Optimal Power Allocation in Cognitive Relay Networks under Different Power Constraints

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***Abstract*—In this paper, we study the optimal power control schemes for fading channels in cognitive relay networks under different power constraints of the primary user. Under the peak power and the average interference power constraints of the primary user, we derive the optimal power allocation strategies to maximize the ergodic achievable rate of the cognitive relay networks when the channel state information is available to both the cognitive transmitter and the receiver. Finally, the numerical results show the feasibility of our proposed power allocation schemes.**

***Index Terms*—Cognitive radio, cooperative communication, power allocation, decode-and-forward (DF), ergodic achievable rate.**

I. Introduction

Current wireless networks are regulated by a fixed spectrum assignment policy, i.e., the spectrum is regulated by governmental agencies and is assigned to license users or services on a long term basis for large geographical regions. Although the fixed spectrum assignment policy generally served well in the past, there is a dramatic increase in the access to the limited spectrum for mobile services in the recent years. This increase is straining the effectiveness of the traditional spectrum policies. The limited available spectrum and the inefficiency in the spectrum usage necessitate a new communication paradigm to exploit the existing wireless spectrum opportunistically [1]. Another method is called spectrum sharing which allows the operation of the secondary system as long as it doesn't affect the transmission of the primary user [2]. In [3], the author proposed the notion of interference temperature. The interference temperature at a receiving antenna provides an accurate measure for the acceptable level of RF interference in the frequency band of interest; any transmission in that band is considered to be harmful if it would increase the noise floor above the interference temperature constraint. In spectrum sharing mode, we have to control the transmitting power of the secondary user in order not to exceed the interference power constraint. In [4], the authors propose a power control scheme in spectrum sharing mode in order to maximize the ergodic rate of the secondary user which consists of only one transmitter and receiver. In [5], the authors consider both the transmitting power of the cognitive user and the interference power constraint of the cognitive user to primary user.

Relay communication have recently emerged as a powerful spatial diversity technique that can improve the performance over conventional point-to-point transmissions. In [6], Cover and Gamal proposed the concept of relay communication. In [7], three transmission protocols, i.e., Amplify-and-Forward (AF), Decode-and-Forward (DF) and coded cooperation were proposed in cooperative wireless networks. Power control scheme in cooperative communication is studied in [8], [9]. Achievable rate of relay networks was studied in [10].

The capacity of cognitive relay network is studied in [11]. In [12] and [13], the authors derive power allocation scheme to maximize the ergodic capacity of a secondary user transmitting in a rayleigh fading channel, where the ergodic capacity is defined as the maximum achievable rate of the secondary user averaged over all the fading states. However, the secondary user in these papers are only formed by one link. In [14], the authors proposed a power control scheme to maximize the instant achievable rate of the secondary user in a cognitive AF relay network.

In this paper, we study the optimal power control schemes for fading channels in cognitive DF relay networks under different power constraints of the primary user. Under the peak power and the average interference power constraints of the primary user, we derive the optimal power allocation strategies to maximize the ergodic achievable rate of the cognitive DF relay networks when the channel state information is available to both the cognitive transmitter and the receiver. Finally, the numerical results show the feasibility of our proposed power allocation schemes.

The rest of the paper is organized as follows. Section II gives the system model and forms the problem. In section II, we derive the power control scheme to maximize the ergodic achievable rate of the cognitive DF relay networks under the different power constraints of the primary user.

II. System Model

In this paper, we consider a simple spectrum-sharing model with one primary user (PU) and one secondary relay network, which is illustrated in Fig. 1. Su-Tx, Su-Relay, Su-Rx, Pu-Tx and Pu-Rx denotes secondary transmitter, secondary relay, secondary receiver, primary transmitter and primary receiver

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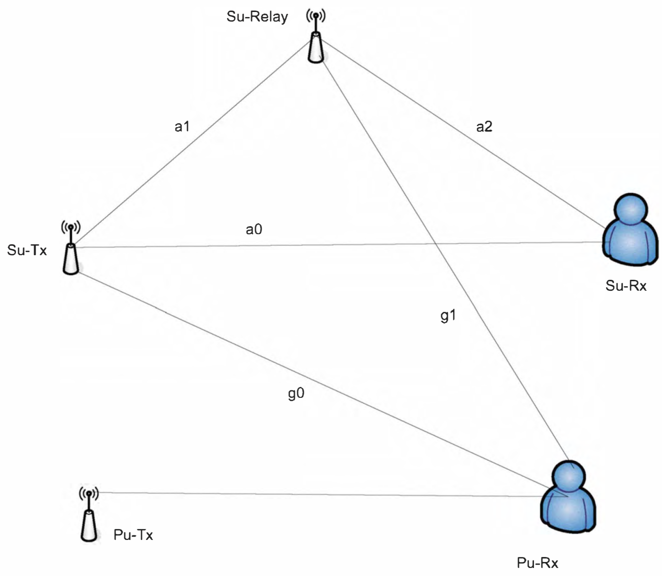

Fig. 1. cognitive relay network model.

respectively. The channel coefficients between the secondary source and relay, the relay and destination, the source and destination are denoted by a_1, a_2 and a_0. The channel coefficients between the secondary source and the primary receiver, the secondary relay and the primary receiver are denoted by g_0 and g_1. The magnitudes of these channel coefficients are assumed to follow an independent Rayleigh distribution. At each receiver, the additive noise is modeled as independent zero-mean, circularly symmetric complex white Gaussian with unit variance. In our model, each frame is divided into two equal time slots. During the first time slot, the source transmits to both the relay and the destination. In the second time slot, the relay forwards the message to the destination if it decodes successfully. Correspondingly, this strategy is also called regenerative relay or digital relay. Let P_s and P_r denote the average transmit power assigned to the source and relay respectively. Then the achievable rate of DF protocol, denoted by R_{DF}, can be expressed as [7]:

$$R_{DF} = \min\left\{\frac{1}{2}\log_2(1+2a_1P_s), \frac{1}{2}\log_2(1+2a_0P_s+2a_2P_r)\right\}. \quad (1)$$

The ergodic achievable rate of the secondary relay network is defined to be:

$$C = \max E\left[\min\left\{\frac{1}{2}\log_2(1+2a_1P_s), \frac{1}{2}\log_2(1+2a_0P_s+2a_2P_r)\right\}\right].$$

Further, we assume that perfect channel state information (CSI) is available at both the transmitter and the receiver of the secondary source and secondary relay. Moreover, we assume that the primary user transmitter (PU-TX) won't cause too much interference to the receiver of the secondary relay and the secondary destination. Thus the interference from the PU-Tx can be neglected.

III. Optimization of Ergodic Achievable Rate under Different Power Constraints

For the secondary user, the peak power constraint is always caused by some physical restrictions. The average transmit power constraint is always more suitable when we mainly consider the power budget. According to the notion of interference temperature [3], we also have interference power constraints. When the primary user provides the service which is susceptive to the delay, we should use the peak power interference. On the other hand, when the service of the primary user can tolerate much delay, we can also use the average interference power constraint.

A. Peak transmit power constraint and peak interference power constraint

Similar in [15], the peak transmitting power of the source and relay is considered wholly, that is, the sum of the transmitting power of cognitive source and relay can't be larger than peak power constraint. For peak interference power constraint, the instantaneous power received by primary receiver can't exceed it. So the overall constraints can be written as:

$$\begin{cases} P_s + P_r \le P, \\ g_0P_s \le Q, \\ g_1P_r \le Q. \end{cases} \quad (2)$$

where P denotes the peak power constraint, and Q denotes the peak interference power constraint.

Theorem 1: The instantaneous CSI based optimal power allocation scheme is:

$$P_s = \begin{cases} \min\left\{P, \frac{Q}{g_0}\right\}, & \text{if } a_1 < a_0, \\ \min\left\{\frac{P}{1+\frac{a_1-a_0}{a_2}}, \frac{Q}{g_0}, \frac{Q}{g_1\frac{a_1-a_0}{a_2}}\right\}, & \text{if } a_1 > a_0. \end{cases} \quad (3)$$

$$P_r = \begin{cases} 0, & \text{if } a_1 < a_0, \\ \frac{a_1-a_0}{a_2}\min\left\{\frac{P}{1+\frac{a_1-a_0}{a_2}}, \frac{Q}{g_0}, \frac{Q}{g_1\frac{a_1-a_0}{a_2}}\right\}, & \text{if } a_1 > a_0. \end{cases} \quad (4)$$

Proof: Case 1: $a_1 < a_0$.
Then $\log_2(1+2a_1P_s) < \log_2(1+2a_0P_s+2a_2P_r)$ always hold. To save transmit power, we set $P_r = 0$. This is equivalent to a direct transmission. In direct transmission, the source transmits to the relay in successive two time slots. Then (2) can be written as:

$$\frac{1}{2}\max E[\log_2(1+2a_0P_s)] \quad (5)$$

subject to:

$$\begin{cases} P_s \le P, \\ g_0P_s \le Q. \end{cases}$$

We can combine the two constraints together. Then we get

$$P_s \le \min\{P, \frac{Q}{g_0}\}.$$

Case 2 :$a_1 > a_0$

Then the problem can be written as:

$$\frac{1}{2}\max E[\min\{\log_2(1+2a_1P_s), \log_2(1+2a_0P_s+2a_2P_r)\}]$$

subject to:

$$\begin{cases} P_s + P_r \le P, \\ g_0P_s \le Q, \\ g_1P_r \le Q. \end{cases} \tag{6}$$

In order to maximize (2), we have $\log_2(1+2a_1P_s) = \log_2(1+2a_0P_s+2a_2P_r)$. Then we have $a_1P_s = a_0P_s + a_2P_r$. So the constraints can be written as

$$\begin{cases} P_s + \frac{a_1-a_0}{a_2}P_s \le P, \\ g_0P_s \le Q, \\ g_1\frac{a_1-a_0}{a_2}P_s \le Q. \end{cases} \tag{7}$$

Therefore, we get $P_s \le \min\left\{\frac{P}{1+\frac{a_1-a_0}{a_2}}, \frac{Q}{g_0}, \frac{Q}{g_1\frac{a_1-a_0}{a_2}}\right\}$ and $P_r = \frac{a_1-a_0}{a_2}P_s$.

From the result, we can see that the power control scheme is related not only to its own channel coefficients a_0, a_1 and a_2, but also related to the channel between the primary user and the secondary relay networks g_0, g_1. Specifically, the relation between a_0 and a_1 is very important. If $a_1 < a_0$, it means that the channel between the secondary source and the secondary relay is worse than the channel between the secondary source and secondary destination. So it is more suitable to use the direct transmission mode. In this case, the relay won't cause any interference to the primary user and we only need to consider the interference caused by the secondary source. So if g_0 is larger than a threshold, that is, the channel between the secondary source and the primary receiver is bad, we can use all the power to transmit. As g_0 is lower than the threshold, the cognitive source should adapt to this change. If the channel a_1 is better than a_0, the secondary relay will be beneficial to the source. Also, we should consider the interference caused to the primary receiver. To maximize the ergodic rate, we should set P_s proportional to P_r. Considering the peak transmit power constraint and the interference power constraint, the power control scheme should be like (3) and (4).

B. Peak transmit power constraint and average interference power constraint

In this subsection, we consider the peak transmit power constraint and average interference power constraint. The interference to the primary user caused by the cognitive source and relay can't exceed $\bar{Q}$ averaged over all the fading states.

Theorem 2: The instantaneous CSI based optimal power allocation scheme is:

When $a_1 < a_0$,

$$P_s = \begin{cases} P, & \text{if } g_0 \le \frac{2a_0}{\mu(1+2a_0P)}, \\ \frac{1}{\mu g_0} - \frac{1}{2a_0}, & \text{if } \frac{2a_0}{\mu(1+2a_0P)} \le g_0 \le \frac{2a_0}{\mu}, \\ 0, & \text{otherwise}. \end{cases}$$
$$P_r = 0.$$

When $a_1 > a_0$,

$$P_s = \begin{cases} \frac{P}{1+\frac{a_1-a_0}{a_2}}, & \text{if } g_0 + g_1\frac{a_1-a_0}{a_2} \le \frac{2a_1}{\mu\left(1+\frac{2a_1P}{1+\frac{a_1-a_0}{a_2}}\right)}, \\ \frac{1}{\mu(g_0+g_1\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1}, & \\ \quad \text{if } \frac{2a_1}{\mu\left(1+\frac{2a_1P}{1+\frac{a_1-a_0}{a_2}}\right)} < g_0 + g_1\frac{a_1-a_0}{a_2} < \frac{2a_1}{\mu}, & \\ 0, \quad \text{otherwise}. \end{cases}$$
$$P_r = \frac{a_1-a_0}{a_2}P_s,$$

where μ is determined by $E[g_0P_s + g_1P_r] = \bar{Q}$.

Proof: Omitted due to limited space.

From the derivation, we can see that when $a_1 < a_0$, we still use the direct transmission mode. In this mode, when g_0 is smaller than a threshold, the secondary source can use power up to P. When g_0 is very large, the secondary source stops transmitting. In this way, it can cut down its average interference to the primary receiver and saves its power. When g_0 is between the two thresholds, the source should adapt its power to the channel. It's noted that this adaption not only relates to g_0, but also relates to a_1. When $a_1 > a_0$, the situation is more complex. Instead of considering g_0, we should consider about $g_0 + g_1\frac{a_1-a_0}{a_2}$. This reveals that the cognitive source and relay both have interference to the primary receiver. However, the relay's transmitting power should be proportional to the source's power. So the interference caused by the relay also has the coefficient $\frac{a_1-a_0}{a_2}$.

C. Average transmit power constraint and peak interference power constraint

In this subsection, we consider the average transmit power constraint and peak interference power constraint. The transmitting power of the cognitive source and relay can't exceed $\bar{P}$ averaged over all the fading states.

Theorem 3: The instantaneous CSI based optimal power allocation scheme is:

When $a_1 < a_0$,

$$P_s = \begin{cases} \frac{Q}{g_0}, & \text{if } \frac{2a_0}{1+2a_0P_s} - \mu > 0, \\ \frac{1}{\mu} - \frac{1}{2a_0}, & \text{if } 0 < \frac{1}{\mu} - \frac{1}{2a_0} < \frac{Q}{g_0}, \\ 0, & \text{otherwise}. \end{cases}$$

When $a_1 > a_0$ and $\frac{Q}{g_0} < \frac{Q}{\frac{a_1-a_0}{a_2}g_1}$,

$$P_s = \begin{cases} \frac{Q}{g_0}, & \text{if } \frac{2a_1}{1+2a_1\frac{Q}{g_0}} - \mu(1+\frac{a_1-a_0}{a_2}) > 0, \\ \frac{1}{\mu(1+\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1}, & \text{if } 0 < \frac{1}{\mu(1+\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1} \le \frac{Q}{g_0}, \\ 0, & \text{otherwise}. \end{cases}$$

When $a_1 > a_0$ and $\frac{Q}{g_0} > \frac{Q}{\frac{a_1-a_0}{a_2}g_1}$,

$$P_s = \begin{cases} \frac{1}{\mu(1+\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1}, & \text{if } 0 < \frac{1}{\mu(1+\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1} \le \frac{Q}{\frac{a_1-a_0}{a_2}g_1}, \\ \frac{Q}{\frac{a_1-a_0}{a_2}g_1}, & \text{if} \frac{2a_1}{1+2a_1\frac{Q}{\frac{a_1-a_0}{a_2}g_1}} - \mu(1+\frac{a_1-a_0}{a_2}) > 0, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$P_r = \begin{cases} 0, & \text{if } a_1 < a_0, \\ \frac{a_1-a_0}{a_2}P_s, & \text{if } a_1 > a_0. \end{cases}$$

where μ is determined by $E[P_s + P_r] = \bar{P}$.

Proof: Case 1: $a_1 < a_0$.

The problem can be written as:

$$\frac{1}{2}\max E[\log_2(1 + 2a_0 P_s)]$$

subject to:

$$\begin{cases} g_0 P_s \le Q, \\ E[P_s] \le \bar{P}. \end{cases}$$

The lagrangian corresponding to the optimization problem is

$$L(P, \lambda, \mu) = E[\log_2(1 + 2a_0 P_s)] - \lambda(g_0 P_s) - Q) - \mu(E[P_s)] - \bar{P}), \quad (8)$$

where $\lambda, \mu \ge 0$ are both lagrange multipliers. From KKT conditions [16], the optimal solutions can be found from the following equations:

$$P_s + P_r \le \bar{P}, \quad (9)$$

$$g_0 P_s \le Q, \quad (10)$$

$$g_1 P_r \le Q, \quad (11)$$

$$\lambda, \mu \ge 0, \quad (12)$$

$$\mu(E[P_s - \bar{P}]) = 0, \quad (13)$$

$$\lambda(g_0 P_s - Q) = 0, \quad (14)$$

$$\frac{\partial L(P, \lambda, \mu)}{\partial P_s} = \frac{2a_0}{1 + 2a_0 P_s} - \lambda g_0 - \mu = 0. \quad (15)$$

First, we will show that under the condition $\frac{2a_0}{1+2a_0\frac{Q}{g_0}} - \mu > 0$, the power allocation for source $P_s = \frac{Q}{g_0}$. We can prove it by contradiction. Suppose that $P_s \neq \frac{Q}{g_0}$. Combining with (10), we know that $P_s < \frac{Q}{g_0}$. From the complementary slackness condition (14), we know $\lambda = 0$. Substituting it into (15), we can see $\frac{2a_0}{1+2a_0 P_s} - \mu = 0$. Comparing it with the condition $\frac{2a_0}{1+2a_0\frac{Q}{g_0}} - \mu > 0$, we can obtain $P_s \ge \frac{Q}{g_0}$. This obviously contradicts to our assumption. So we can determine $P_s = \frac{Q}{g_0}$, when $\frac{2a_0}{1+2a_0\frac{Q}{g_0}} - \mu \ge 0$. Under other conditions, the proof is nearly the same to the case 1 in subsection B.

Case 2: $a_1 > a_0$.

From $(a_1 - a_0)P_s = a_2 P_r$, the problem can be written as:

$$\frac{1}{2}\max E[\log_2(1 + 2a_1 P_s)],$$

subject to:

$$\begin{cases} g_0 P_s \le Q, \\ g_1 \frac{a_1-a_0}{a_2} P_s \le Q, \\ E[P_s(1 + \frac{a_1-a_0}{a_2})] \le \bar{P}. \end{cases}$$

Scenario 1: $\frac{Q}{g_0} < \frac{Q}{\frac{a_1-a_0}{a_2}g_1}$. So $P_s \leqslant \frac{Q}{g_0}$. Then the lagrangian corresponding to this scenario is:

$$L(P, \lambda, \mu) = E[\log_2(1 + 2a_1 P_s)] - \lambda(g_0 P_s - Q) - \mu(E[P_s(1 + \frac{a_1 - a_0}{a_2})] - \bar{P}), \quad (16)$$

where $\lambda, \mu \geqslant 0$ are both lagrange multipliers. From KKT conditions [16], the optimal solutions can be found from the following equations:

$$E[P_s + P_r] \le \bar{P}, \ (17)$$

$$g_0 P_s \le Q, \ (18)$$

$$\lambda \ge 0, \mu \ge 0, \ (19)$$

$$\mu(E[P_s + P_r - \bar{P}]) = 0, \ (20)$$

$$\lambda(g_0 P_s - Q) = 0, \ (21)$$

$$\frac{\partial L(P, \lambda, \mu)}{\partial P_s} = \frac{2a_0}{1 + 2a_0 P_s} - \lambda g_0 - \mu(1 + \frac{a_1 - a_0}{a_2}) = 0. \ (22)$$

First, we will show that under condition $\frac{2a_1}{1+2a_1\frac{Q}{g_0}} - \mu(1 + \frac{a_1-a_0}{a_2}) > 0$, the power allocation are $P_s = \frac{Q}{g_0}, P_r = \frac{a_1-a_0}{a_2}P_s$. We can prove it by contradiction. Suppose that $P_s \neq \frac{Q}{g_0}$. Combining with (18), we know that $P_s < \frac{Q}{g_0}$. From the complementary slackness condition (21), we know $\lambda = 0$. Substituting it into (22), we can see $\frac{2a_1}{1+2a_1 P_s} - \mu(1+\frac{a_1-a_0}{a_2}) = 0$, or equivalently, $g_0 = \frac{2a_0}{\mu(1+2a_0 P)}$. Comparing it with the condition $\frac{2a_1}{1+2a_1\frac{Q}{g_0}} - \mu(1+\frac{a_1-a_0}{a_2}) > 0$, we can obtain $P_s \ge \frac{Q}{g_0}$. This obviously contradicts to our assumption. So we can determine $P_s = \frac{Q}{g_0}$, when $\frac{2a_1}{1+2a_1\frac{Q}{g_0}} - \mu(1 + \frac{a_1-a_0}{a_2}) > 0$. Under other conditions, the proof is nearly the same to case 1 in subsection B.

Scenario 2: $\frac{Q}{g_0} > \frac{Q}{\frac{a_1-a_0}{a_2}g_1}$. As in scenario 1, we can get

1) $P_s = \frac{1}{\mu(1+\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1}$ and $0 < P_s \le \frac{Q}{\frac{a_1-a_0}{a_2}g_1}$;
2) $P_s = \frac{Q}{\frac{a_1-a_0}{a_2}g_1}$ and $\frac{2a_1}{1+2a_1 P_s} - \mu(1 + \frac{a_1-a_0}{a_2}) > 0$;
3) $P_s = 0$ and $\frac{1}{\mu(1+\frac{a_1-a_0}{a_2})} - \frac{1}{2a_1} \leqslant 0$.

In this case, it is a little different from the section B. If $a_1 < a_0$, then only the cognitive source transmits. In this situation, P_s is upper-bounded by Q/g_0. As we have the average transmit power constraint, the power scheme has great relationship with the cognitive source to destination channel a_0. And the source transmit power should only adapt to a_0. When $a_1 < a_0$, the case becomes more complex. As the cognitive source and relay both have peak interference power constraint, we should first compare the two constraints. Then as in the case 1, we should make both nodes to transmit adapt to the channel coefficients.

D. Average transmit power constraint and average interference power constraint

In this subsection, we consider the average transmit power constraint and average interference power constraint. The transmitting power of the cognitive source and relay can't exceed $\bar{P}$ averaged over all the fading states. Also, the interference to the primary user caused by the cognitive source and relay can't exceed $\bar{Q}$ averaged over all the fading states.

$$\frac{1}{2}\max E[\min\{\log_2(1+2a_1P_s), \log_2(1+2a_0P_s+2a_2P_r)\}],$$

subject to:

$$\begin{cases} E[P_s+P_r] \le \bar{P}, \\ E[g_0P_s+g_1P_r] \le \bar{Q}. \end{cases}$$

Though the Lagrangian method is feasible, it is rather tedious to determine the parameters λ and μ. So it's more convenient to use the decoupling method [17]. We can decouple the original problem into two sub-problems:

Sub-problem 1: Maximize (2) subject to $E[P_s+P_r] \le \bar{P}$;

Sub-problem 2: Maximize (2) subject to $E[g_0P_s+g_1P_r] \le \bar{Q}$.

The details on the solution of the two sub-problems are omitted due to limited space. Based on the solution of the two sub-problems, we can solve the original optimization problem in the following way. First, we solve sub-problem 1 to get the optimal value P_s^1 and P_r^1, then we put the optimal value into sub-problem 2 to test whether the constraint holds. If yes, we can conclude that it is really the global optimal value; if not, then we solve sub-problem 2 to get the optimal value P_s^2 and P_r^2. Similarly, we can test whether it is feasible for sub-problem 1. If yes, it is global optimal; if not, then we should use the Lagrangian method to solve this problem. By this way, we can reduce the complexity of solving the original problem.

IV. Numerical Results

In our Monte-Carlo simulations, all the channels are Rayleigh fading. We set all the coefficients a_0, a_1, a_2, g_0, g_1 to be exponentially distributed. Also, the additive noise is modeled as independent zero-mean, circularly symmetric complex white Gaussian with unit variance.

Fig. 2 depicts the ergodic achievable rate of the cognitive relay networks versus P for the first case of Section III. The scenario without interference power constraints is also simulated for comparison. We can see from the figure that when P is small, the curves are almost the same with each other, regardless of the value of Q. This indicates that P is the bottleneck that restricts the performance of the secondary relay networks when P is small. When P increases, the capacities for different Q gradually become different. When Q is very large, the capacity approaches the case without interference power constraints.

Fig. 3 depicts the ergodic achievable rate of the cognitive relay networks versus $\bar{Q}$ for the second case of Section III. From the figure, we can see that P seriously constrain the rate of the system. When P is 0dB, and $\bar{Q}$ is almost more than 0dB, the ergodic rate becomes almost unchanged. This is

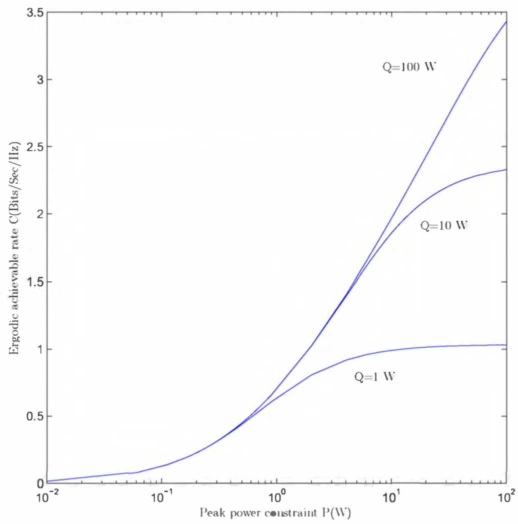

Fig. 2. ergodic achievable rate of cognitive relay networks under both peak transmit and interference power constraint vs. P peak.

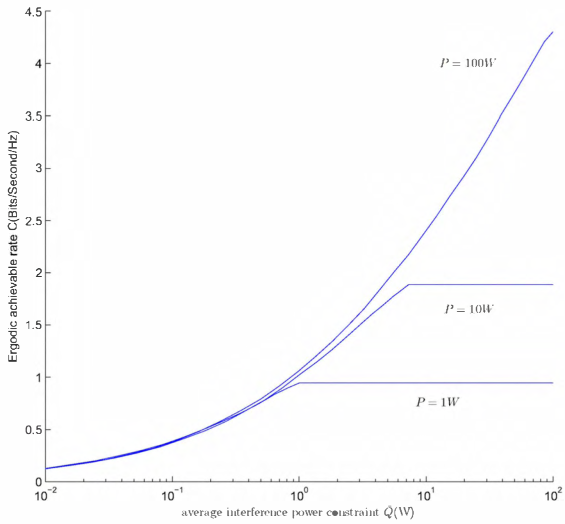

Fig. 3. ergodic achievable rate of cognitive relay networks under peak transmit and average interference power constraint vs. Q average.

also similar when P is 10dB. So we conclude that P is small compared to $\bar{Q}$. Then P becomes the key effect to determine the performance of the system. When P is larger than $\bar{Q}$, then $\bar{Q}$ can play a great role in determining the performance. As we can see that when P is 20dB, the ergodic achievable rate increases as the $\bar{Q}$ increases.

Fig. 4 depicts the ergodic achievable rate versus $\bar{P}$ for the third case of Section III. A little different from Fig. 2, when $\bar{P}$ is smaller than Q, the capacity grows quite fast as $\bar{P}$ grows. But when $\bar{Q}$ exceeds Q, the rate will grow more slowly. When $\bar{P}$ is more than 1.5 times $\bar{Q}$, the curves become different straight lines for different Q. This is because that the optimal

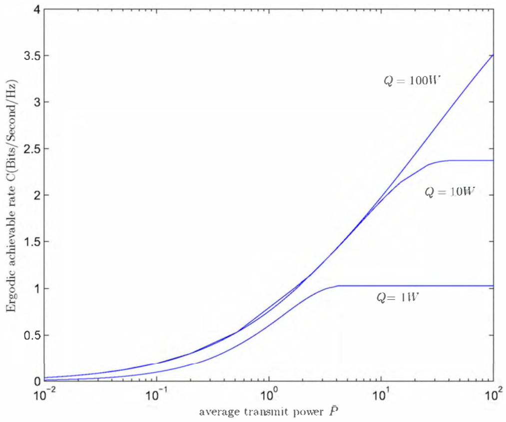

Fig. 4. ergodic achievable rate of cognitive relay networks under average transmit and peak interference power constraint vs. P average.

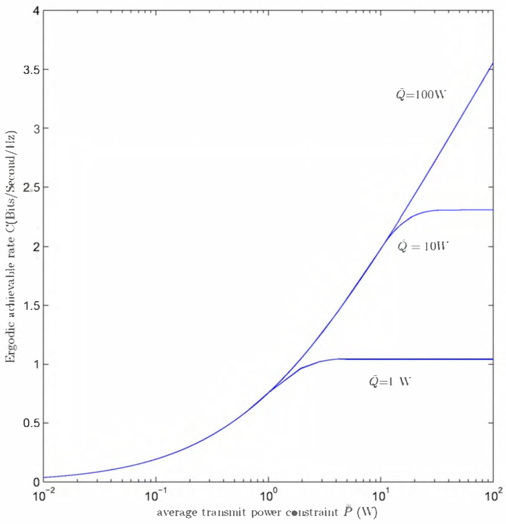

Fig. 5. ergodic achievable rate of cognitive relay networks under both average transmit and interference power constraint vs. P average.

P_s will always either be $\frac{Q}{g_0}$ or $\frac{Q}{\frac{a_1-a_0}{a_2}g_1}$, and the ergodic achievable rate under this condition must be unchanged.

Fig. 5 depicts the ergodic achievable rate versus $\bar{P}$ for the last case of Section III. When $\bar{P}$ is small, the three curves overlap each other. We can see that when $\bar{P}$ is smaller than $\bar{Q}$, the rate is almost the same. This indicates that the solution for sub-problem 1 is always global optimal. When $\bar{P}$ is only a little bigger than $\bar{Q}$, the global optimal solution is determined by both P and Q. After $\bar{P}$ is larger than a threshold, the rate remains to be constant. This indicates that $\bar{Q}$ constrains the performance and the solution to sub-problem 2 is global optimal. We can also see that for $\bar{Q} = 20$dB, the rate increases only with $\bar{P}$, because that the solution to sub-problem 1 is always feasible for sub-problem 2.

V. Conclusion

In this paper, we present the optimal power allocation schemes to achieve the ergodic achievable rate of the secondary DF relay network in spectrum sharing model. Both transmit and interference power constraints are studied. As both power constraints have peak power constraints and average power constraints, four cases are studied. We use the lagrange method and decoupling method to derive the power allocation scheme for the secondary source and relay. We analyze the impacts of the channel gains to the power allocation schemes. We do simulations under different power constraints for the primary user and show their own features of the ergodic achievable rate. The simulation results verified our theoretical analysis.

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