Integer-Forcing Linear Receiver Design over MIMO Channels[†]

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Abstract—Motivated by recently presented integer-forcing linear receiver architecture, we propose algorithms to design optimal integer-forcing coefficient matrix such that the total achievable rate is maximized.

Index Terms—multiple-input multiple-output, linear receiver, lattice codes, compute-and-forward, multiple-access channel.

I. INTRODUCTION

Network coding [1], originally designed for wired networks, is a generalized routing approach that breaks the traditional assumption of simply forwarding data, and allows intermediate nodes to send out functions of their received packets, by which the multicast capacity given by the max-flow min-cut theorem can be achieved. [2]-[4] made the important observation that, for multicasting, intermediate nodes can simply send out a linear combination of their received packets. Linear network coding with random coefficients is considered in [5]. In order to address the broadcast nature of wireless transmission, physical layer network coding [6] was proposed to embrace interference in wireless networks in which intermediate nodes attempt to decode the modulo-two sum (XOR) of the transmitted messages. Several network coding realizations in wireless networks are discussed in [7]-[11].

Compute-and-forward (CPF) strategy [12]-[13] is a promising new approach to physical-layer network coding for general wireless networks, beneficial from both network coding and lattice codes [14]-[15]. The main idea is that a relay will decode a linear function of transmitted messages according to the observed channel coefficients rather than ignoring the interference as noise. Upon utilizing the algebraic structure of lattice codes [16]-[17], i.e., the integer combination of lattice codewords is still a codeword, the intermediate relay node decodes and forwards an integer combination of original messages. With enough linear independent equations, the destination can recover the original messages respectively. Subsequent works to design CPF network coding coefficients are discussed in [18] for multi-source multi-relay channels and in [19] for two-way relay channels.

Recently, as a research extension from the idea of CPF strategy, a new linear receiver technique called *integer forcing* (IF) receiver for MIMO system has been proposed in [20]-[21]. In MIMO communication, the destination often utilizes linear

receiver architecture to reduce implementation complexity with some performance sacrifice compared with maximum-likelihood (ML) receiver. The standard linear detection methods include zero-forcing (ZF) technique and the minimum mean square error (MMSE) technique [22]. In the newly proposed IF linear receiver, instead of attempting to recover a transmitted codeword directly, each IF decoder recovers a different integer combination of the lattice codewords according to a designed IF coefficient matrix. If the IF coefficient matrix is of full rank, these linear equations can be solved for the original messages.

However, the works of [20]-[21] only give the optimization criteria regarding IF coefficient matrix without explicit methods to solve it. In this paper, we present algorithms to design IF coefficient matrix with full rank such that the total achievable rate is maximized. First, we will generate feasible searching set instead of the whole integer searching space based on Fincke-Pohst method [23], which has been applied to lattice code decoder in [24]-[25]. Then we try to pick up integer vectors within our searching set to construct the full rank IF coefficient matrix, while in the meantime, the total achievable rate is maximized.

II. SYSTEM MODEL

Fig. 1. System Diagram

We consider the classic MIMO channel with L transmit antennas and N receive antennas. Each transmit antenna delivers an independent data stream which encoded separately to form the transmitted codewords. As any MIMO complex channel can be represented by its real-valued equivalence [26], for ease of analysis, we will work on the real MIMO channel for notational convenience. The system diagram is shown in Fig. 1.

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Without loss of generality, in one information codeword transmission, each antenna has a length- k information vector

$$\mathbf{w}_m = [w_m(1), w_m(2), \dots, w_m(k)], \quad (1)$$

where $\mathbf{w}_m \in \mathbb{F}_p^k$, $m = 1, 2, \dots, L$; $\mathbb{F}_p = \{0, 1, \dots, p-1\}$ is a prime size finite field. Each antenna is equipped with an encoder

$$\mathcal{E}_m : \mathbb{F}_p^k \rightarrow \mathbb{R}^n, \quad (2)$$

that maps the length- k message $\mathbf{w}_m$ into a length- n lattice codeword $\mathbf{x}_m \in \mathbb{R}^n$. The codeword satisfies the power constraint of

$$\frac{1}{n} \|\mathbf{x}_m\|^2 \leq P. \quad (3)$$

After mapping message $\mathbf{w}_m$ into a lattice codeword

$$\mathbf{x}_m = [x_m(1), x_m(2), \dots, x_m(n)], \quad (4)$$

antenna m will transmit one information codeword $\mathbf{x}_m$ in a total of n transmission realizations. Assume a slow fading model where the channel remains constant over the entire codeword transmission and channel state information is only available at the receiver.

In the i th transmission realization, the received vector is,

$$\begin{aligned} \mathbf{y}^{[i]} &= \sum_{m=1}^L \mathbf{h}_m x_m(i) + \mathbf{z}^{[i]} \\ &= \mathbf{H} \mathbf{x}^{[i]} + \mathbf{z}^{[i]}, \end{aligned} \quad (5)$$

where

$$\mathbf{x}^{[i]} = [x_1(i), x_2(i), \dots, x_L(i)]^T, \quad (6)$$

denotes the information codewords from all L antennas in the i th transmission; $\mathbf{h}_m \in \mathbb{R}^N$ is real valued fading channel vector from antenna m to the receiver; the equivalent channel matrix

$$\mathbf{H}_{N \times L} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_L], \quad (7)$$

$\mathbf{z}^{[i]} \in \mathbb{R}^N$ is additive Gaussian noise. The entries of all channel vectors are generated i.i.d. according to a normal distribution $\mathcal{N}(0, 1)$.

In a linear receiver architecture, the receiver will project $\mathbf{y}^{[i]}$ with some matrix $\mathbf{B} \in \mathbb{R}^{L \times N}$ to get the effective received vector for further decoding,

$$\begin{aligned} \tilde{\mathbf{y}}^{[i]} = \mathbf{B} \mathbf{y}^{[i]} &= \mathbf{B} \mathbf{H} \mathbf{x}^{[i]} + \mathbf{B} \mathbf{z}^{[i]} \\ &= \mathbf{A} \mathbf{x}^{[i]} + \tilde{\mathbf{z}}^{[i]}. \end{aligned} \quad (8)$$

The standard linear detection methods include the zero-forcing (ZF) receiver and the minimum mean square error (MMSE) receiver,

$$\begin{aligned} \mathbf{B}_{ZF} &= (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T & \mathbf{A}_{ZF} &= \mathbf{I}_L \\ \mathbf{B}_{MMSE} &= (\mathbf{H}^T \mathbf{H} + \frac{1}{P} \mathbf{I}_L)^{-1} \mathbf{H}^T & \mathbf{A}_{MMSE} &= \mathbf{B}_{MMSE} \mathbf{H}, \end{aligned}$$

where $(\cdot)^T$ denotes transpose operation and $\mathbf{I}_L$ is $L \times L$ identity matrix. The ZF technique nullifies the interference such that $\mathbf{A}_{ZF} = \mathbf{I}_L$ with the effect of noise enhancement. The MMSE receiver maximizes the post-detection signal-to-interference plus noise ratio (SINR) and mitigates the

noise enhancement effects. However, both ZF and MMSE receiver have been proved to be largely suboptimal in terms of diversity-multiplexing tradeoff [27].

We recall the important algebraic structure of lattice codes, that the integer combination of lattice codewords is still a codeword [16]-[17]. Instead of restricting matrix $\mathbf{A}$ to be identity, we may allow $\mathbf{A}$ to be some full rank matrix with integer coefficients, i.e.

$$\mathbf{A}_{IF} \in \mathbb{Z}^{L \times L}. \quad (9)$$

Then, we can first separately recover linear combinations of transmitted lattice codewords with coefficients drawn from matrix $\mathbf{A}_{IF}$. After that, these equations can be easily solved for the original messages.

Hence, in integer forcing (IF) receiver [20]-[21], the receiver will try to design an equalization matrix $\mathbf{B}_{IF} \in \mathbb{R}^{L \times N}$, such that after the projection process of (8), the resulting IF matrix $\mathbf{A}_{IF}$ satisfies that $\mathbf{A}_{IF} \in \mathbb{Z}^{L \times L}$ and the achievable rate is maximized. We summarize the results regarding IF receiver in [20]-[21] in the following theorem. Interested readers can look into [20]-[21] for detailed proofs of Theorem 2.1.

Theorem 2.1: Let $\mathbf{A}_{IF} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L]^T$ and $\mathbf{B}_{IF} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_L]^T$. For each pair of $(\mathbf{a}_m, \mathbf{b}_m)$, the following computation rate is achievable,

$$\mathcal{R}_m = \frac{1}{2} \log \left(\frac{P}{\|\mathbf{b}_m\|^2 + P \|\mathbf{H}^T \mathbf{b}_m - \mathbf{a}_m\|^2} \right). \quad (10)$$

For a fixed IF coefficient matrix $\mathbf{A}_{IF}$, the computation rate is maximized by choosing¹

$$\mathbf{b}_m^T = \mathbf{a}_m^T \mathbf{H}^T \left(\mathbf{H} \mathbf{H}^T + \frac{1}{P} \mathbf{I}_L \right)^{-1}. \quad (11)$$

□

According to Theorem 2.1, we plug in the optimal $\mathbf{b}_m$ of (11) into the computation rate $\mathcal{R}_m$ of (10), which will result in

$$\mathcal{R}_m = \frac{1}{2} \log \left(\frac{1}{\mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m} \right), \quad (12)$$

where

$$\mathbf{Q} \triangleq \mathbf{I}_L - \mathbf{H}^T \left(\mathbf{H} \mathbf{H}^T + \frac{1}{P} \mathbf{I}_L \right)^{-1} \mathbf{H}. \quad (13)$$

Then, the total achievable rate of the IF receiver is

$$\begin{aligned} \mathcal{R}_{total} &\triangleq \max_{|\mathbf{A}| \neq 0} L \min_m \mathcal{R}_m \\ &= \max_{|\mathbf{A}| \neq 0} \min_m L \log \left(\frac{1}{\mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m} \right), \end{aligned} \quad (14)$$

Hence, the design criteria for optimal IF coefficient matrix $\mathbf{A}_{IF}$ is

$$\begin{aligned} \mathbf{A}_{IF} &= \arg \max_{|\mathbf{A}| \neq 0} \min_m \frac{L}{2} \log \left(\frac{1}{\mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m} \right) \\ &= \arg \min_{|\mathbf{A}| \neq 0} \max_m \mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m, \end{aligned} \quad (15)$$

¹The optimal projection matrix is $\mathbf{B}_{IF} = \mathbf{A}_{IF} \mathbf{H}^T (\mathbf{H} \mathbf{H}^T + \frac{1}{P} \mathbf{I}_L)^{-1}$.

where matrix $\mathbf{Q}$ is defined in (13). It means that we need to find integer vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L$ to construct a full rank matrix $\mathbf{A}_{IF}$, such that the maximum value of $\mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m$ is minimized.

Solving this optimization problem is critical as it dominates the total achievable rate of the desired IF receiver that sources can reliably communicate with the destination. As it needs to return L integer vectors to construct the IF coefficient matrix $\mathbf{A}_{IF}$ with full rank, no explicit solution is presented in the previous works. In this paper, we will propose algorithms to design this optimal $\mathbf{A}_{IF}$.

III. PROPOSED ALGORITHMS

To approach the optimization problem of (15), first we need to generate some feasible searching set

$$\Omega \subset \mathbb{Z}^L, \quad (16)$$

for $\mathbf{a}_m \in \Omega$, $m = 1, 2, \dots, L$, instead of the whole searching space $\mathbf{a}_m \in \mathbb{Z}^L$. Then, we will find L linearly independent vectors within this searching set Ω to construct the optimal IF coefficient matrix $\mathbf{A}_{IF}$.

Accordingly, we propose the following strategy with two steps. In the first step, we generate the searching set Ω based on Fincke-Pohst (FP) method [23], such that the integer vectors $\mathbf{t} \in \mathbb{Z}^L$ with top $|\Omega|$ minimum $\mathbf{t}^T \mathbf{Q} \mathbf{t}$ values are within.

In the second step, we pick up $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L \in \Omega$, to construct the full rank IF coefficient matrix

$$\mathbf{A}_{IF} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L]^T, \quad (17)$$

while in the meantime, the maximum value of $\mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m$ is minimized. Then, equivalently, this optimal $\mathbf{A}_{IF}$ will maximize the total achievable rate.

A. FP Based Candidate Set Searching Algorithm

We attempt to find the candidate set Ω such that integer vectors with top $|\Omega|$ minimum $\mathbf{t}^T \mathbf{Q} \mathbf{t}$ values are within. The procedure of enumerating all vectors $\mathbf{t} \in \mathbb{Z}^L$ ($\mathbf{t} \neq \mathbf{0}$) in Ω , such that

$$\mathbf{t}^T \mathbf{Q} \mathbf{t} \leq C, \quad (18)$$

for a given positive constant C is based on FP method and derived as follows.

We operate Cholesky factorization of matrix $\mathbf{Q}$ which yields

$$\mathbf{Q} = \mathbf{U}^T \mathbf{U}, \quad (19)$$

where $\mathbf{U}$ is an upper triangular matrix. Low complexity Cholesky factorization algorithm is explained in [28].

Let u_{ij} , $i, j = 1, 2, \dots, L$, be entries of matrix $\mathbf{U}$ and

$$\mathbf{t} = [t_1, t_2, \dots, t_L]^T. \quad (20)$$

Then, the searching vector $\mathbf{t}$ that makes $\mathbf{t}^T \mathbf{Q} \mathbf{t} \leq C$ can be expressed as

$$\begin{aligned} \mathbf{t}^T \mathbf{Q} \mathbf{t} &= \|\mathbf{U} \mathbf{t}\|_F^2 = \sum_{i=1}^L \left(u_{ii} t_i + \sum_{j=i+1}^L u_{ij} t_j \right)^2 \\ &= \sum_{i=1}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \\ &= \sum_{i=k}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 + \sum_{i=1}^{k-1} g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \\ &\leq C \end{aligned} \quad (21)$$

where

$$g_{ii} = u_{ii}^2, \quad g_{ij} = u_{ij}/u_{ii}, \quad (22)$$

for $i = 1, 2, \dots, L$, $j = i+1, \dots, L$.

To satisfy (21), since every summation item is non-negative, it is equivalent to consider for every $k = L, L-1, \dots, 1$ in (21), i.e.,

$$\begin{cases} g_{LL} t_L \leq C \\ \sum_{i=L-1}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \leq C \\ \vdots \\ \sum_{i=k}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \leq C \\ \vdots \\ \sum_{i=2}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \leq C \\ \sum_{i=1}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \leq C. \end{cases} \quad (23)$$

Then, we can start work backwards to find bounds for vector entries $t_L, t_{L-1}, \dots, t_1$ one by one.

To evaluate the element t_k , $k \in \{L, L-1, \dots, 1\}$, of the searching vector $\mathbf{t}$, referring to (23) we consider

$$\sum_{i=k}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \leq C, \quad (24)$$

that leads to equation (25) on the top of next page.

If we denote

$$\Delta_k = \sum_{j=k+1}^L g_{kj} t_j, \quad (26)$$

$$\begin{aligned} C_k &= C - \sum_{i=k+1}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \\ &= C_{k+1} - g_{k+1,k+1} (\Delta_{k+1} + t_{k+1})^2, \end{aligned} \quad (27)$$

the bounds for t_k can be expressed as

$$LB_k \leq t_k \leq UB_k, \quad (28)$$

where

$$\begin{aligned} UB_k &= \left\lceil \sqrt{\frac{C_k}{g_{kk}}} - \Delta_k \right\rceil, \\ LB_k &= \left\lfloor -\sqrt{\frac{C_k}{g_{kk}}} - \Delta_k \right\rfloor. \end{aligned} \quad (29)$$

$$\left| -\sqrt{\frac{1}{g_{kk}} \left(C - \sum_{i=k+1}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \right) - \sum_{j=k+1}^L g_{kj} t_j} \right| \leq a_k \leq \left| \sqrt{\frac{1}{g_{kk}} \left(C - \sum_{i=k+1}^L g_{ii} \left(t_i + \sum_{j=i+1}^L g_{ij} t_j \right)^2 \right) - \sum_{j=k+1}^L g_{kj} t_j} \right|. \quad (25)$$

Note that for given radius $\sqrt{C}$ and the matrix $\mathbf{U}$, the bounds for t_k only depends on the previous evaluated $t_{k+1}, t_{k+2}, \dots, t_L$.

The entries $t_L, t_{L-1}, \dots, t_1$ are chosen as follows: for a chosen t_L , we can choose t_{L-1} satisfying its bounds requirements as in (28) for $k = L - 1$. If such t_{L-1} does not exist, we go back and choose other t_L . Then search for t_{L-1} that meets its bounds requirement for this new t_L . If t_L and t_{L-1} are chosen, we follow the same procedure to choose t_{L-2} , and so on. When a set of $t_L, t_{L-1}, \dots, t_1$ is chosen and satisfies all corresponding bounds requirements, one candidate vector $\mathbf{t} = [t_1, t_2, \dots, t_L]^T$ is obtained. We record all candidate vectors satisfying $\mathbf{t}^T \mathbf{Q} \mathbf{t} \leq C$ in Ω .

Regarding the positive constant C , we set it based on the binary vector obtained by applying the direct sign operator of the real minimum-eigenvalue eigenvector of $\mathbf{Q}$, denoted as $\mathbf{t}_{quant}$, such that

$$C = \mathbf{t}_{quant}^T \mathbf{Q} \mathbf{t}_{quant}. \quad (30)$$

By setting the searching sphere radius this way, it is big enough to have several searching vectors falls inside, while in the meantime small enough to have not too many searching vectors within.

We summarize our proposed algorithm for the searching set Ω based on Fincke-Pohst method as follows.

Algorithm 1 FP Based Candidate Set Searching Algorithm

Input: Matrix $\mathbf{Q}$.

Output: The searching set Ω .

Step 1: Calculate the binary quantized vector obtained by applying the direct sign operator of the real minimum-eigenvalue eigenvector of $\mathbf{Q}$, denoted as $\mathbf{t}_{quant}$, and set C as

$$C = \mathbf{t}_{quant}^T \mathbf{Q} \mathbf{t}_{quant}.$$

Step 2: Performing Cholesky factorization of matrix $\mathbf{Q}$ yields

$$\mathbf{Q} = \mathbf{U}^T \mathbf{U},$$

where $\mathbf{U}$ is an upper triangular matrix.

Let u_{ij} , $i, j = 1, 2, \dots, L$ denote the entries of matrix $\mathbf{U}$. Set

$$g_{ii} = u_{ii}^2, \quad g_{ij} = u_{ij}/u_{ii},$$

for $i = 1, 2, \dots, L$, $j = i + 1, \dots, L$.

Step 3: Construct search set

$$\Omega = \{ \mathbf{t} : \mathbf{t}^T \mathbf{Q} \mathbf{t} \leq C, \mathbf{t} \neq \mathbf{0}, \mathbf{t} \in \mathbb{Z}^L \},$$

according to the following FP procedure.

(i) Start from $\Delta_L = 0$, $C_L = C$, $k = L$ and $\Omega = \emptyset$.

(ii) Set the upper bound UB_k and the lower bound LB_k as follows

$$UB_k = \left\lfloor \sqrt{\frac{C_k}{g_{kk}}} - \Delta_k \right\rfloor, \\ LB_k = \left\lceil -\sqrt{\frac{C_k}{g_{kk}}} - \Delta_k \right\rceil$$

and $t_k = LB_k - 1$.

(iii) Set $t_k = t_k + 1$. For $t_k \leq UB_k$, go to (v); else go to (iv).

(iv) If $k = L$, terminate and output Ω ; else set $k = k + 1$ and go to (iii).

(v) For $k = 1$, go to (vi); else set $k = k - 1$, and

$$\Delta_k = \sum_{j=k+1}^L g_{kj} t_j,$$

$$C_k = C_{k+1} - g_{k+1,k+1} (\Delta_{k+1} + t_{k+1})^2$$

then go to (ii).

(vi) If $\mathbf{t} = \mathbf{0}$ terminate, else we get a candidate vector $\mathbf{t} \neq \mathbf{0}$ that satisfies all the bounds requirements and put it inside Ω , i.e. $\Omega = \{\Omega, \mathbf{t}\}$. Go to (iii).

B. Constructing IF Coefficient Matrix $\mathbf{A}_{IF}$

According to our proposed FP Based Candidate Set Searching Algorithm, we get the feasible searching set Ω for IF vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L$. Define a function $f(\mathbf{t}) \triangleq \mathbf{t}^T \mathbf{Q} \mathbf{t}$. We sort the vectors in the searching set such that

$$\Omega = \{ \mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \dots, \mathbf{t}^{[|\Omega|]} : f(\mathbf{t}^{[1]}) \leq f(\mathbf{t}^{[2]}) \leq \dots \leq f(\mathbf{t}^{[|\Omega|]}) \}. \quad (31)$$

Choose L linear independent vectors within this sorted set by

$$\begin{cases} \mathbf{a}_1 = \mathbf{t}^{[i_1]}, \\ \mathbf{a}_2 = \mathbf{t}^{[i_2]}, \\ \vdots \\ \mathbf{a}_L = \mathbf{t}^{[i_L]}, \end{cases} \quad (32)$$

for some $i_1 < i_2 < \dots < i_L$. Then, the optimization of (15) becomes

$$\begin{aligned} \mathbf{A}_{IF} &= \arg \min_{|\mathbf{A}| \neq \mathbf{0}} \max_m \mathbf{a}_m^T \mathbf{Q} \mathbf{a}_m \\ &= \arg \min_{|\mathbf{A}| \neq \mathbf{0}} \mathbf{a}_L^T \mathbf{Q} \mathbf{a}_L. \end{aligned} \quad (33)$$

Hence, we attempt to find the last coefficient vector $\mathbf{a}_L$, such that $\mathbf{a}_L^T \mathbf{Q} \mathbf{a}_L$ is minimized, if $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{L-1}$ are chosen from the sorted set Ω of (31) as vectors in front of $\mathbf{a}_L$.

For example, we can start with $\mathbf{a}_L = \mathbf{t}^{[L]}$, and check whether $\mathbf{a}_1 = \mathbf{t}^{[1]}, \mathbf{a}_2 = \mathbf{t}^{[2]}, \dots, \mathbf{a}_L = \mathbf{t}^{[L]}$ are linear independent. If it satisfies this criteria, $\mathbf{A}_{IF} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L]^T$ is exactly the optimal IF coefficient matrix. If not, move to $\mathbf{a}_L = \mathbf{t}^{[L+1]}$ and construct a cut set

$$\Omega_{cut} = \{\mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \dots, \mathbf{t}^{[L]}\}. \quad (34)$$

Check whether we can find $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{L-1} \in \Omega_{cut}$ such that the constructed $\mathbf{A}_{IF}$ is of full rank. We continue the process until we find the full rank $\mathbf{A}_{IF}$.

We summarize this procedure to constructing the full rank optimal matrix $\mathbf{A}_{IF}$ with searching set Ω as follows.

Algorithm 2 IF Coefficient Matrix Constructing Algorithm

Input: Searching set Ω .

Output: The IF coefficient matrix $\mathbf{A}_{IF}$ with full rank that gives the maximum total achievable rate $\mathcal{R}_{total}$.

Step 1: Define a function $f(\mathbf{t}) \triangleq \mathbf{t}^T \mathbf{Q} \mathbf{t}$ and sort the vectors in the searching set such that

$$\Omega = \{\mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \dots, \mathbf{t}^{[|\Omega|]} : f(\mathbf{t}^{[1]}) \leq f(\mathbf{t}^{[2]}) \leq \dots \leq f(\mathbf{t}^{[|\Omega|]})\}$$

Initiate $i_L = L$.

Step 2: If $i_L > |\Omega|$, go to Step 4. Else, let $\mathbf{a}_L = \mathbf{t}^{[i_L]}$. Construct the cut set

$$\Omega_{cut} = \{\mathbf{t}^{[1]}, \mathbf{t}^{[2]}, \dots, \mathbf{t}^{[i_L-1]}\}. \quad (35)$$

Then, search through $\binom{i_L-1}{L-1}$ possibilities, to see whether we can find $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{L-1} \in \Omega_{cut}$ such that the constructed $\mathbf{A}_{IF}$ is of full rank.

Step 3: Once we find one full rank matrix $\mathbf{A}_{IF}$, terminate and output this $\mathbf{A}_{IF}$. Else, $i_L = i_L + 1$ and go to Step 2.

Step 4: If we cannot find $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L$ within searching set Ω to construct full rank matrix $\mathbf{A}_{IF}$, we expand the C value setting in (31) as $C = 2C$ and re-generate the searching set Ω by our proposed FP Based Candidate Set Searching Algorithm. Then, go to Step 1.

IV. EXPERIMENTAL STUDIES

In this section, we will present numerical results to evaluate the performance of our proposed algorithms for IF receiver design. The standard linear detection methods of ZF and MMSE receivers are included for comparisons. We also take account in the channel capacity of

$$C = \frac{1}{2} \log \det(\mathbf{I}_N + P \mathbf{H} \mathbf{H}^T), \quad (36)$$

which represents the upper bound for all receiver structures, linear or non-linear including joint ML.

In Fig. 2, we show the rate comparisons over MIMO channels with $L = N = 2$ and average of 1000 randomly

Fig. 2. Average rate comparison with $L = N = 2$

Fig. 3. Average rate comparison with $L = N = 4$

generated channel realizations. We can see that the traditional linear detection technique of ZF and MMSE give poor results. Regarding linear IF receiver with IF coefficient matrix $\mathbf{A}_{IF}$ designed by our proposed algorithms, the average rate is significantly improved and get closer to the channel capacity.

We repeat our experimental with $L = N = 4$ in Fig. 3. Similar conclusions can be drawn. The linear IF receiver still outperforms other linear receiver structures (ZF or MMSE) and follows towards the upper bound.

V. CONCLUSIONS

In this paper, we consider the problem of IF linear receiver design with respect to the channel conditions. We present algorithms to design the IF full rank coefficient matrix with integer elements, such that the total achievable rate is maximized, based on Fincke-Pohst method. First, we will generate feasible candidate integer vector set instead of the whole integer searching space based on Fincke-Pohst method. Then we try to pick up integer vectors within our searching set to construct the full rank IF coefficient matrix, while in the meantime, the total achievable rate is maximized. Numerical studies show the comparisons of other traditional linear receivers.

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