Low Complexity Network Error Correction Based on Nonbinary LDPC Codes over Matrix Channels

Yang Yu, and Wen Chen *Senior Member, IEEE,*
Network Coding and Transmission Laboratory
Shanghai Jiao Tong University, Shanghai, 200240, China
Email: {yuyang83, wenchen}@sjtu.edu.cn

***Abstract*—This paper presents a low-complexity network error correction (NEC) code for wireless relay networks (WRN), where the non-binary low density parity check (LDPC) code is jointly designed with a random network code. We give different transmission schemes under which the decoding complexity of the network code can be much reduced by dividing the transfer matrix into smaller decodable sub-matrices. We also propose a complexity optimization algorithm to the non-binary LDPC code based on an upper bound of message error probability. Simulations show that the complexity optimized codes can outperform the threshold optimized codes in higher SNR regime.**

***Index Terms*—Network error correction, non-binary LDPC, ADT networks, performance-complexity tradeoff.**

I. Introduction

Network coding is a powerful tool to increase network throughput and to achieve the capacity of networks. For an error free network, it has been shown that a random network code can be constructed if the coefficient field size $q > |\mathcal{T}|$, where $|\mathcal{T}|$ is the number of destinations [1]. However, for a network with errors, the resulting network solution may not be static to certain links error pattern $\mathcal{F}$ [2]. Cai *et al.* first show how to correct links error by establishing the relationship between network error correction (NEC) and the algebraic coding theory [3]. Later, Zhang studies the basic property of maximum distance separable NEC, and gives minimum rank decoding principle for the general decoding problem [4]. The network coding in wireline networks has been extensively studied. However, wireless network coding faces many difficulties, such as broadcast, interference, noise and fading. It is hard to give capacity-achieving network solutions. A physical layer network coding (PLNC) scheme is proposed for wireless relay networks (WRN) in [5], [6] to utilize the broadcasting and interfering nature of WRN, which shows that, by judiciously choosing linear error-correcting codes, intermediate nodes can directly recover the combinations of packets. Further, in order to simplify the analysis of WRN, Koetter *et al.* put forward a network equivalence theory to model the behavior of WRN components in terms of noiseless wireline models in combinatorial essence [7]. Meanwhile, by hard-coding noise, Avestimehr *et al.* give a deterministic approximation of Gaussian WRN, which is also known as ADT networks [8]. In ADT networks, noise is eliminated. The resulting Gaussian channels only passes the signals that are above the noise level, which can be seen as a sequence of symbols at different *signal levels*. The same signal level at relays from different transmissions superimpose together. The ADT capacity, C_A, is the cut-set bound of multicast networks.

A more tractable procedure, which is shown of great potential in analyzing and designing capacity achieving strategies for WRN, is also given in [8]. By this procedure, the multicast WRN can be approximated by eliminating Gaussian noise from every channels, then the exact analysis of information flows can be made by tracking different signal levels at the intermediate nodes. In the end, the exact analytic result for ADT network is converted to WRN with little perturbation. This procedure much simplifies the analysis of WRN and makes designing capacity approaching strategies for large WRN possible. In this paper we use this procedure to design our low complexity NEC codes with excellent performance. In ADT networks, it is natural to apply quantize-map-and-forward (QMF) strategy [8] to signals transmissions. However, under QMF, carry-overs are ignored when signals at the same level superimpose with each other, which is hard to be operated at relays. In order to use the exact analytic results, we drop the quantization step and apply PLNC at intermediate nodes. As a result, a map-and-forward (MF) strategy is used in WRN instead of QMF. In addition, signals are transmitted in different time slots over half-duplex channels under MF strategy as the signal levels in ADT networks.

A matrix channel is proposed in [9] to formulate the packet network under linear network coding (LNC), and an *error trapping* technique is also present to record errors which can be separated by the Gaussian-elimination. However, decoding NEC based on error trapping, will lead to high trapping failure rate when the size q of Galois field $GF(q)$ and the rank r of transmitted matrix is not sufficiently large. In fact, q and r are both bounded by C_A, which makes the error trapping codes not practical. To guarantee reliable transmissions, we turn to LDPC codes which are excellent error-correcting codes that perform close to shannon limits with lower decoding complexity than turbo codes. Moreover, investigation over $GF(q)$, $q = 2^p$, shows that q-ary LDPC codes have potentially better performance than binary LDPC codes [10]. In order to design NEC code with excellent performance, we adopt the non-binary LDPC codes. On the other hand, a major concern of q-ary LDPC is the decoding complexity. A straight-forward implementation of sum-product algorithm to q-ary LDPC has computational complexity dominated by $O(q^2)$. The FFT (fast Fourier transform) q-ary sum-product algorithm (FFT-QSPA)

978-1-4673-5939-9/13/$31.00 ©2013 IEEE

can reduce this complexity to $O(q \log q)$. And the extension min-sum (EMS) algorithm in [11] further reduces the complexity to $O(n_m \log n_m)$, where n_m is much smaller than q. A log-domain decoder, which is mathematically equivalent to the conventional sum-product decoder, can be found in [12]. Section III presents a complexity optimization algorithm to guarantee faster convergence rate of the non-binary LDPC code, and give different transmission schemes to reduce the decoding complexity of the network code.

II. System Model

We start our discussion in the standard framework: acyclic graph $\mathcal{N} = (\mathcal{V}, \mathcal{E})$ with node set $\mathcal{V}$ and link set $\mathcal{E}$. Let $\mathcal{S}, \mathcal{T} \subseteq \mathcal{V}$ be the source and destinations sets, respectively, C_A be the ADT capacity for $\mathcal{N}$. A matrix channel can be expressed as $\mathbf{Y} = \mathbf{AX} + \mathbf{Z}$, where $\mathbf{Y} = (y_{ij})_{b\times\Omega}$ is the received signal, $\mathbf{A} = (a_{ij})_{b\times b}$ is the transfer matrix consisting of global encoding vectors, $\mathbf{X} = (x_{ij})_{b\times\Omega}$ is the transmitted signal and $\mathbf{Z} = (z_{ij})_{b\times\Omega}$ is the links error. In addition, A is a non-singular matrix. More details can be found in [9]. Nodes in $\mathcal{N} = (\mathcal{V}, \mathcal{E})$ are working under half duplex mode.

The proposed NEC is encoded and signaled in serial manner. That is, the information bits are first transformed into their non-binary form and then be encoded by a q-ary LDPC encoder. Then each q-ary element $\alpha \in GF(q)$ is represented by a matrix $(\mathbf{b}_i)_{1\times\Omega}$ where $\mathbf{b}_i$ is a column vector of size b with each coordinate taken from sub-field $GF(\log M)$. Then α can be signaled by a constellation with size M. In each time slot, we transmit b symbols. A q-ary symbol will be transmitted in Ω time slots, where $\Omega = p/(b \cdot \log M)$.

For a non-binary LDPC codeword $\mathbf{x} = (x_1, x_2, ..., x_n)$ over $GF(2^p)$, with each x_i of pR information bits which passes through a interleaver π, is mapped into $p/\log M$ symbols. Then, we obtain the transmitted matrix as $\mathbf{X} = [\varphi(x_1), \varphi(x_2), ..., \varphi(x_n)]$. And PLNC is applied at relays.

$$\varphi : \alpha_i \to \begin{pmatrix} \varphi(\alpha_i,1,1) & \varphi(\alpha_i,1,2) & \cdots & \varphi(\alpha_i,1,\Omega) \\ \varphi(\alpha_i,2,1) & \varphi(\alpha_i,2,2) & \cdots & \varphi(\alpha_i,2,\Omega) \\ \vdots & \ddots & \ddots & \vdots \\ \varphi(\alpha_i,b,1) & \varphi(\alpha_i,b,2) & \cdots & \varphi(\alpha_i,b,\Omega) \end{pmatrix}.$$

Decoding of NEC is just the reverse operation of the encoding procedure. That is, we first decode the random network code formulated by the matrix channel then decode the q-ary LDPC code. Note that, information transmitted over the network can not exceed the ADT capacity. So $pR < \Omega C_A$. Then

$$b \times \log|\mathcal{T}| < b \log M = p/\Omega < C_A/R, \tag{1}$$

Eq. (1) implies that $2^{b\Omega \log|\mathcal{T}|} < q < 2^{C_A\Omega/R}$, within which a network code can be constructed [1], [8]. If $b\Omega = 1$, the interleaver is not necessary, since interleaving the coded symbols amounts to shuffling the columns of the parity check matrix. Hence interleaving can be absorbed into code design.

III. Code design

In this section, we first give different transmission schemes, under which the decoding complexity of the network code can be much reduced, then propose a check-irregular extrinsic information transfer (EXIT) chart based on an upper bound of message error probability for irregular q-ary LDPC codes. Finally, we present a complexity optimization algorithm based on the EXIT chart to analyze and design low decoding complexity q-ary LDPC codes which can outperform the threshold optimized codes in lower error rate regime.

A. Network Code

Consider destination $t \in \mathcal{T}(s)$, source $s \in \mathcal{S}$ and $b < \frac{C_A}{R \log M}$. If the network solution is solvable, the network decoder estimates the transmitted matrix $\mathbf{X}$ by $\hat{\mathbf{X}} = \mathbf{A^{-1}Y}$. This decoder requires a decoding complexity of order $O(b^2)$ for every column of X in general. However, when it comes to WRN under MF strategy, we show that the complexity can be much reduced by properly choosing transmitting schedules at relays, which is illustrated by following examples.

Example 1: In Fig. 1, nonzero encoding coefficients α_i, β_i, γ_i are randomly chosen from $GF(q)$. Relays transmit symbols while maintaining the order as they are in the source-transmit vector. At destination, we divide $\mathbf{A}$ into 2 overlapped sub-matrices, $\mathbf{A_1}$, $\mathbf{A_2}$, which can be decoded in serial manner (first $\mathbf{A_1}$, then $\mathbf{A_2}$) by using the network decoder. The overall decoding complexity will be reduced from $O(6^2)$ to $O(2\times 4^2)$.

Actually, Example 1 is a worst case in the sense that all the symbols are transmitted using the same time slots at relays. Example 2 will show that the decoding complexity can be further reduced when there is only partial interference happening among close nodes.

Example 2: In Fig. 2, we transmit signals using different time slots at different relays according to the deterministic capacity (signal levels) of the Gaussian channels. As a result, only partial interference happens at the same signal levels. At destination, $\mathbf{A}$ is divided into even smaller separated decodable sub-matrices, $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{A_4}$. These matrices can be decoded in parallel. The overall decoding complexity is reduced to $O(2 \times 2^2 + 2)$ in general.

From the examples, we know that, to achieve a low decoding complexity performance, we need to randomize the local encoding vectors. In addition, the relays must, using different time slots when it is possible, send different symbols while maintaining the order as they are in the source-transmit vectors. As a result, interfering links will lead to separated small divisions of decodable sub-matrices of A. For *coherent network coding* [2], where the knowledge of the network topology is already given, these division operations can be done offline. The total complexity will be reduced to $O(\sum\limits_i (b/r_i)^2)$, where r_i can be much larger than 1. Note that each sub-matrix is also non-singular matrix. Then the division does not change the decoding capacity for the network code.

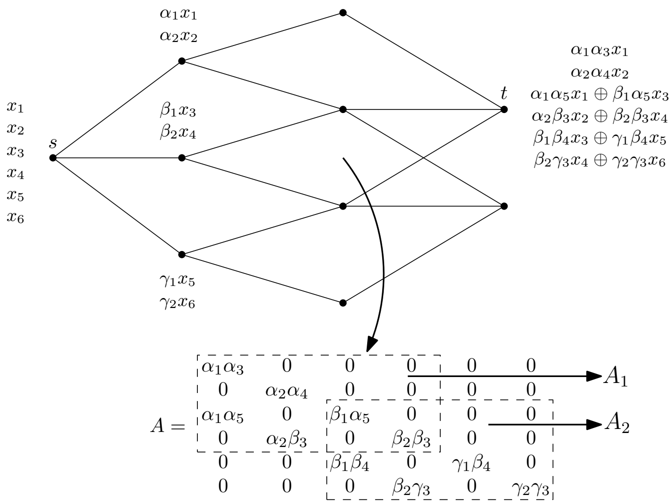

Fig. 1. A worst case when total interference happens.

B. Complexity-Optimized q-ary LDPC Codes

An LDPC code is called regular if the column and row weight of the parity check matrix is constant, respectively. Irregular LDPC codes can outperform regular LDPC codes. For large field order, average columns weight $\bar{d}_v$ of the best q-ary LDPC [10], [12] will tend to 2, which is also called q-ary cycle LDPC codes [13]. Irregular q-ary LDPC codes with small $\bar{d}_v$, *i.e.* $2 < \bar{d}_v < 3$, can outperform other LDPC codes [10]. In this paper, we propose a complexity optimization algorithm to reduce the decoding complexity of the non-binary LDPC code with small d_v by using an upper bound of message error probability.

Performance complexity tradeoff (PCT) analysis based on EXIT chart is first proposed in [14] for *check-regular binary LDPC* codes, which shows that complexity-optimized LDPC codes significantly outperform threshold-optimized LDPC codes at long block length. Here, we generalize this analysis to irregular q-ary LDPC codes, and design low-complexity non-binary LDPC code over matrix channel. We consider the irregular LDPC codes, from an edge-perspective, characterized by a variable degree distribution

$$\lambda(x) = \sum_{i \geqslant 2} \lambda_i x^{i-1}$$

and a check degree distribution

$$\rho(x) = \sum_{i \geqslant 2} \rho_i x^{i-1}$$

, where λ_i or ρ_i is the fraction of edges belonging to degree-i variable or check node. Using this characterization, code rate R can be given as

$$R = 1 - \frac{\int_0^1 \rho(x)dx}{\int_0^1 \lambda(x)dx}.$$

From Gallager's formula [15], we know that for a degree-k check node, the probability of either no errors or of errors adding to 0 (mod-q) in one of the $k-1$ parity check sets is

$$Q_{out,k} = \frac{1 + (q-1)(1 - \frac{qp_{in}}{q-1})^{k-1}}{q}, \tag{2}$$

where p_{in} is the input error probability of messages from a variable node to a check node. The reasons we adopt the Gallager's formula to extend the PCT analysis to non-binary LDPC codes are as follows. *(i)*. This formula has been shown of great potential in designing excellent irregular LDPC codes for soft decision decoders in [16], where they show that given the degree distributions, one can construct decoding graphs for any number of nodes with the correct edge fractions, under *belief propagation* algorithm, by using Gallager's formula. The designed results can be directly applied to soft decision decoders. *(ii)* For practical considerations, this formula simplifies the analysis of convergence behavior of q-ary LDPC codes and makes the design of complexity-optimized q-ary LDPC codes possible.

In [14], [17], a check-regular EXIT chart based on one-step density evolution for binary LDPC codes is given. Now, we give an irregular EXIT chart to analyze q-ary LDPC using Gallager's formula. For an irregular-check-degree depth-one tree, we define Q_{out} as.

$$Q_{out} = \sum_{k \geqslant 2} \mathbf{Pr}(\text{check degree} = k)\mathbf{Pr}(Q_{out,k} | \text{check degree} = k) = \sum_{k \geqslant 2} \rho_k Q_{out,k}.$$

For a variable with degree $d_v = i$, the output message error probability $p_{i,out} = f_i(p_{in})$, which is described below:

$$f_i(p_{in}) = p_0 - p_0 \sum_{l=l_0}^{i-1} \binom{i-1}{l} Q_{out}^l (1 - Q_{out})^{i-1-l} + (1-p_0)(q-1) \sum_{l=l_0}^{i-1} \binom{i-1}{l} \left(\frac{1 - Q_{out}}{a-1}\right)^l \left(1 - \frac{1 - Q_{out}}{a-1}\right)^{i-1-l}, \tag{3}$$

where p_0 is the initial error probability from the channel. Second additive term of E.q (3) is the probability of message received in error in the variable and then corrected, while the third additive term is the probability that l_0 check nodes agree on the same error message. l_0 is the smallest integer chosen to minimize p_{out}, subject to $l_0 > (i-1)/2$, for which

$$\frac{1 - p_0}{p_0} \leqslant \frac{Q_{out}^{l_0}(q-1)^{i-2}}{(1 - Q_{out})^{(2l_0+1-i)}(q - 2 - Q_{out})^{(i-1-l_0)}}. \tag{4}$$

Consider irregular variable degree distribution, we have

$$p_{out} = \sum_{i \geqslant 2} \lambda_i f_i(p_{\text{in}}). \tag{5}$$

Note that E.q (2)-(5) are strictly upper bounds for message error probability. However, this simplifies the analysis of q-ary LDPC codes and makes the construction of complexity-optimized q-ary LDPC possible. From [12], [14], [18], [19], we know that the overall decoding complexity is proportional to NE, where N is the number of decoding iteration and E is the number of edges in Tanner graph. Since each codeword encodes Rnp information bits, the decoding complexity per information bit is $O(\frac{NE}{Rnp})$. Then we obtain the decoding complexity is

$$K = \frac{NE}{Rnp} = \frac{N(1-R)}{Rp\sum\frac{\rho_i}{i}},$$

where

$$N = \int_{p_t}^{p_0}\left(p\ln\left(\frac{p}{\sum\lambda_i f_i(p)}\right)\right)^{-1}dp,$$

and p_t is the target error probability [14]. So, complexity optimization is equivalent to finding the unique local minimum of K in general, because the convexity can not be always guaranteed. Next, we show how to design complexity-optimized q-ary LDPC codes with good performance.

The fact that q-ary LDPC codes with small mean column weight $\bar{d}_v$ can outperform other LDPC codes, has been known for years [10]. And non-binary LDPC codes of consecutive variable degrees are sufficient to achieve high performance if the optimized coefficient are chosen which has been shown in [10], [13], [20]. Here, by adopting the non-binary LDPC code with consecutive variable degrees, we give a general method to improve the decoding performance while maintaining $2 < \bar{d}_v < 3$. We say $H_1 \cong H_2$ if H_2 is a *column-permuted* form of H_1. Let H be the parity check matrix of our q-ary LDPC codes, and $H \cong [H_C, H_W]$, where H_C is the cycle sub-parity-matrix, and H_W is the sub-parity-matrix with column weight $d_v = 3$. We consider consecutive check degrees as suggested in [14]. Let

$$\bar{d}_c = \bar{d}_v/(1-R)$$

be the mean row weight, $\tau_1 = \lfloor\bar{d}_c\rfloor$, $\tau_2 = \lceil\bar{d}_c\rceil$. Given rate R, we calculate the degree distributions as

$$\lambda_2 = \frac{2(3-\bar{d}_v)}{\bar{d}_v},$$

$$\rho_{\tau_1} = \frac{\tau_1(\bar{d}_c-\tau_2)}{\bar{d}_c(\tau_1-\tau_2)}.$$

Setting a target rate $R_0, R_0 \geqslant R$, the optimization algorithm in [14] can be modified as

$$\begin{aligned}
\text{minimize} \quad & \frac{1-R_0}{R_0\log(q)\sum\frac{\rho_i}{i}}\int_{p_t}^{p_0}\left(p\ln\left(\frac{p}{\sum\lambda_i f_i(p)}\right)\right)^{-1}dp.\\
\text{subject to} \quad & p < \sum\lambda_i f_i(p);\\
& \bar{d}_c \leqslant \frac{\bar{d}_v}{1-R_0};\\
& \lambda_2 = \frac{2(3-\bar{d}_v)}{\bar{d}_v}, \rho_{\tau_1} = \frac{\tau_1(\bar{d}_c-\tau_2)}{\bar{d}_c(\tau_1-\tau_2)};\\
& \sum_{i=2}^{3}\lambda_i = \sum_{i=\tau_1}^{\tau_2}\rho_i = 1;\\
& |\bar{d}_v - \bar{d}_1| < \varepsilon_1, |\bar{d}_c - \bar{d}_2| < \varepsilon_2.
\end{aligned} \tag{6}$$

where ε_1 and ε_2 are the maximum permissible changes, which are carefully set to small numbers to guarantee finding the unique local minimum.

Note that, this *irregular* algorithm is different to the *quasi-regular* optimization in [14] in the sense that we update $\bar{d}_1$ and $\bar{d}_2$ in each step and find corresponding degree distributions. $\bar{d}_i$ can be initialized with the value that a threshold-optimized codeword suggests [12], [14]. The constraint $p < \sum\lambda_i f_i(p)$ is substantial for which this optimization algorithm is valid. Table I gives the required smallest $\bar{d}_v$, $T_{\bar{d}_v}$ for different code rate R. More importantly, a mild condition, *i.e.* $\{\lambda_i | f(p_{in}) \geqslant e^2 p_{in}\}$, is given in [14], under which $f(p_{in})$ is a convex function of λ_i. The complexity-optimized q-ary LDPC codes, resulting from our irregular algorithm, has a little lower threshold than the original one, but converges faster at higher SNR regime. If the message error probability is sufficiently small, then $Q_{out,k} \approx 1-(k-1)p_{in}$, $Q_{out} \approx 1-(\tau_1+\rho_{\tau_2}-1)p_{in}$, and $Q_{out}^2 \approx 1-2(\tau_1+\rho_{\tau_2}-1)p_{in}$, where τ_1 and τ_2 is the check degrees. In addition, the element EXIT charts of the designed q-ary LDPC codes are

$$\begin{aligned}
f_2(p_{in}) &= 1-(2-p_0)Q_{out},\\
f_3(p_{in}) &= p_0 + \frac{1+p_0}{q-1}(1-2Q_{out}+Q_{out}^2) - Q_{out}^2.
\end{aligned}$$

Then, we have

$$f(p_{in}) \approx (p_0-1) + (2-\lambda_2 p_0)(\tau_1+\rho_{\tau_2}-1)p_{in}. \tag{7}$$

It is easy to verify that Eq. (7) does not always satisfy the convex condition. Numerical simulations nevertheless suggest that, there exists a unique local optimum.

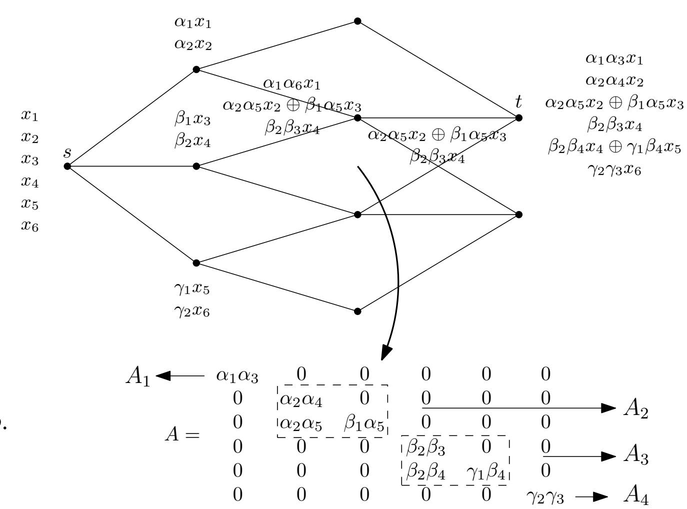

Fig. 2. Separated decodable sub-matrices when partial interference happens.

TABLE I
THE SMALLEST $\bar{d}_v$ REQUIRED FOR DIFFERENT RATES

R	1/6	1/5	1/4	1/3	1/2	2/3
$T_{\bar{d}_v}$	2.37	2.40	2.48	2.56	2.70	2.81

IV. SIMULATIONS

In this section, we use PEG algorithm to construct the non-binary Tanner graph.

A. Simulation 1 (PCT Algorithm)

Performance comparison between non-binary LDPC codes constructed by different methods can be found in [10], [13], [20]. Here, we only consider the performance optimized LDPC codes and show how to reduce the decoding complexity of a given code. Consider a threshold optimized q-ary LDPC code characterized by $\lambda(x) = 0.4793x^2 + 0.5207x^3$ and $\rho(x) = 0.0536x^2 + 0.9464x^3$. We expect that the optimized code will reach a decoding error rate of 10^{-4} faster at a smaller number of iterations, while maintaining the excellent performance as the original one. Let $C_r(q, N, K)$ and $C_t(q, N, K)$ be the original and optimized codes, respectively, where N is the number of iterations and K is the decoding complexity according to equation (6). Fig. 3 shows that the optimized code characterized by $\lambda(x) = 0.5x^2 + 0.5x^3$ and $\rho(x) = 0.0833x^2 + 0.9167x^3$, can outperform the original one with faster convergence rate at a small N. $C_t(4, 15, 155)$ even has a better performance than $C_r(4, 35, 386)$. When N increases, C_r is "chasing" C_t in the low error rate regime. When decoding complexity is high enough, C_t will lose its advantage compared to C_r. This justifies our PCT algorithm for q-ary LDPC codes. And the convergence process has been accelerated by 59.8% regarding the decoding complexity.

B. Simulation 2 (Codes Comparison)

We compare different codes, $C(R, \bar{d}_v, q)$ with 2400 bits and $N = 50$, in Fig. 4. It can be seen that $C(1/4, 2.3, 8)$ performs the best while $C(1/4, 2.3, 4)$ has a higher error floor. The performance of regular Mackay-(3,6) code are improved by irregularizing degree distributions and diminishing $\bar{d}_v$, as shown by the curves of $C(1/2, 2.6, 4)$ and $C(1/2, 2.7, 4)$. These results demonstrate the good performance of codes constructed by our methods.

C. Simulation 3 (Concatenated NEC System)

We simulate Example 2 by using M-PSK PLNC at the intermediate nodes, and use the optimized code in simulation 1. The results for the low complexity NEC code is illustrated in Fig. 5. We show frame (block) error rate (FER) over symbol-to-noise rate. It can be seen that, this low complexity NEC code performs well in our simulations. Smaller $b\Omega$ results in a higher order constellation, which have flatter curves under lower SNR, and wider performance differences for different $\bar{d}_v$.

V. DISCUSSION

The global encoding matrix $\mathbf{A}$ considered in this paper is a non-singular matrix. However if the $\mathbf{A}$ is a singular matrix, the network code can not be fully decoded. That is, only partial transmitted symbols are decoded in the network decoder. If we replace the un-decoded symbols with 0 and the rank of $\mathbf{A}$ is r, we can find non-binary LDPC codes of $R < \min\{r/b, \Omega C_A/p\}$ with sufficient block length and number of decoding iterations to correct the errors. In addition, as to multi-source relay network, if we place a virtual source before the sources to represent the centralized control within the sources, then the multi-source relay network is transformed into a single source relay network. As a result, the proposed algorithms can be easily extended into their multi-source forms.

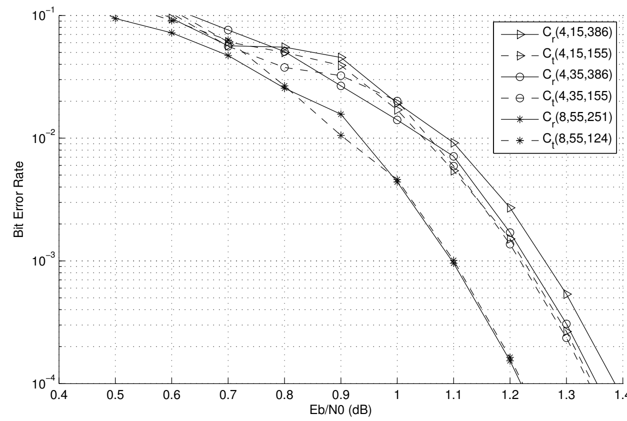

Fig. 3. Performance comparison while complexity increasing.

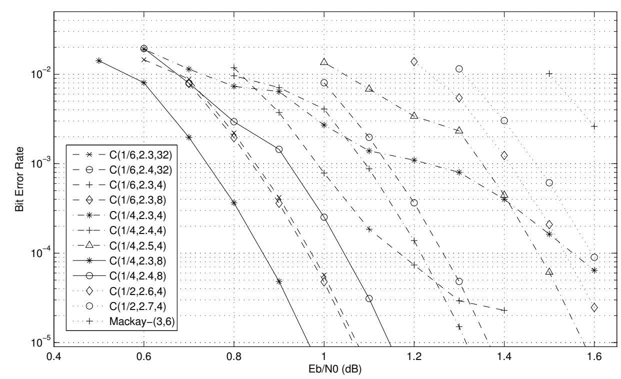

Fig. 4. Comparison of different q-ary LDPC codes with 2400 bits.

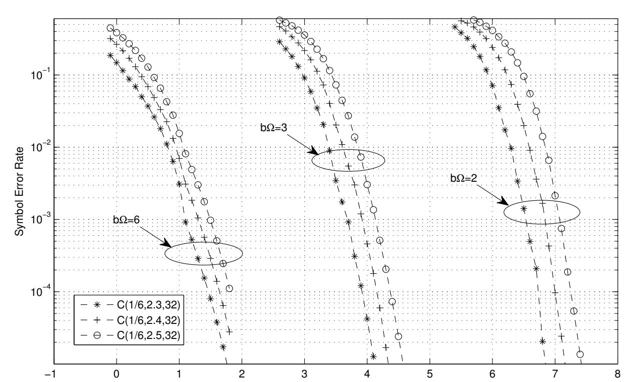

Fig. 5. Performance comparison of the proposed NEC system under different transmission schemes.

VI. Conclusion

In this paper, the nonbinary LDPC codes are adopted to be transmitted over the matrix channel which results in a jointly designed NEC code for the WRN. For the network code, by dividing the transfer matrix into smaller decodable sub-matrices, we give different transmission schedules under which the decoding complexity is reduced from $O(b^2)$ to $O(\sum_i (b/r_i)^2)$ where r_i can be much larger than 1. For the non-binary LDPC codes, we give an irregular EXIT chart based on an upper bound of message error probability and a general method to construct non-binary LDPC codes that attain exceptional performance for matrix channels. Further, a complexity optimization algorithm based on the EXIT chart is present to reduce the decoding complexity of nonbinary LDPC codes. Simulations show that the complexity optimized codes can outperform the threshold optimized codes in higher SNR regime.

Acknowledgment

This work is supported by the National 973 Project #2012CB316106 and #2009CB824904, by NSF China #60972031 and #61161130529.

References

[1] T. Ho, M. Medard, J. Shi, M. Effros, and D. R. Karger, "On randomized network coding," in *In Proceedings of 41st Annual Allerton Conference on Communication, Control, and Computing*, 2003.

[2] R. Koetter and M. Medard, "An algebraic approach to network coding," *IEEE/ACM Transactions on Networking*, vol. 11, pp. 782–795, 2003.

[3] N. Cai and R. Yeung, "Network coding and error correction," in *Information Theory Workshop, 2002. Proceedings of the 2002 IEEE*, oct. 2002, pp. 119 – 122.

[4] Z. Zhang, "Linear network error correction codes in packet networks," *Information Theory, IEEE Transactions on*, vol. 54, no. 1, pp. 209 –218, jan. 2008.

[5] S. Zhang, S. chang Liew, and P. P. Lam, "Physical-layer network coding," in *in ACM Mobicom 06*, 2006.

[6] B. Nazer and M. Gastpar, "Reliable physical layer network coding," *Proceedings of the IEEE*, vol. 99, no. 3, pp. 438 –460, march 2011.

[7] R. Koetter, M. Effros, and M. Medard, "A theory of network equivalence part i: Point-to-point channels," *Information Theory, IEEE Transactions on*, vol. 57, no. 2, pp. 972 –995, feb. 2011.

[8] A. Avestimehr, S. Diggavi, and D. Tse, "Wireless network information flow: A deterministic approach," *Information Theory, IEEE Transactions on*, vol. 57, no. 4, pp. 1872 –1905, april 2011.

[9] D. Silva, F. Kschischang, and R. Koetter, "Communication over finite-field matrix channels," *Information Theory, IEEE Transactions on*, vol. 56, no. 3, pp. 1296 –1305, march 2010.

[10] M. C. Davey and D. J. MacKay, *Error-correction using LDPC Codes*. Cambridge Press, 1998.

[11] D. Declercq and M. Fossorier, "Decoding algorithms for nonbinary ldpc codes over gf(q)," *Communications, IEEE Transactions on*, vol. 55, no. 4, pp. 633 –643, april 2007.

[12] G. Li, I. Fair, and W. Krzymien, "Density evolution for nonbinary ldpc codes under gaussian approximation," *Information Theory, IEEE Transactions on*, vol. 55, no. 3, pp. 997 –1015, march 2009.

[13] J. Huang, S. Zhou, and P. Willett, "Nonbinary ldpc coding for multi-carrier underwater acoustic communication," *Selected Areas in Communications, IEEE Journal on*, vol. 26, no. 9, pp. 1684 –1696, december 2008.

[14] B. Smith, M. Ardakani, W. Yu, and F. Kschischang, "Design of irregular ldpc codes with optimized performance-complexity tradeoff," *Communications, IEEE Transactions on*, vol. 58, no. 2, pp. 489 –499, february 2010.

[15] R. G. Gallager, *Low-Density Parity-Check Codes*. MIT Press, 1963.

[16] M. Luby, M. Mitzenmacher, M. Shokrollahi, and D. Spielman, "Improved low-density parity-check codes using irregular graphs," *Information Theory, IEEE Transactions on*, vol. 47, no. 2, pp. 585 –598, feb 2001.

[17] M. Ardakani and F. Kschischang, "A more accurate one-dimensional analysis and design of irregular ldpc codes," *Communications, IEEE Transactions on*, vol. 52, no. 12, pp. 2106 – 2114, dec. 2004.

[18] Y. Yu and W. Chen, "Design of low complexity non-binary ldpc codes with an approximated performance-complexity tradeoff," *IEEE Communications Letters*, vol. 16, no. 4, pp. 514 –517, april 2012.

[19] Y. Yu, W. Chen, and L. Wei, "Design of convergence-optimized non-binary ldpc codes over binary erasure channel," *IEEE Wireless Communications Letters*, vol. 1, no. 4, pp. 336 –339, august 2012.

[20] C. Poulliat, M. Fossorier, and D. Declercq, "Design of regular (2,dc)-ldpc codes over gf(q) using their binary images," *Communications, IEEE Transactions on*, vol. 56, no. 10, pp. 1626 –1635, october 2008.