j,2}, \ldots, x_{j,K} \; \right)^T$.

The number of users interfered in each subcarrier is d_f, which is much less than the total number of users J. This characteristic allows the utilization of near optimal multiuser detection based on MPA.

B. Codebook with Diagonal Covariance Matrix

To derive the SCMA capacity, it is found that the correlation between the elements of a codeword result in the difficulty to obtain the capacity. Then, we try to utilize the characteristic of the SCMA codebook structure. For an example, SCMA structure with 6 codebooks and 4 subcarriers can be represented by a factorgraph

$$\boldsymbol{F} = \begin{pmatrix} a_0 & a_1^* & b_2 & 0 & 0 & 0 \\ b_0 & 0 & 0 & a_2 & a_1^* & 0 \\ 0 & b_1^* & 0 & b_2 & 0 & a_0 \\ 0 & 0 & a_2 & 0 & b_1^* & b_0 \end{pmatrix}. \tag{3}$$

$\boldsymbol{F}$ represents the mapping relationship between the codebooks and subcarriers. The non-zero entries a_0, b_0, a_1^*, b_1^*, a_2, b_2 denote the complex symbols, and the position of the non-zero entries means the connection of the corresponding codebook and subcarrier. The different letters in $\boldsymbol{F}$ denote the correlation between different codebooks. According to the codebook structure, we consider the covariance matrix $\boldsymbol{K}_{\boldsymbol{x}}$ of the codeword selected from the codebook. Then

$$\boldsymbol{K}_{\boldsymbol{x}} = \begin{pmatrix} E(\boldsymbol{x}_1\boldsymbol{x}_1^*) & \ldots & E(\boldsymbol{x}_1\boldsymbol{x}_J^*) \\ \vdots & \ddots & \vdots \\ E(\boldsymbol{x}_J\boldsymbol{x}_1^*) & \ldots & E(\boldsymbol{x}_J\boldsymbol{x}_J^*) \end{pmatrix}, \tag{4}$$

where $\boldsymbol{x}_j$ denotes a $K \times 1$ column vector, and each entry denotes a $K \times K$matrix.

For two arbitrary users, their inputs are independent. In other words, non-diagonal entries are all-zero matrices. The expansion of the diagonal entries is showed in formula (5).

TABLE I: Non-zero dimensions of the original 4-point constellation table

incoming bits	first dimension	second dimension
(0,1)	(0.5257,0.5257)	(0.8507,-0.8507)
(1,1)	(0.5257,-0.5257)	(0.8507,0.8507)
(1,0)	(-0.5257,0.5257)	(-0.8507,-0.8507)
(0,0)	(-0.5257,-0.5257)	(-0.8507,0.8507)

There is a correlation among the non-zero elements of SCMA codewords. We try to figure out whether the non-diagonal entries of $E(\boldsymbol{x}_j\boldsymbol{x}_j^*)$ can be zero in some cases. It is found that if the non-zero dimensional constellation is central symmetric in each layer, then formula (4) can be reduced to a diagonal matrix. Besides, this kind of codebook also meet the design principles mentioned in [7], which means that this characteristic dosen't sacrifice the performance. A two non-zero dimensions of the 4-points original constellation designed with this method is shown in Table. I.

The other constellations can be attained from the original constellation with the constellation operators such as phase rotation, dimensional permutation and complex conjugate [1]. For the example in Fig. 1, the typical functions to generate other constellations are

$$a_1 = a_0 * \exp\left(j\frac{\pi}{6}\right), \qquad b_1 = b_0 * \exp\left(j\frac{\pi}{6}\right); \tag{6}$$

$$a_2 = a_0 * \exp\left(j\frac{\pi}{3}\right), \qquad b_2 = b_0 * \exp\left(j\frac{\pi}{3}\right). \tag{7}$$

Then, the covariance of input $\boldsymbol{x}$ can be rewritten as the diagonal matrix (8).

C. SCMA Capacity

Since the SCMA transmission model in (2) shares the form of the fading MU-MIMO, we can derive SCMA capacity using some techniques in dericing the MU-MIMO channel capacity by David Tse [11]. Then consider the mutual information,

$$\begin{aligned} & I(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}) \\ \leq\ & \log((\pi e)^K \det(N_0\boldsymbol{I}_K + \boldsymbol{H}\boldsymbol{K}_x\boldsymbol{H}^*)) - K\log(\pi e N_0) \\ =\ & \log det(\boldsymbol{I}_K + \frac{1}{N_0}\boldsymbol{H}\boldsymbol{K}_x\boldsymbol{H}^*), \end{aligned} \tag{9}$$

where N_0 denotes the average power of noise. When the input $\boldsymbol{x}$ follows Gaussian distribution, the equality in (9) holds. Then, substitute the covariance expression mentioned in Eq. (8) into Eq. (9),

$$\begin{aligned} & I(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}) \\ =\ & \log\det(\boldsymbol{I}_K + \frac{1}{N_0}\boldsymbol{H}\boldsymbol{K}_x\boldsymbol{H}^*) \\ =\ & \log\det\begin{pmatrix} 1+\frac{\sum_{j=1}^J h_{j,1}^2 E(x_{j,1}^2)}{N_0} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1+\frac{\sum_{j,1}^J h_{j,k}^2 E(x_{j,k}^2)}{N_0} \end{pmatrix} \\ =\ & \sum_{k=1}^K \log\left(1+\frac{\sum_{j=1}^J h_{j,k}^2 E(x_{j,k}^2)}{N_0}\right). \end{aligned} \tag{10}$$

Allocate average power $\sqrt{p_{j,k}}$ to the codeword of the jth user in the kth dimension. As the diagonal elements of $\boldsymbol{K}_x$ can be either zero or non-zero, we can define a variable $\sigma_{j,k}$ to show the connection between users and subcarriers, i.e., $\sigma_{j,k} = 1$ means user j is connected to subcarrier k, otherwise, $\sigma_{j,k} = 0$. We can time $\sigma_{j,k}$ to the user's power to show the connection between user and subcarrier. Then obtain $E(x_{j,k}^2) = \sigma_{j,k}p_{j,k}$. Therefore, the capacity of uplink SCMA system is

$$C \leq \sum_{k=1}^K \log\left(1+\frac{\sum_{j=1}^J h_{j,k}^2\sigma_{j,k}p_{j,k}}{N_0}\right). \tag{11}$$

Note that there are both binary variables and continuous variables in formula (11).

III. Joint Codebook assignment and power allocation

This section presents the optimization problem of maximizing the system sum-rate by jointly designing codebook assignment and power allocation. It shows that this problem is a non-convex Mixed-Integer Nonlinear Programming (MINLP) problem.

A. Problem Formulation

Our objective is to maximize the system sum-rate, under the constraints of users' maximum transmit power and SCMA codebook structure. The optimization problem is formulated as

$$E(\boldsymbol{x}_j\boldsymbol{x}_j^*) = \begin{pmatrix} E(x_{j,1}^2) & E(x_{j,1}x_{j,2}^*) & \dots & E(x_{j,1}x_{j,K-1}^*) & E(x_{j,1}x_{j,K}^*) \\ E(x_{j,2}x_{j,1}^*) & E(x_{j,2}^2) & \dots & E(x_{j,2}x_{j,K-1}^*) & E(x_{j,2}x_{j,K}^*) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ E(x_{j,K-1}x_{j,1}^*) & E(x_{j,K-1}x_{j,2}^*) & \dots & E(x_{j,K-1}^2) & E(x_{j,K-1}x_{j,K}^*) \\ E(x_{j,K}x_{j,1}^*) & E(x_{j,K}x_{j,2}^*) & \dots & E(x_{j,K}x_{j,K-1}^*) & E(x_{j,K}^2) \end{pmatrix}. \tag{5}$$

$$\boldsymbol{K}_{\boldsymbol{x}} = diag\begin{pmatrix} E(x_{1,1}^2) & \dots & E(x_{1,K}^2) & \dots & E(x_{J,1}^2) & \dots & E(x_{J,K}^2) \end{pmatrix}. \tag{8}$$

$$\max_{p_{j,k}\sigma_{j,k}} \quad \sum_{k=1}^{K} \log\left(1+\frac{\sum_{j=1}^{J} h_{j,k}^2 \sigma_{j,k} p_{j,k}}{N_0}\right), \tag{12}$$

$$s.t. \quad \sum_{k=1}^{K} \sigma_{j,k} p_{j,k} \le P_j, \qquad \forall j, \tag{13}$$

$$p_{j,k} \ge 0, \qquad \forall j,k, \tag{14}$$

$$\sum_{j=1}^{J} \sigma_{j,k} = d_f, \qquad \forall k, \tag{15}$$

$$\sum_{k=1}^{K} \sigma_{j,k} = N, \qquad \forall j, \tag{16}$$

$$\sigma_{j,k} \in \{0,1\}, \qquad \forall j,k, \tag{17}$$

where $\sigma \triangleq \{\sigma_{j,k},\}_{j\times k}$ is a $J\times K$ matrix, in which there are N non-zero entries in each row and d_f non-zero entries in each column. Moreover, matrix σ represents a codebook assignment strategy. If a codebook assignment scheme is determined, matrix σ will be uniquely determined.

Consider the uplink SCMA system, each user will transmit data to N subcarriers according to the non-zero elements of the codebook. Constraint (13) shows that the sum of the power among the N subcarriers must be no more than the user's local power constraint P_j. Constraint (14) shows that the power allocation is non-negative. Moreover, from Constraints (15) to (17), according to the codebook structure, there are only N dimensions that are non-zero in the K-dimensional codeword of each user, and only d_f users will interfere in each subcarriers.

In the integer programming part, the optimal codebook assignment scheme can be obtained by traversal searching. However, the complexity of this method will increase exponentially with the increasing of users, which means that it is impossible to utilize traversal searching with massive users. Then one has to turn to sub-optimal methods. The sub-optimal methods can be generally classified into two categories: One is based on Greedy algorithm or some heuristic algorithms, which complexity is proportional to the size of searching group, and the solution is close to the optimum with great searching group; The other one is to relax the discrete variables to continuous variables and obtain the solution, then change the variable back [12], which will inevitably introduce approximation error and result in a sub-optimal solution.

B. Codebook Assignment

Based on the objective function of maximizing the sum-rate $\sum_{k=1}^{K}\log(1+\frac{\sum_{j=1}^{J} h_{j,k}^2\sigma_{j,k}p_{j,k}}{N_0})$, in uplink system, the rate on each subcarrier is calculated respectively and summed up. From this formula, we can't distinguish the contribution of each user. So it is necessary to find the local optimal solution for each user.

Define $\varepsilon_{j,k} = h_{j,k}^2\sigma_{j,k}p_{j,k}$, and $\varepsilon_{0,k} = 0$. The rate on subcarrier k can be written as

$$\begin{aligned}
&\log\left(1+\frac{\sum_{j=1}^{J} h_{j,k}^2\sigma_{j,k}p_{j,k}}{N_0}\right)\\
=\ &\log\left(\frac{N_0+\varepsilon_{1,k}+\cdots+\varepsilon_{J,k}}{N_0}\right)\\
=\ &\log\left(\frac{N_0+\varepsilon_{1,k}}{N_0}\right)+\log\left(\frac{N_0+\varepsilon_{1,k}+\varepsilon_{2,k}}{N_0+\varepsilon_{1,k}}\right)\\
&+\cdots+\log\left(\frac{N_0+\varepsilon_{1,k}+\cdots+\varepsilon_{J,k}}{N_0+\varepsilon_{1,k}+\cdots+\varepsilon_{J-1,k}}\right)\\
=\ &\log\left(1+\frac{\varepsilon_{1,k}}{I_{1,k}+N_0}\right)+\cdots+\log\left(1+\frac{\varepsilon_{J,k}}{I_{J,k}+N_0}\right),
\end{aligned} \tag{18}$$

where $I_{j,k} = \sum_{i=1}^{j}\varepsilon_{i-1,k}$ is the interference for user j on subcarrier k.

Define the individual rate $\omega_{j,k}$ for user j on subcarrier k, which will be used in the codebook assignment. Then the expression of $\omega_{j,k}$ is

$$\omega_{j,k} = \log\left(1+\frac{\varepsilon_{j,k}}{I_{j,k}+N_0}\right). \tag{19}$$

Then we propose a joint codebook assignment and power allocation algorithm, which has three steps: Firstly, utilize the equal power allocation to calculate the weight $\omega_{j,k}$ and execute codebook assignment which will be introduced in next section; Secondly, when codebook assignment, i.e., σ is determined, the objective function becomes a concave function. Then the optimal power allocation result can be obtained by convex optimization; Thirdly, replace the equal power allocation with the derived optimal power allocation and execute the codebook assignment again, and then repeat optimal power allocation and codebook assignment iteratively.

This section describes the codebook assignment in detail. The main idea of codebook assignment is to determine the mapping matrix $\boldsymbol{F}$ to maximize the sum-rate. The initial state of the mapping matrix is a $J \times K$ all-zero matrix. We can explain the codebook assignment algorithm by three steps:

- Step 1: Calculate the individual rate $\omega_{j,k}$ for each user j on each subcarrer k according to $\boldsymbol{F}$. The individual rate represents the individual contribution to the sum-rate, and it is relevant to the interference from other users. In other words, the individual rate will determine the connection between a user and a subcarrier.
- Step 2: Form a $J \times K$ matrix $\boldsymbol{W} = \{\omega_{jk}\}$. Choose the largest individual rate and change the corresponding entry of the mapping matrix $\boldsymbol{F}$ to 1. This choice can lead to a largest contribution to the sum-rate.
- Step 3: Set chosen entries of $\boldsymbol{W}$ to 0 and go back to the first step until each row of the mapping matrix $\boldsymbol{F}$ has d_f non-zero elements and each column of the mapping matrix has N non-zero elements.

To distinguish the users' states, we define the user's freedom as $\mathcal{U}$. Each user's initial freedom is N, the number of non-zero elements in codeword, and the user's freedom minus 1 when selected. We define the contain value of a subcarrier as $\mathcal{P}$. Each subcarrier's initial contain value is d_f, the maximum number of users allowed to collide in the subcarrier, and the subcarrier's contain value minus 1 when selected a user. The codebook selection algorithm is terminated when every subcarrier's contain value is 0 or every user's freedom is 0. In addition, a codebook can't be utilized by more than one user. Thus, any two columns of the mapping matrix are different. The algorithm is summarized in Algorithm I:

Algorithm 1 Codebook Assignment Algorithm

Initialization:

Put all users into a user set $\mathcal{M}$;

Set contain value for each subcarrier $\mathcal{P}_k = d_f$;

Set freedom for each user $\mathcal{U}_j = N$;

The number of codebooks can be assigned is $c = J$;

Procedure:

1: For each subcarrier k with $\mathcal{P}_k \neq 0$, select the user with maximal individual rate $\omega_{j,k}$ among the users with positive freedom. Then compare the selected individual rate for all subcarriers with non-zero contain value, and choose subcarrier k and user j with the largest individual rate.

2: If $\mathcal{U}_j$ is equal to N, the user j will choose a codebook and the number of codebooks will minus 1, i.e., $c = c - 1$; Execute $\mathcal{P}_k = \mathcal{P}_k - 1$ and $\mathcal{U}_j = \mathcal{U}_j - 1$, and set the binary variable $\sigma_{j,k} = 1$.

3: If $\mathcal{U}_j$ is equal to 0, the user j selects the corresponding codebook and is removed from the user set $\mathcal{M}$.

4: If $\mathcal{P}_k = 0, \forall k$, the program is terminated; Otherwise, go to Step 1.

C. Power Allocation

When the codebook assignment is determined, the matrix σ becomes a fixed matrix. Therefore, there are no discrete variables in the optimization problem, and the optimization problem can be reduced to

$$\max_{p_{j,k}\sigma_{j,k}} \quad \sum_{k=1}^{K} \log\left(1 + \frac{\sum_{j=1}^{J} h_{j,k}^2 \sigma_{j,k} p_{j,k}}{N_0}\right), \tag{20}$$

$$s.t. \quad \sum_{k=1}^{K} \sigma_{j,k} p_{j,k} \leq P_j, \qquad \forall j, \tag{21}$$

$$p_{j,k} \geq 0, \qquad \forall j, k. \tag{22}$$

The objective function is concave. We try to get the optimal solution by Lagrange dual method. The Lagrange function can be written as

$$L(p,\lambda) = \sum_{k=1}^{K} \log\left(1 + \frac{\sum_{j=1}^{J} h_{j,k}^2 \sigma_{j,k} p_{j,k}}{N_0}\right) + \sum_{j=1}^{J} \lambda_j \left(\sum_{k=1}^{K} \sigma_{j,k} p_{j,k} - P_j\right), \tag{23}$$

where λ is the Lagrange operator. Let $\frac{\partial L}{\partial p_{j,k}} = 0$. We can get

$$\sum_{j=1}^{J} h_{j,k}^2 \sigma_{j,k} p_{j,k} = \frac{h_{j,k}^2}{\lambda_j} - N_0. \tag{24}$$

It is observed that one can't separate $p_{j,k}$ from $\sum_{j=1}^{J} h_{j,k}^2 \sigma_{j,k} p_{j,k}$. In other words, the closed form solution of the optimizing problem can not be obtained. As a result, we utilize the module CVX of matlab to find the numerical solution of the optimization problem.

IV. Simulation Results

Consider a single-cell uplink SCMA system where 6 users are randomly distributed in an area of circle with the radius of 1 km. The channels between the base station and users are generated with a normalized Rayleigh fading component and a distance-dependent path loss, modeled as $PL(\text{dB}) = 128.1 + 37.6 \log_{10}(d) + 20$ with 8dB log-normal shadowing [13], where d is the distance from the user to the base station. Suppose that there are 6 codebooks designed using the method provided in [7], and each user is allocated with a specific codebook and a codebook can't be utilized by more than one user. The overloading factor is 150%. We suppose that the system bandwith is $15kHz$. There are 4 subcarriers, and the number of non-zero elements in a codeword is 2. Considering that the constraint of uplink SCMA system is the power of users' terminals, probably mobile phones. Suppose that the users' transmit power interval is from $-4dbm$ to $14dbm$. The power density of noise is $-174dbm/hz$, and the interference from other cell is not considered.

In Fig. 2, we compare the throughput for 4 different cases of codebook assignment and power allocation:

1) The proposed codebook assignment and optimal power allocation: Codebooks assignment uses the proposed codebook assignment algorithm and users' power is allocated on each subcarrier by the proposed power allocation result.
2) Random codebook assignment and equal power allocation: Codebooks are assigned to users randomly and users'power is allocated equally on each subcarrier.
3) Random codebook assignment and optimal power allocation: Codebooks are assigned to users randomly and users' power is allocated on each subcarrier with the proposed power allocation result.
4) The proposed codebook assignment and equal power allocation: Codebooks are assigned to users using the proposed codebook assignment algorithm and users'power is allocated equally on each subcarrier.

Fig. 2 shows that the proposed codebook assignment with the proposed power allocation perform best among the four cases. In addition, it is found that the gain of the codebook assignment optimization is greater than that of power allocation optimization. With the increasing transmit power, the performance gaps of the four cases are getting bigger.

The codebooks are designed by using the method introduced in [7]. This kind of codebooks can make the input's covariance

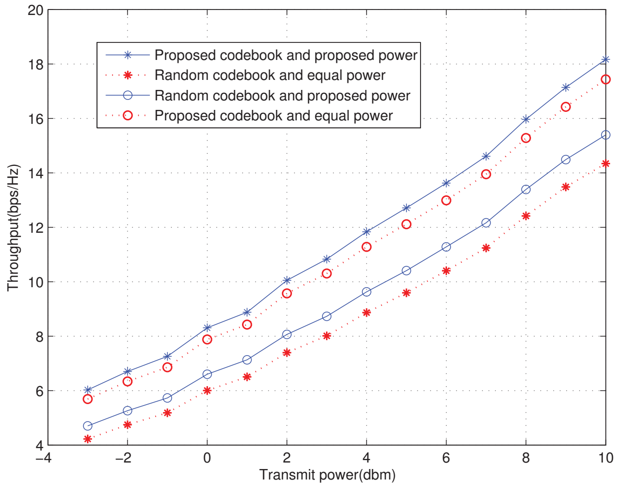

Fig. 2: Performance of 4 cases of codebook assignment and power allocation.

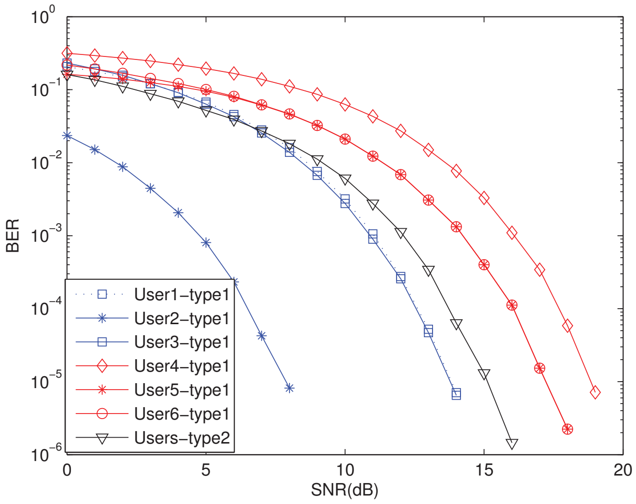

Fig. 3: Link performance with dynamic codebook adjustment.

matrix diagonal. With the codebook assignment and power allocation, the performance of throughput is optimized. So it is necessary to study the link performance. We adjust the codebook with the power allocation results and observe the link performance. In Fig. 3, type 1 means the link performance with the adjusted codebooks, and each user's performance has great difference; Type 2 means the link performance with the original codebooks without optimal power allocation and users' performance in this type are nearly identical. We can find that, in Type 1, user 1, user 2 and user 3 have better performance than users of type 2, but user 4, user 5 and user 6 are worse. This is because that after the power allocation, maybe less than N dimensions are allocated power, and others are all zero. As a result, some users are using one subcarrier and have no interference from other users. In other words, the factor graph of the codebook structure is irregular.

V. Conclusion

When the covariance matrix of the input codewords is diagonal, this paper derives the SCMA uplink capacity. By maximizing the capacity, this paper jointly optimizes the codebook assignment and power allocation. Based on the individual rate of each user on each subcarrier, an algorithm to maximize the uplink sum-rate is developed. Meanwhile an optimal power allocation method is established using convex optimization. Based on these results, a joint codebook assignment and power allocation algorithm is established in this paper. Simulations show that the proposed algorithm can greatly improve the SCMA uplink capacity.

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