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Network Coding in Multiple Access Relay Channel with Multiple Antenna Relay

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***Abstract*—Network coding is a paradigm for modern communication networks by allowing intermediate nodes to mix messages received from multiple sources. In this paper, we carry out a study on network coding in multiple access relay channel (MARC) with multiple antenna relay. Under the same transmission time slots constraint, we investigate several different transmission strategies applicable to the system model, including direct transmission, decode-and-forward, digital network coding, digital network coding with Alamouti space time coding, analog network coding, and compare the error rate performance. Interestingly, simulation studies show that in the system model under investigation, the schemes with network coding do not show any performance gain compared with the traditional schemes with same time slots consumption.**

***Index Terms*—multiple access relay channel, network coding, cooperative, space-time coding.**

I. Introduction

In the past decade, network coding (NC) [1] has rapidly emerged as a major research area in electrical engineering and computer science. Originally designed for wired networks, network coding is a generalized routing approach that breaks the traditional assumption of simply forwarding data, and allows intermediate nodes to send out functions of their received packets, by which the multicast capacity given by the max-flow min-cut theorem can be achieved. Subsequent works of [2]-[3] made the important observation that, for multicasting, intermediate nodes can simply send out a linear combination of their received packets. Linear network coding with random coefficients is considered in [4].

In order to address the broadcast nature of wireless transmission, physical layer network coding (PLNC) [5] was proposed to embrace interference in wireless networks where intermediate nodes attempt to decode the modulo-two sum (XOR) of the transmitted messages. Compute-and-forward network coding, based on the linear structure of lattice codes, is proposed in [6]-[7] and subsequent works follow in [8]-[9]. Analog network coding (ANC) is presented in [10] where relays simply amplify-and-forward the received mixed signals. Linear network coding with user cooperation is proposed in [11]. Several other network coding realizations in wireless networks are discussed in [12]-[13].

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Regarding network coding in wireless multiple access relay channels, throughput analysis is given in [14] under collision model. Complex field network coding is presented in [15]. Analog network coding mappings in multiple access relay channels with and without direct links is discussed in [16]. Multiple-access relay channels with compute-and-forward relays are studied in [17]. Regarding network coding with MIMO space time coding technique, Alamouti scheme [18] is applied to decode-and-forward (DF) network coding for two-way relay channels with multiple antenna relay in [19]-[20].

The previous works demonstrate that network coding can be beneficial. For example, in two-way relay channel or multiple access relay channel with two sources, network coding can be applied with less time slots than some traditional schemes so that time resources can be more efficiently utilized. In this paper, we carry out a study on network coding application in multiple access relay channel (MARC) with four sources and multiple antenna relay under same time slots constraint. We set up this system model intentionally to investigate several different transmission strategies applicable, including direct transmission, decode-and-forward, digital network coding, digital network coding with Alamouti space time coding and analog network coding. We describe in details those different schemes with the same transmission time slots constraint and compare the error rate performance. Interestingly, simulation studies show that under this scenario with same transmission time slots constraint, those schemes with network coding do not show any performance gain compared with the traditional schemes, which is different from previous beneficial network coding study with less time slots consumption. Hence, network coding technique will be more favorable in scenarios that can save transmission time slots.

II. System Model

Consider the system model setting with four sources $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$ and $\mathcal{S}_4$ communicating with destination $\mathcal{D}$ via a relay $\mathcal{R}$ with direct links from sources to the destination, as shown in Fig. 1. We assume that the sources $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, $\mathcal{S}_4$ and the destination $\mathcal{D}$ are equipped with single antenna, while relay $\mathcal{R}$ is equipped with two antennas.

We will discuss several possible transmission strategies for this system model. One realization of the information transmission will be performed within four time slots for all the schemes.

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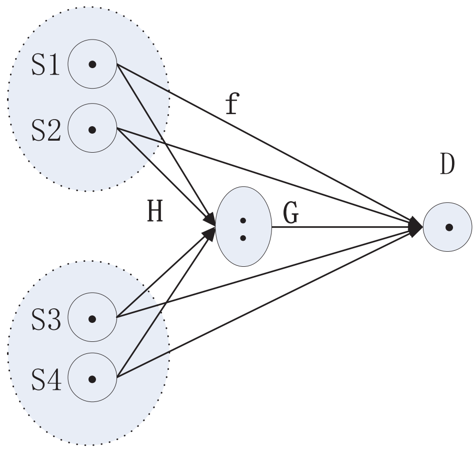

Fig. 1. System Model

A. Scheme 1: Direct Transmission (DT)

In this scheme, we assume the relay will keep silent during all the transmission realizations and the sources will communicate to the destination one by one: $\mathcal{S}_1$ will transmit in the first time slot; $\mathcal{S}_2$ will transmit in the second time slot, and so on so forth. The transmit data vector of all source nodes is denoted by

$$\mathbf{x} = [x_1, x_2, x_3, x_4]^T, \tag{1}$$

where $x_i \in \mathbb{R}$ is the transmit data symbol[1] from node $\mathcal{S}_i$ which has been normalized to $E\{|x_i|^2\} \leq 1$. E_x is the power constraint for data symbol transmission. Let $f_i \in \mathbb{R}$ be the direct link channel coefficient between source $\mathcal{S}_i$ to destination $\mathcal{D}$. f_i is an independent Gaussian random variable with variance σ_f^2. Additive Gaussian noise follows normal distribution $\mathcal{N}(0, 1)$.

The received signals $\mathbf{y}_D = [y_{D1}, y_{D2}, y_{D3}, y_{D4}]^T$ at the destination $\mathcal{D}$ during the four time slots can be expressed as

$$\mathbf{y}_D = \underbrace{\sqrt{E_x}\begin{bmatrix} f_1 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & 0 \\ 0 & 0 & 0 & f_4 \end{bmatrix}}_{\mathbf{A}_{DT}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{D1} \\ n_{D2} \\ n_{D3} \\ n_{D4} \end{bmatrix}}_{\mathbf{z}}, \tag{2}$$

or equivalently

$$\mathbf{y}_D = \mathbf{A}_{DT}\mathbf{x} + \mathbf{z}. \tag{3}$$

Note that $\mathbf{x}$ is the transmit data vector of four sources and $\mathbf{x} \in \Omega_{\mathbf{x}}$, where $\Omega_{\mathbf{x}}$ is the data vector alphabet set. Hence the decoding procedure will simply be

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\Omega_{\mathbf{x}}} ||\mathbf{y}_D - \mathbf{A}_{DT}\mathbf{x}||^2. \tag{4}$$

B. Scheme 2: Decode-and-Forward (DF)

In this scheme, one realization of the information transmission is performed in two phases. The first phase (with two time slots) is the transmission from sources to relay $\mathcal{R}$. We first separate the four sources into two groups. $\mathcal{S}_1$, $\mathcal{S}_2$ belong to one group and $\mathcal{S}_3$, $\mathcal{S}_4$ belong to another group. In the first time slot, sources $\mathcal{S}_1$ and $\mathcal{S}_2$ will transmit to the relay; while in the second time slot, sources $\mathcal{S}_3$ and $\mathcal{S}_4$ will transmit to the relay. The second phase (also with two time slots) is the transmission from relay $\mathcal{R}$ to destination $\mathcal{D}$.

The received signals at destination $\mathcal{D}$ at the end of first time slot and second time slot are

$$y_D^{[1]}(1) = \sqrt{E_x} f_1 x_1 + \sqrt{E_x} f_2 x_2 + n_D^{[1]}(1) \tag{5}$$
$$y_D^{[1]}(2) = \sqrt{E_x} f_3 x_3 + \sqrt{E_x} f_4 x_4 + n_D^{[1]}(2) \tag{6}$$

where the superscript $\{\cdot\}^{[1]}$ denotes the first phase. We can rewrite (5) and (6) into

$$\mathbf{y}_D^{[1]} = \sqrt{E_x}\underbrace{\begin{bmatrix} f_1 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & f_4 \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \mathbf{n}_D^{[1]}, \tag{7}$$

where $\mathbf{y}_D^{[1]} = [y_D^{[1]}(1), y_D^{[1]}(2)]^T$; $\mathbf{n}_D^{[1]}$ is i.i.d. Gaussian noise with normal distribution.

The received signals at relay $\mathcal{R}$ at the end of first time slot and second time slot are

$$\mathbf{y}_R^{(1)} = \sqrt{E_x}\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{R1} \\ n_{R2} \end{bmatrix}, \tag{8}$$
$$\mathbf{y}_R^{(2)} = \sqrt{E_x}\begin{bmatrix} h_{13} & h_{14} \\ h_{23} & h_{24} \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_{R3} \\ n_{R4} \end{bmatrix}, \tag{9}$$

where $\mathbf{y}_R^{(1)}$ is the received vector at relay with two antennas in the first time slot and $\mathbf{y}_R^{(2)}$ is the received vector at relay with two antennas in the second time slot; n_{Ri}, $i = 1, 2, 3, 4$ is the additive Gaussian noise; h_{ri} is the channel coefficient between source node $\mathcal{S}_i$ and relay antenna r, an independent Gaussian random variable with variance σ_h^2. Additive noise elements are generated i.i.d. according to a normal distribution $\mathcal{N}(0, 1)$.

With equations (8) and (9), we can easily obtain

$$\begin{aligned} \mathbf{y}_R &= \begin{bmatrix} \mathbf{y}_R^{(1)} \\ \mathbf{y}_R^{(2)} \end{bmatrix} = \begin{bmatrix} y_{R1} \\ y_{R2} \\ y_{R3} \\ y_{R4} \end{bmatrix} \\ &= \sqrt{E_x}\underbrace{\begin{bmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{21} & h_{22} & 0 & 0 \\ 0 & 0 & h_{13} & h_{14} \\ 0 & 0 & h_{23} & h_{24} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{R1} \\ n_{R2} \\ n_{R3} \\ n_{R4} \end{bmatrix}}_{\mathbf{n}_R}, \end{aligned} \tag{10}$$

or equivalently

$$\mathbf{y}_R = \sqrt{E_x}\mathbf{H}\mathbf{x} + \mathbf{n}_R, \tag{11}$$

which is the received signal at relay $\mathcal{R}$ at the end of first transmission phase.

In the second phase, with the decode-and-forward (DF) scheme, relay $\mathcal{R}$ will first decode for four sources

$$\hat{\mathbf{x}}_R = \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \\ \hat{x}_{R3} \\ \hat{x}_{R4} \end{bmatrix} = \arg\min_{\mathbf{x}\in\Omega_{\mathbf{x}}} ||\mathbf{y}_R - \sqrt{E_x}\mathbf{H}\mathbf{x}||^2. \tag{12}$$

[1]For source $\mathcal{S}_i$, x_i is the transmitted symbol after modulation based on the transmitted bit b_i, where $b_i \in \{0, 1\}$.

Then, with two antennas and power constraint E_R, relay $\mathcal{R}$ will transmit $[\hat{x}_{R1}, \hat{x}_{R2}]^T$ in the third time slot and transmit $[\hat{x}_{R3}, \hat{x}_{R4}]^T$ in the fourth time slot with two antennas.

The received signals at destination $\mathcal{D}$ at the end of third time slot and fourth time slot are

$$\begin{aligned} y_D^{[2]}(1) &= \sqrt{E_R} g_1 \hat{x}_{R1} + \sqrt{E_R} g_2 \hat{x}_{R2} + n_D^{[2]}(1) \quad (13) \\ y_D^{[2]}(2) &= \sqrt{E_R} g_1 \hat{x}_{R3} + \sqrt{E_R} g_2 \hat{x}_{R4} + n_D^{[2]}(2), \quad (14) \end{aligned}$$

where the superscript $\{\cdot\}^{[2]}$ denotes the second phase. g_r, $r = 1, 2$, is the channel coefficient between relay antenna r and destination $\mathcal{D}$, an independent Gaussian random variable with variance σ_g^2. Additive Gaussian noise elements are generated i.i.d. according to a normal distribution $\mathcal{N}(0, 1)$. We can rewrite (13) and (14) into

$$\mathbf{y}_D^{[2]} = \sqrt{E_R} \underbrace{\begin{bmatrix} g_1 & g_2 & 0 & 0 \\ 0 & 0 & g_1 & g_2 \end{bmatrix}}_{\mathbf{G}} \hat{\mathbf{x}}_R + \mathbf{n}_D^{[2]}. \quad (15)$$

With the received signals in the first phase (7) and in the second phase (15) at destination $\mathcal{D}$, we have

$$\begin{cases} \mathbf{y}_D^{[1]} &= \sqrt{E_x} \begin{bmatrix} f_1 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & f_4 \end{bmatrix} \mathbf{x} + \mathbf{n}_D^{[1]} \\ &= \sqrt{E_x} \mathbf{F} \mathbf{x} + \mathbf{n}_D^{[1]}, \\ \mathbf{y}_D^{[2]} &= \sqrt{E_R} \begin{bmatrix} g_1 & g_2 & 0 & 0 \\ 0 & 0 & g_1 & g_2 \end{bmatrix} \hat{\mathbf{x}}_R + \mathbf{n}_D^{[2]} \\ &= \sqrt{E_R} \mathbf{G} \hat{\mathbf{x}}_R + \mathbf{n}_D^{[2]}. \end{cases} \quad (16)$$

If we construct the matrix $\mathbf{A}_{DF}$ as

$$\mathbf{A}_{DF} \triangleq \begin{bmatrix} \sqrt{E_x} \mathbf{F} \\ \sqrt{E_R} \mathbf{G} \end{bmatrix}, \quad (17)$$

then the decoding procedure will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} ||\mathbf{y}_D - \mathbf{A}_{DF} \mathbf{x}||^2. \quad (18)$$

We note that in this DF scheme, the system model with four sources is actually equivalent to two separate multiple access relay channels (MARC) with two sources. Each separate MARC system is working in two time slots. For example, for a sub-system with sources $\mathcal{S}_1$ and $\mathcal{S}_2$ in Fig. 2, in the first time slot, $\mathcal{S}_1$ and $\mathcal{S}_2$ transmit simultaneously; in the second time slot, relay $\mathcal{R}$ will forward the decoded $\hat{x}_1$ and $\hat{x}_2$ with two antennas.

C. Scheme 3: Digital Network Coding (DNC)

In this scheme, the first transmission phase will be the same as in the decode-and-forward scheme. In other words, at the end of first phase, the received signals at the destination $\mathcal{D}$ will be (7) and the received signals at the relay $\mathcal{R}$ will be (10)-(11).

Then, relay $\mathcal{R}$ will first decode for four sources the same way as the DF scheme, equation (12) and formulate $\hat{x}_{R1} \oplus \hat{x}_{R2}$, $\hat{x}_{R3} \oplus \hat{x}_{R4}$[2]. For BPSK modulation, we will have the following

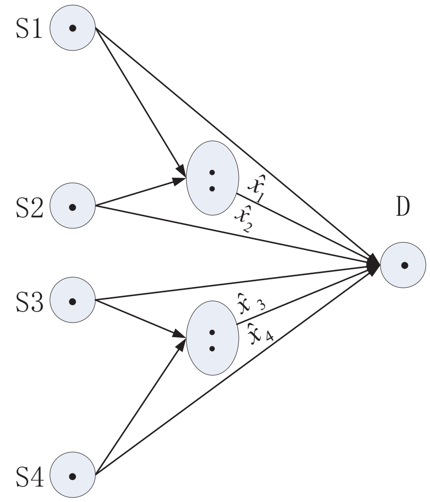

Fig. 2. The equivalent two separate MARC with two sources for DF

relationship.

$$\begin{aligned} \hat{x}_{R1} \oplus \hat{x}_{R2} &= -\hat{x}_{R1} * \hat{x}_{R2} \quad (19) \\ \hat{x}_{R3} \oplus \hat{x}_{R4} &= -\hat{x}_{R3} * \hat{x}_{R4}. \quad (20) \end{aligned}$$

According to digital network coding strategy, with two antennas and power constraint E_R, the relay will transmit $[\hat{x}_{R1} \oplus \hat{x}_{R2}, \hat{x}_{R1} \oplus \hat{x}_{R2}]^T$ and $[\hat{x}_{R3} \oplus \hat{x}_{R4}, \hat{x}_{R3} \oplus \hat{x}_{R4}]^T$ in the following two time slots,

$$\begin{aligned} [y_D^{[2]}(1), y_D^{[2]}(2)] &= \sqrt{E_R}\, [g_1, g_2] \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} & \hat{x}_{R3} \oplus \hat{x}_{R4} \\ \hat{x}_{R1} \oplus \hat{x}_{R2} & \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix} \\ &\quad + [n_D^{[2]}(1), n_D^{[2]}(2)], \end{aligned}$$

which can be arranged as

$$\mathbf{y}_D^{[2]} = \sqrt{E_R} \begin{bmatrix} g_1 + g_2 & 0 \\ 0 & g_1 + g_2 \end{bmatrix} \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix} + \mathbf{n}_D^{[2]}. \quad (21)$$

Recall the received signals in the first phase (7) and in the second phase (21) at destination $\mathcal{D}$, we have

$$\begin{cases} \mathbf{y}_D^{[1]} = \sqrt{E_x} \begin{bmatrix} f_1 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & f_4 \end{bmatrix} \mathbf{x} + \mathbf{n}_D^{[1]} = \mathbf{F}\mathbf{x} + \mathbf{n}_D^{[1]}, \\ \mathbf{y}_D^{[2]} = \sqrt{E_R} \begin{bmatrix} g_1 + g_2 & 0 \\ 0 & g_1 + g_2 \end{bmatrix} \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix} + \mathbf{n}_D^{[2]}. \end{cases} \quad (22)$$

we can easily see that, according to this DNC scheme, the system is actually equivalent to two separate multiple access relay channels (MARC) with two sources as in Fig. 3. Each two sources MARC system is working in two time slots, while in the first time slot sources transmitting and in the second time slot the relay will forward the XOR symbol of the decoded messages.

The decoding procedure can also be processed according to those two separate MARC system. For sources $\mathcal{S}_1$ and $\mathcal{S}_2$, we have

$$\begin{cases} y_D^{[1]}(1) &= \sqrt{E_x} f_1 x_1 + \sqrt{E_x} f_2 x_2 + n_D^{[1]}(1) \\ y_D^{[2]}(1) &= \sqrt{E_R}(g_1 + g_2)(-\hat{x}_{R1} * \hat{x}_{R2}) + n_D^{[2]}(1), \end{cases} \quad (23)$$

[2]The relay actually first demodulates $\hat{x}_{Ri}$ to information bit $\hat{b}_{Ri}$, then calculate $\hat{b}_{R1} \oplus \hat{b}_{R2}$, $\hat{b}_{R3} \oplus \hat{b}_{R4}$, and finally modulates them again. We simply denote the modulated $\hat{b}_{R1} \oplus \hat{b}_{R2}$, $\hat{b}_{R3} \oplus \hat{b}_{R4}$ as $\hat{x}_{R1} \oplus \hat{x}_{R2}$, $\hat{x}_{R3} \oplus \hat{x}_{R4}$.

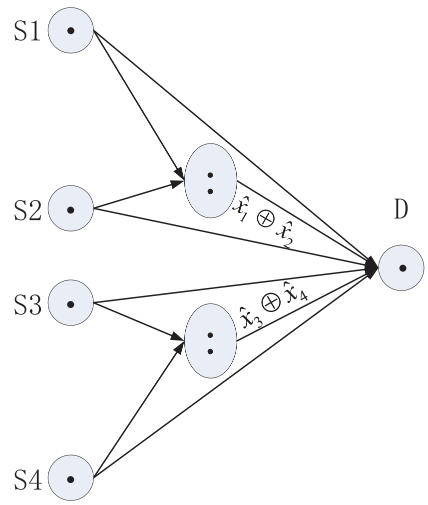

Fig. 3. The equivalent two separate MARC with two sources for DNC

and will decode $[\hat{x}_1, \hat{x}_2]^T$ as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \arg\min_{x_1,x_2} ||y_D^{[1]}(1) - \sqrt{E_x} f_1 x_1 - \sqrt{E_x} f_2 x_2||^2 + ||y_D^{[2]}(1) + \sqrt{E_R}(g_1 + g_2)(x_1 * x_2)||^2. \quad (24)$$

For sources $\mathcal{S}_3$ and $\mathcal{S}_4$ we can decode in a similar way,

$$\begin{bmatrix} \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \arg\min_{x_3,x_4} ||y_D^{[1]}(2) - \sqrt{E_x} f_3 x_3 - \sqrt{E_x} f_4 x_4||^2 + ||y_D^{[2]}(2) + \sqrt{E_R}(g_1 + g_2)(x_3 * x_4)||^2. \quad (25)$$

D. Scheme 4: Digital Network Coding with Alamouti space time coding (DNC-Alamouti)

In this scheme, the first transmission phase will also be the same as in the decode-and-forward scheme. In other words, at the end of first phase, the received signals at the destination $\mathcal{D}$ will be (7) and the received signals at the relay $\mathcal{R}$ will be (10)-(11).

Then, relay $\mathcal{R}$ will first decode for four sources the same way as the DF scheme, equation (12) and formulate $\hat{x}_{R1} \oplus \hat{x}_{R2}$ and $\hat{x}_{R3} \oplus \hat{x}_{R4}$. The transmission from relay $\mathcal{R}$ will follow the Alamouti space time coding. In the third time slot, the relay will transmit $[\hat{x}_{R1} \oplus \hat{x}_{R2}, \hat{x}_{R3} \oplus \hat{x}_{R4}]^T$ and in the fourth time slot, the relay will transmit $[-(\hat{x}_{R3} \oplus \hat{x}_{R4})^*, (\hat{x}_{R1} \oplus \hat{x}_{R2})^*]^T$. Denote the corresponding received signals at destination $\mathcal{D}$ in the second phase (with two time slots) as $y_D^{[2]}(1)$ and $y_D^{[2]}(2)$, then

$$\left[y_D^{[2]}(1), y_D^{[2]}(2)\right] = \sqrt{E_R}[g_1, g_2] \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} & -(\hat{x}_{R3} \oplus \hat{x}_{R4})^* \\ \hat{x}_{R3} \oplus \hat{x}_{R4} & (\hat{x}_{R1} \oplus \hat{x}_{R2})^* \end{bmatrix} + [n_D^{[2]}(1), n_D^{[2]}(2)]. \quad (26)$$

After receiving signals from relay $\mathcal{R}$ in the second phase, destination $\mathcal{D}$ arranges the received signals into a vector $\mathbf{y}_D^{[2]} = \left[y_D^{[2]}(1), -y_D^{[2]}(2)^*\right]^T$, which can be rewritten as

$$\mathbf{y}_D^{[2]} = \begin{bmatrix} y_D^{[2]}(1) \\ -y_D^{[2]}(2)^* \end{bmatrix} = \sqrt{E_R} \begin{bmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{bmatrix} \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix} + \mathbf{n}_D^{[2]}, \quad (27)$$

where $\mathbf{n}_D^{[2]} = [n_D^{[2]}(1), -n_D^{[2]}(2)^*]^T$.

Recall the received signals in the first phase (7) and in the second phase (27) at destination $\mathcal{D}$, we have

$$\begin{cases} \mathbf{y}_D^{[1]} = \sqrt{E_x} \begin{bmatrix} f_1 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & f_4 \end{bmatrix} \mathbf{x} + \mathbf{n}_D^{[1]} = \mathbf{F}\mathbf{x} + \mathbf{n}_D^{[1]}, \\ \mathbf{y}_D^{[2]} = \sqrt{E_R} \begin{bmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{bmatrix} \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix} + \mathbf{n}_D^{[2]}. \end{cases} \quad (28)$$

Equivalently, the received signals at destination during the two phases can be written as

$$\begin{cases} y_{D1} = \sqrt{E_x} f_1 x_1 + \sqrt{E_x} f_2 x_2 + n_{D1} \\ y_{D2} = \sqrt{E_x} f_3 x_3 + \sqrt{E_x} f_4 x_4 + n_{D2} \\ y_{D3} = -\sqrt{E_R} g_1 (\hat{x}_{R1} * \hat{x}_{R2}) - \sqrt{E_R} g_2 (\hat{x}_{R3} * \hat{x}_{R4}) + n_{D3} \\ y_{D4} = \sqrt{E_R} g_2^* (\hat{x}_{R1} * \hat{x}_{R2}) - \sqrt{E_R} g_1^* (\hat{x}_{R3} * \hat{x}_{R4}) + n_{D4}, \end{cases}$$

where $[y_{D1}, y_{D2}, y_{D3}, y_{D4}] = [(\mathbf{y}_D^{[1]})^T, (\mathbf{y}_D^{[2]})^T]$ and $[n_{D1}, n_{D2}, n_{D3}, n_{D4}] = [(\mathbf{n}_D^{[1]})^T, (\mathbf{n}_D^{[2]})^T]$.

The correspondent decoding procedure will be

$$\begin{aligned} \hat{\mathbf{x}} = \arg \min_{x_1,x_2,x_3,x_4} & ||y_{D1} - \sqrt{E_x} f_1 x_1 - \sqrt{E_x} f_2 x_2||^2 \\ & + ||y_{D2} - \sqrt{E_x} f_3 x_3 - \sqrt{E_x} f_4 x_4||^2 \\ & + ||y_{D3} + \sqrt{E_R} g_1 (\hat{x}_{R1} * \hat{x}_{R2}) + \sqrt{E_R} g_2 (\hat{x}_{R3} * \hat{x}_{R4})||^2 \\ & + ||y_{D4} - \sqrt{E_R} g_2^* (\hat{x}_{R1} * \hat{x}_{R2}) + \sqrt{E_R} g_1^* (\hat{x}_{R3} * \hat{x}_{R4})||^2. \end{aligned} \quad (29)$$

E. Scheme 5: Analog Network Coding (ANC)

In this scheme, relay $\mathcal{R}$ will utilize analog network coding to process the received signals. First, after receiving $\mathbf{y}_R$ of (10)-(11), relay $\mathcal{R}$ constructs the following signal vector

$$\begin{aligned} \mathbf{t} &= \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} = \begin{bmatrix} \beta_1 y_{R1} \\ \beta_2 y_{R2} \\ \beta_3 y_{R3} \\ \beta_4 y_{R4} \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix}}_{\mathbf{B}} (\sqrt{E_x}\mathbf{H}\mathbf{x} + \mathbf{n}_R), \end{aligned} \quad (30)$$

where β_r, $r = 1, 2, 3, 4$ is the scaling factor at relay $\mathcal{R}$ given by

$$\beta_r = \sqrt{\frac{1}{E\{|y_{Rr}|^2\}}} = \sqrt{\frac{1}{E_x ||\mathbf{h}_r||^2 + 1}}. \quad (31)$$

Note that $\mathbf{h}_r^T$ is the r-th row vector of matrix $\mathbf{H}$ in (10), i.e.,

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3, \mathbf{h}_4]^T. \quad (32)$$

Table 1: Comparison of Different Schemes

	DT	DF	DNC	DNC-Alamouti	ANC
Time Slot 1	S1: x_1	$S1: x_1$ $S2: x_2$	$S1: x_1$ $S2: x_2$	$S1: x_1$ $S2: x_2$	$S1: x_1$ $S2: x_2$
Time Slot 2	S2: x_2	$S3: x_3$ $S4: x_4$	$S3: x_3$ $S4: x_4$	$S3: x_3$ $S4: x_4$	$S3: x_3$ $S4: x_4$
Time Slot 3	S3: x_3	$R: \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix}$	$R: \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R1} \oplus \hat{x}_{R2} \end{bmatrix}$	$R: \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix}$	$R: \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$
Time Slot 4	S4: x_4	$R: \begin{bmatrix} \hat{x}_{R3} \\ \hat{x}_{R4} \end{bmatrix}$	$R: \begin{bmatrix} \hat{x}_{R3} \oplus \hat{x}_{R4} \\ \hat{x}_{R3} \oplus \hat{x}_{R4} \end{bmatrix}$	$R: \begin{bmatrix} -(\hat{x}_{R3} \oplus \hat{x}_{R4})^* \\ (\hat{x}_{R1} \oplus \hat{x}_{R2})^* \end{bmatrix}$	$R: \begin{bmatrix} t_3 \\ t_4 \end{bmatrix}$

Then, relay $\mathcal{R}$ will transmit t_1, t_2, t_3 and t_4 in two time slots as follows,

$$\left[y_D^{[2]}(1), y_D^{[2]}(2)\right] = \sqrt{E_R}\; [g_1, g_2] \begin{bmatrix} t_1 & t_3 \\ t_2 & t_4 \end{bmatrix} + [n_D^{[2]}(1), n_D^{[2]}(2)], \tag{33}$$

which can be arranged as

$$\begin{aligned} \mathbf{y}_D^{[2]} &= \begin{bmatrix} y_D^{[2]}(1) \\ y_D^{[2]}(2) \end{bmatrix} \\ &= \sqrt{E_R} \underbrace{\begin{bmatrix} g_1 & g_2 & 0 & 0 \\ 0 & 0 & g_1 & g_2 \end{bmatrix}}_{\mathbf{G}} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} + \mathbf{n}_D^{[2]} \\ &= \sqrt{E_R}\sqrt{E_x}\mathbf{GBHx} + \sqrt{E_R}\mathbf{GBn}_R + \mathbf{n}_D^{[2]}. \end{aligned} \tag{34}$$

Combining the received signals in the first phase (7) and in the second phase (34) at destination $\mathcal{D}$, we have

$$\mathbf{y}_D = \underbrace{\sqrt{E_x}\begin{bmatrix} \mathbf{F} \\ \sqrt{E_R}\mathbf{GBH} \end{bmatrix}}_{\mathbf{A}_{ANC}} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{n}_D^{[1]} \\ \sqrt{E_R}\mathbf{GBn}_R + \mathbf{n}_D^{[2]} \end{bmatrix}}_{\mathbf{z}}. \tag{35}$$

which can be decoded as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\Omega_{\mathbf{x}}} ||\mathbf{y}_D - \mathbf{A}_{ANC}\mathbf{x}||^2. \tag{36}$$

The information send in each time slot of those five discussed schemes are shown in Table 1.

III. Simulation Studies

In this section, we present numerical results to evaluate the performance of different schemes used in the system model. Let $E_x = E_R$, i.e., the transmission power constraints at sources and relay are equivalent. The channels between source and relay, relay and destination and source to destination all have an equal channel gain, i.e., $\sigma_f^2 = \sigma_h^2 = \sigma_g^2 = 1$. With the average of 100000 randomly generated channel realizations, we show in Fig. 4 the error rate of the transmission signal vector $\mathbf{x}$ defined in (1) for five possible schemes. All schemes are subject to four time slots constraint.

We can observe in Fig. 4 that direct transmission (DT) gives the poorest performance since other schemes can benefit from the help of relay. Interestingly, the traditional decode-and-forward (DF) scheme gives the best performance and is

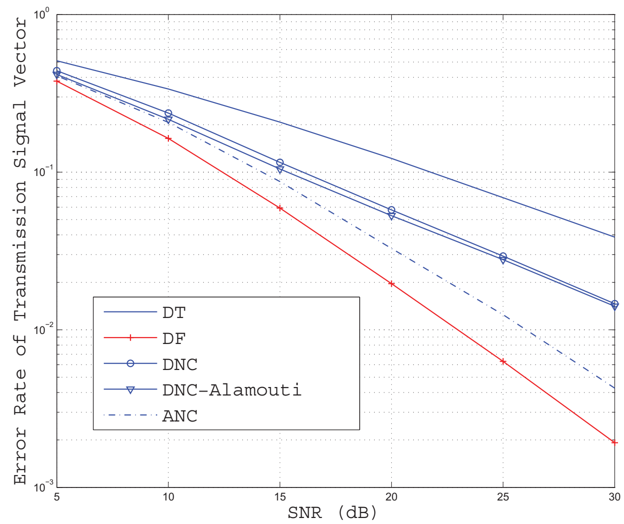

Fig. 4. Simulation Comparison of five schemes, $\sigma_f^2 = \sigma_h^2 = \sigma_g^2 = 1$

superior to digital network coding (DNC), DNC with Alamouti space time coding and analog network coding (ANC). In other words, for this system model and transmission time slots constraint, those schemes with network coding do not show any performance gain compared with the traditional DF scheme. In current literature it has shown that network coding improves the system performance in two-way relay channel and multiple access relay channel with two sources since the schemes with network coding can save transmission time slots. However, for our system model, which we set up purposely to investigate the performance of combining digital network coding with Alamouti space time coding scheme, those schemes with network coding cannot save transmission time slots, i.e., they need the same four time slots to complete the transmission. In the traditional DF transmission, each relay antenna has clean decoded signals to transmit, while in schemes with network coding each relay antenna is transmitting mixed decoded signals, which degrades the performance.

Furthermore, we investigate the performance of all possible schemes with more channel setup conditions. For example, if the direct links from sources to destination have the worst channel with $\sigma_f^2 = 0.1$ while $\sigma_h^2 = \sigma_g^2 = 1$, we show the error

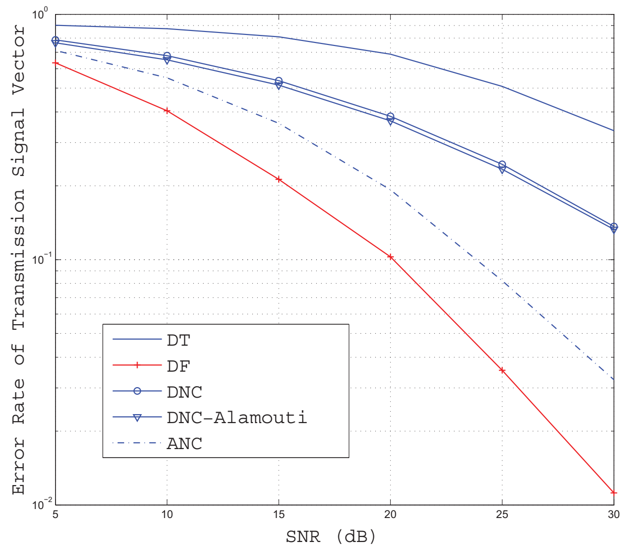

Fig. 5. Simulation Comparison of five schemes: $\sigma_f^2 = 0.1$, $\sigma_h^2 = \sigma_g^2 = 1$,

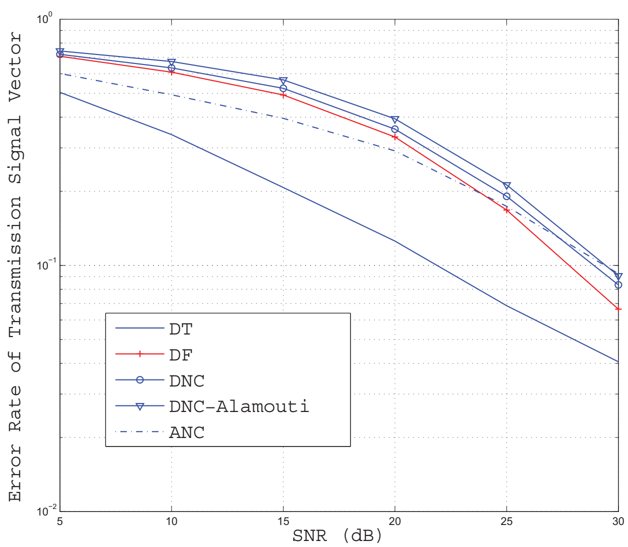

Fig. 6. Simulation Comparison of five schemes: $\sigma_h^2 = 0.1$, $\sigma_f^2 = \sigma_g^2 = 1$

rate of the transmission signal vector comparisons in Fig. 5. We can see that the traditional DF scheme still gives best performance, while DT, DNC and DNC-Alamouti schemes suffers more since at the destination, decoding procedures with those schemes rely on the information transmitted on the direct links more.

Another channel gain setup is that the links between the sources and the relay have the worst conditions, i.e., $\sigma_h^2 = 0.1$, $\sigma_f^2 = \sigma_g^2 = 1$. In this case, the relay cannot help much since it cannot get enough accurate information. Hence, from the simulation result in Fig. 6, we can see that DT gives the best performance as expected; while in other schemes that need help from the relay, give inferior performance.

IV. Conclusions

A study on network coding in multiple access relay channel (MARC) with multiple antenna relay is presented. We set up a multiple access relay system model with four sources intentionally, to investigate five different schemes under four time slots transmission constraint and compare the error rate performance. Interestingly, simulation studies show that those schemes with network coding (including combining network coding with space time coding technique) do not show any performance gain compared with the traditional schemes, which indicates that network coding may not be favorable for the system model if traditional schemes can also be implemented with the same time slots constraint.

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