Robust Linear Beamformer Designs for MIMO Relaying Broadcast Channel with Max-Min Fairness

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***Abstract*—In this paper, we address the robust source and relay matrices design for the multiple-input multiple-output (MIMO) relaying broadcast channels (BC) with imperfect channel state information at the transmitter (CSIT). Our objective is to maximize the minimum achievable rate among all users, which dominates the quality of service (QoS) performance of the system. In the proposed scheme, we first set up an equivalent problem, and then relax the constraints of the new problem to decouple it into three tractable subproblems. Finally, an iterative algorithm is proposed to jointly optimize the source and relay matrices. The advantage of the proposed scheme is demonstrated by numerical experiments.**

***Index Terms*—Source and relay matrices design; MIMO; relaying broadcast channels; QoS; imperfect CSIT.**

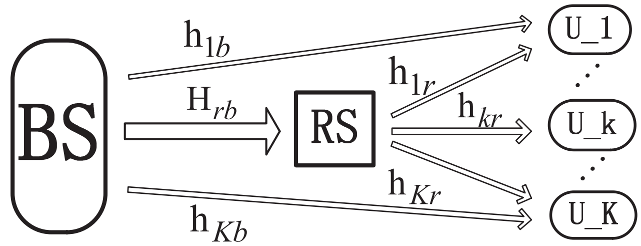

Fig. 1. The MIMO relaying broadcast channel with one base station (source), one fixed relay station, and K mobile users.

I. Introduction

Recently, MIMO relaying broadcast channel (BC) has attracted much research interest. For a MIMO relaying BC, there are two independent channel links between source and receivers; i.e., *source-relay-receivers* links, and *source-receivers* direct links (DLs). Many works have investigated the linear strategy for MIMO relaying BC with perfect CSIT. In [1], an strategy with a Tomlinson-Harashima precoder at source and a linear beamformer at relay is presented. In [2], a joint source and relay design to minimize the weighted sum-power consumption under the QoS-constraints is presented. In [3], a singular value decomposition (SVD) combining zero forcing (ZF) scheme is presented. In [4], the authors propose to use the quadratic programming to joint source and relay precoding design to maximize the system capacity. In [5], the authors propose a scheme based on duality of MIMO MAC and BC to maximize the system capacity. All these works are assumed that the source have perfect CSIT and did not consider the DLs.

Recently, Phuyal *et al.* in [6] has considered a ZF scheme with DLs contribution to deal with the power control problem under the perfect CSIT assumption. In our previous works [7] [8], we also consider the DLs in design but only consider the scenario with perfect CSIT to maximize the sum-rate. In practical scenarios, the DLs' contribution in spatial diversity to MIMO relaying BC should not be ignored, and, furthermore, perfect CSIT may not be available at source due to many practical factors such as quantization error, limited feedback and so on. In [9] [10], the authors have considered the beamforming design with imperfect CSIT to minimize the sum MSE (or maximize the sum rate) of the system, but, both works ignored the DLs contribution. In practical systems, the minimum of the achievable rate (or maximum of the MSE) among all users is a key factor for determining QoS of the system.

In this paper, we study the robust linear source precoding matrix (PM) and relay beamforming matrix (BM) design to maximize the minimum of the achievable rate among all users with imperfect CSIT. Based on alternating optimization method, a robust joint source and relay linear precoding scheme is proposed.

Notations: $\mathbb{E}(\cdot)$, $\mathrm{Tr}(\cdot)$, $(\cdot)^{-1}$, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^\dagger$, and $\det(\cdot)$ denote expectation, trace, inverse, transpose, conjugate, conjugate transpose, and determinant, respectively. i.i.d. stands for independent and identically distributed. $\mathbf{I}$ is the identity matrix with appropriate dimensions. $\mathrm{diag}()$ is a diagonal matrix. $\log$ is of base 2. $\mathcal{C}^{M\times N}$ represents the set of $M \times N$ matrices over complex field, and $\sim \mathcal{CN}(x, y)$ means satisfying a circularly symmetric complex Gaussian distribution with mean x and covariance y.

II. System Model

We consider a MIMO relaying broadcast channel with a base station (BS), a fixed relay station (RS) and K single-antenna mobile users as Fig. 1. It is assumed that both BS and RS are equipped with M $(K \leq M)$ antennas to serve K

single-antenna users simultaneously. We consider a two-phase scheme with a non-regenerative and half-duplex RS.

Let $\mathbf{P} \triangleq [\mathbf{p}_1, \cdots, \mathbf{p}_K]$ denote the source PM where $\mathbf{p}_k \in \mathcal{C}^{M\times 1}$ is a precoding vector acting on the transforming symbol $s_k \sim \mathcal{CN}(0,1)$ for user k, and the symbols for different users be independent from each other. During the first phase, BS broadcasts the precoded data streams to RS and users by applying a linear PM $\mathbf{P}$. During the second phase, RS forwards the received signal vector to users after a linear BM $\mathbf{F}$. Then, the received signal vector at user k can be expressed in matrix-form as

$$\underbrace{\begin{bmatrix} y_{1k} \\ y_{2k} \end{bmatrix}}_{\mathbf{y}[k]} = \underbrace{\begin{bmatrix} \mathbf{h}_{kb} \\ \mathbf{h}_{kr}\mathbf{F}\mathbf{H}_{rb} \end{bmatrix}}_{\mathbf{H}_k} \mathbf{p}_k s_k + \underbrace{\begin{bmatrix} n_{1k} \\ \mathbf{h}_{kr}\mathbf{F}\mathbf{n}_r + n_{2k} \end{bmatrix}}_{\mathbf{G}_k} + \sum_{i=1, i\neq k}^{K} \begin{bmatrix} \mathbf{h}_{kb} \\ \mathbf{h}_{kr}\mathbf{F}\mathbf{H}_{rb} \end{bmatrix} \mathbf{p}_i s_i, \quad (1)$$

where y_{ik} is the received signal during the ith phase at user k, vector $\mathbf{h}_{ij}$ (or matrix $\mathbf{H}_{ij}$) represents the channel coefficient from the transmitter j to receiver i, and $n_i \sim \mathcal{CN}(0,1)$ ($i = 1k,\ 2k$ and r) are the Gaussian noise signals at user k during the first and second phase, and at relay, respectively. The power constraints at BS and RS can be expressed, respectively, as

$$\sum_{k=1}^{K} \mathrm{Tr}(\mathbf{p}_k\mathbf{p}_k^\dagger) = \mathrm{Tr}(\mathbf{P}\mathbf{P}^\dagger) \le P_b \quad (\text{ BS }), \quad (2a)$$

$$\mathrm{Tr}(\mathbf{F}\mathbf{H}_{rb}\mathbf{P}\mathbf{P}^\dagger\mathbf{H}_{rb}^\dagger\mathbf{F}^\dagger + \mathbf{F}\mathbf{F}^\dagger) \le P_r \quad (\text{ RS }). \quad (2b)$$

For the CSIT, each user is assumed to has perfect CSI, but, the BS has only imperfect CSI due to limited feedback from users. We consider [11]

$$\mathbf{h}_{kb} = \widehat{\mathbf{h}}_{kb} + \sigma_{e,kb}\widetilde{\mathbf{h}}_{kb}, \quad (3a)$$

$$\mathbf{h}_{kr} = \widehat{\mathbf{h}}_{kr} + \sigma_{e,kr}\widetilde{\mathbf{h}}_{kr}, \quad (3b)$$

$$\mathbf{H}_{rb} = \widehat{\mathbf{H}}_{rb} + \sigma_{e,rb}\widetilde{\mathbf{H}}_{rb}, \quad (3c)$$

where $\mathbf{X}_x$, $\widehat{\mathbf{X}}_x$, and $\widetilde{\mathbf{X}}_x$ represent the true channel vector, the estimated channel vector and the estimation error vector, respectively. Moreover, both $\mathbf{X}_x$ and $\widetilde{\mathbf{X}}_x$ are assumed to have i.i.d. complex Gaussian elements and each element $\sim \mathcal{CN}(0, \sigma_x^2)$. $\sigma_{e,x}$ denotes the CSI error factor which is known to the BS. Then, the BS PM and RS BM need to be designed based on the imperfect CSI knowledge. Hence, assuming Gaussian signaling for source, the achievable rate for the kth user during two phases is given as

$$R_k = \log\det\left(\mathbf{I} + \mathbf{H}_k\mathbf{p}_k\mathbf{p}_k^\dagger\mathbf{H}_k^\dagger\mathbf{R}_{\bar{k}}^{-1}\right), \quad (4)$$

where $\mathbf{R}_{\bar{k}} = \sum_{i=1, i\neq k}^{K} \mathbf{H}_k\mathbf{p}_i\mathbf{p}_i^\dagger\mathbf{H}_k^\dagger + \mathbf{G}_k\mathbf{G}_k^\dagger$.

The main objective of this paper is to design the PM $\mathbf{P}$ and and BM $\mathbf{F}$ to maximize the minimum of the achievable rate based on the imperfect CSIT. Therefore, the optimization problem can be formulated as

$$[\mathbf{P}, \mathbf{F}] = \arg\max_{\mathbf{P},\mathbf{F}} \min_{k\in\{1,\cdots,K\}} \widehat{R}_k = \mathbb{E}\left[R_k \,\middle|\, \widehat{\mathbf{X}}_x, \sigma_{e,x}^2\right] \quad (5a)$$

$$\text{s.t.}: \quad (2a) \quad \text{and} \quad (2b). \quad (5b)$$

Let

$$\begin{aligned} \mathbf{\Pi}_k &\triangleq \mathbf{p}_k\mathbf{p}_k^\dagger, \\ \widehat{\mathbf{H}}_k^T &\triangleq \left[\widehat{\mathbf{h}}_{kb}^T \quad \left(\widehat{\mathbf{h}}_{kr}\mathbf{F}\widehat{\mathbf{H}}_{rb}\right)^T\right], \\ \mathbf{D}_k &\triangleq \mathrm{diag}\left(\sigma_{e,kb}^2\sigma_{kb}^2\mathrm{Tr}(\mathbf{\Pi}_k),\right. \\ &\quad \left.\sigma_{e,kr}^2\sigma_{kr}^2\mathrm{Tr}\left(\mathbf{F}\left(\mathbf{H}_{rb}\mathbf{\Pi}_k\mathbf{H}_{rb}^\dagger + \sigma_{e,rb}^2\sigma_{rb}^2\mathrm{Tr}(\mathbf{\Pi}_k)\mathbf{I}\right)\mathbf{F}^\dagger\right)\right), \\ \widehat{\mathbf{R}}_{\bar{k}} &\triangleq \sum_{i\neq k}\left(\widehat{\mathbf{H}}_k\mathbf{\Pi}_i\widehat{\mathbf{H}}_k^\dagger + \mathbf{D}_i\right) + \\ &\quad \mathrm{diag}\left(1, 1 + \widehat{\mathbf{h}}_{kr}\mathbf{F}\mathbf{F}^\dagger\widehat{\mathbf{h}}_{kr}^\dagger + \sigma_{e,kr}^2\sigma_{kr}^2\mathrm{Tr}\left(\mathbf{F}\mathbf{F}^\dagger\right)\right). \end{aligned}$$

Then, we have

$$\widehat{R}_k = \log\det\left(\mathbf{I} + \left(\widehat{\mathbf{H}}_k\mathbf{\Pi}_k\widehat{\mathbf{H}}_k^\dagger + \mathbf{D}_k\right)\widehat{\mathbf{R}}_{\bar{k}}^{-1}\right),$$

Note that, we have here used the following property:

$$\mathbb{E}\left[\widetilde{\mathbf{h}}_x\mathbf{A}\mathbf{A}^\dagger\widetilde{\mathbf{h}}_x^\dagger\right] = \sigma_x^2\mathrm{Tr}(\mathbf{A}\mathbf{A}^\dagger)\ \mathbf{I}, \quad \left(\widetilde{\mathbf{h}}_x = \widetilde{\mathbf{h}}_{kb},\ \widetilde{\mathbf{h}}_{kr}\right). \quad (6)$$

Obviously, the optimization problem in (5) is a non-linear and non-convex problem, and it is difficult to directly obtain the optimum closed-form solution. Therefore, we first find a tight lower bound of the $\widehat{R}_k$, and then set up another optimization problem based on the tight lower bound of the $\widehat{R}_k$ to move forward. To find a tight lower bound of the $\widehat{R}_k$, we have the following inequalities

$$\begin{aligned} \widehat{R}_k &\ge \log\det\left(\mathbf{I} + \widehat{\mathbf{H}}_k\mathbf{\Pi}_k\widehat{\mathbf{H}}_k^\dagger\widehat{\mathbf{R}}_{\bar{k}}^{-1}\right) \\ &\overset{a}{=} \log\left(1 + \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\widehat{\mathbf{R}}_{\bar{k}}^{-1}\widehat{\mathbf{H}}_k\mathbf{p}_k\right) \\ &\overset{b}{=} -\log\left(1 - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\left(\widehat{\mathbf{R}}_{\bar{k}} + \widehat{\mathbf{H}}_k\mathbf{\Pi}_k\widehat{\mathbf{H}}_k^\dagger\right)^{-1}\widehat{\mathbf{H}}_k\mathbf{p}_k\right) \\ &\overset{c}{\ge} -\log\left(1 - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\widehat{\mathbf{R}}_{\Sigma}^{-1}\widehat{\mathbf{H}}_k\mathbf{p}_k\right) \\ &\triangleq -\log\widehat{e}_k, \end{aligned} \quad (7)$$

where $\widehat{\mathbf{R}}_\Sigma = \widehat{\mathbf{R}}_{\bar{k}} + \widehat{\mathbf{H}}_k\mathbf{\Pi}_k\widehat{\mathbf{H}}_k^\dagger + \mathbf{D}_k$. (a) comes from the fact that $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$, (b) follows from the Woodbury identity $(\mathbf{A} + \mathbf{U}\mathbf{B}\mathbf{V})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{U}\left(\mathbf{B}^{-1} + \mathbf{V}\mathbf{A}^{-1}\mathbf{U}\right)^{-1}\mathbf{V}\mathbf{A}^{-1}$, and (c) is due to $\mathbf{D}_k \ge 0$. Since the differences generated in the first inequality and (c) depends on $\mathbf{D}_k$, which will be small when $\sigma_{e,x}^2 \ll 1$, the lower bound is tight. Specifically, $\widehat{R}_k = -\log\widehat{e}_k$, if $\sigma_{e,x}^2 = 0$, ($x = kb,\ kr$ and rb). Therefore, based on (7), the optimization problem based on lower bound can be expressed as following

$$\min_{\mathbf{P},\mathbf{F}} \max_{k\in\{1,2,\cdots,K\}} \{\ \log\widehat{e}_k\ \}, \quad (8a)$$

$$\text{s.t.}: (2a) \text{ and } (2b). \quad (8b)$$

To solve this min-max optimization problem, we need the following theorem.

Theorem 1: The optimal solution for the following problem is also the solution for the min-max problem formulated in (8):

$$\min_{\mathbf{P},\mathbf{F}} \sum_{k=1}^{K} \widehat{e}_k, \tag{9a}$$
$$\text{s.t.}: \widehat{e}_1 = \cdots = \widehat{e}_K, \tag{9b}$$
$$(2a) \text{ and } (2b). \tag{9c}$$

Proof: If the $\mathbf{P}^{\text{opt}}$ and $\mathbf{F}^{\text{opt}}$ are the optimal matrices for problem in (8), they must make sue that $\widehat{e}_1 = \cdots = \widehat{e}_K$ is the smallest achievable value. Therefore, they are also the optimal matrices for the problem in (9), and vice versa. ■

III. Source and relay matrices Design

To find the optimal source and relay matrices for the aforementioned problem, we first relax the constraints of the new optimization problem, and then decouple it into three tractable optimization sub-problem. Finally, we summarize a general iterative algorithm for source PM and relay BM. We first relax the constraints of the problem formulated in (9) as following

$$\min_{\mathbf{P},\mathbf{F}} \sum_{k=1}^{K} \widehat{e}_k, \tag{10a}$$
$$\text{s.t.}: \ \text{Tr}(\mathbf{p}_k\mathbf{p}_k^\dagger) = c_k, \quad k \in \{1, 2, \cdots, K\} \tag{10b}$$
$$(2b), \tag{10c}$$

where $\mathbf{c} \triangleq \{c_k\}_{k=1}^{K}$ is a predetermined constant vector which should be chosen to satisfy the power budget constraint.

Remark 1: Due to the fact that $\widehat{e}_k(c_k') < \widehat{e}_k(c_k)$ for all $c_k' > c_k$ with the optimal beamforming struct of the problem in (10), we can adjust the predetermined constant vector $\mathbf{c}$ to meet the equivalent condition in (9b) by an iterative method. To solve the relaxed optimization problem in (10), we need the following theorem.

Theorem 2: Let $\xi_k \triangleq 1 - \mathbf{a}_k\widehat{\mathbf{H}}_k\mathbf{p}_k - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger + \mathbf{a}_k\widehat{\mathbf{R}}_\Sigma\mathbf{a}_k^\dagger$, where $\mathbf{a}_k$ is a row vector variable. Then, the optimal solution for the following problem is also the solution for the relaxed problem formulated in (10):

$$\min_{\{\mathbf{P},\mathbf{F},\mathbf{A}\triangleq\{\mathbf{a}_k\}_{k=1}^{K}\}} \sum_{k=1}^{K} \xi_k, \tag{11a}$$
$$\text{s.t.}: \ (10b) \text{ and } (2b). \tag{11b}$$

Proof: For any $\mathbf{P}$ and $\mathbf{F}$, it can readily find the optimal $\mathbf{a}_k$ which is equal to $\mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\widehat{\mathbf{R}}_\Sigma^{-1}$. Then, substituting $\mathbf{a}_k = \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\widehat{\mathbf{R}}_\Sigma^{-1}$ into ξ_k, we can obtain the $\widehat{e}_k = \xi_k$. ■

The problem formulated in (11) is also a non-linear problem, and its closed-form solution is still intractable. However, for fixed two of the three matrices (i.e., $\mathbf{P}, \mathbf{F}$ and $\mathbf{A}$), the rest one can be optimized [8]. In fact, in the proof of Theorem 2, given the $\mathbf{P}$ and $\mathbf{F}$, the optimal $\mathbf{a}_k$ is

$$\mathbf{a}_k = \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\widehat{\mathbf{R}}_\Sigma^{-1}. \tag{12}$$

Secondly, for fixed $\mathbf{A}$ and $\mathbf{F}$, the optimization problem in (11) with respect to (w.r.t.) $\mathbf{P}$ can be adjusted as following

$$\min_{\mathbf{P}} \sum_{k=1}^{K} \left(1 - \mathbf{a}_k\widehat{\mathbf{H}}_k\mathbf{p}_k - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger + \mathbf{a}_k\widehat{\mathbf{R}}_\Sigma\mathbf{a}_k^\dagger\right), \tag{13a}$$
$$\text{s.t.}: \ (10b) \text{ and } (2b). \tag{13b}$$

This optimization problem can be transformed into a convex quadratically constrained quadratic program (QCQP) problem which can be efficiently solved by using the available software package [12] [13]. However, if we fix $\mathbf{F}$ and ignore the power change at relay due to the change of the source precoder, we can obtain a serial independent optimization subproblems, and each of the subproblems is only w.r.t. one column vector of the $\mathbf{P}$, i.e., $\mathbf{p}_k$, which can be solved by KKT conditions method. Note that this method is simpler than solving the QCQP problem with software package. Hence, we set up the following optimization subproblems w.r.t. each column vector of $\mathbf{P}$ from (13) by eliminating the power constraint at relay for a fixed $\mathbf{F}$. The optimization subproblem for the $\mathbf{p}_k$ $(k = 1, 2, \cdots, K)$ can be written as

$$\min_{\mathbf{p}_k} \ 1 - \mathbf{a}_k\widehat{\mathbf{H}}_k\mathbf{p}_k - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger + \sum_{i=1}^{K} \mathbf{a}_i\widehat{\mathbf{R}}_\Sigma\mathbf{a}_i^\dagger, \tag{14a}$$
$$\text{s.t.}: \ \text{Tr}(\mathbf{p}_k\mathbf{p}_k^\dagger) = c_k. \tag{14b}$$

It is very easy to verify that the optimization problem in (14) is a convex problem which can be solved by KKT conditions method. Thus, we can readily obtain the Lagrangian function of (14) as

$$\mathcal{L}(\mathbf{p}_k) = -\mathbf{a}_k\widehat{\mathbf{H}}_k\mathbf{p}_k - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger + \sum_{i=1}^{K} \mathbf{a}_i\widehat{\mathbf{H}}_i\mathbf{p}_k\mathbf{p}_k^\dagger\widehat{\mathbf{H}}_i^\dagger\mathbf{a}_i^\dagger + \lambda\left(\text{Tr}\left(\mathbf{p}_k\mathbf{p}_k^\dagger\right) - c_k\right).$$

Then, the first-order necessary condition of $\mathcal{L}$ w.r.t. $\mathbf{p}_k^*$ yields

$$\mathbf{p}_k(\lambda) = \left(\sum_{i=1}^{K} \widehat{\mathbf{H}}_i^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\widehat{\mathbf{H}}_i + \lambda\mathbf{I}\right)^{-1} \widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger, \tag{15}$$

where $\lambda \geq 0$ is the Lagrangian multiplier which should satisfy the KKT complementarity conditions for power budget constraint, i.e., $\text{Tr}\left(\mathbf{p}_k(\lambda)\mathbf{p}_k^\dagger(\lambda)\right) = c_k$, $k = 1, 2, \cdots, K$.

Thirdly, for fixed $\mathbf{A}$ and $\mathbf{P}$, the optimization problem in (11) w.r.t. $\mathbf{F}$ can be adjusted as following

$$\min_{\mathbf{F}} \sum_{k=1}^{K} \left(-\mathbf{a}_k\widehat{\mathbf{H}}_k\mathbf{p}_k - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger + \mathbf{a}_k\widehat{\mathbf{R}}_\Sigma\mathbf{a}_k^\dagger\right), \tag{16a}$$
$$\text{s.t.}: \quad (2b). \tag{16b}$$

This optimization problem is easy to be verified to be a convex problem. Thus, the Lagrangian function for $\mathbf{F}$ is given as

$$\mathcal{L}(\mathbf{F}) = \sum_{k=1}^{K} \left(1 - \mathbf{a}_k\widehat{\mathbf{H}}_k\mathbf{p}_k - \mathbf{p}_k^\dagger\widehat{\mathbf{H}}_k^\dagger\mathbf{a}_k^\dagger + \mathbf{a}_k\widehat{\mathbf{R}}_\Sigma\mathbf{a}_k^\dagger\right) + \mu\left(\text{Tr}(\mathbf{F}\widehat{\mathbf{H}}_{rb}\mathbf{P}\mathbf{P}^\dagger\widehat{\mathbf{H}}_{rb}^\dagger\mathbf{F}^\dagger + \mathbf{F}\mathbf{F}^\dagger) - P_r\right). \tag{17}$$

Before dealing with the KKT conditions, we first substitute $\mathbf{a}_k \triangleq [a_{1k}\ a_{2k}]$ and $\widehat{\mathbf{H}}_k \triangleq \begin{bmatrix} \widehat{\mathbf{h}}_{kb} \\ \widehat{\mathbf{h}}_{kr}\mathbf{F}\widehat{\mathbf{H}}_{rb} \end{bmatrix}$ into (17) to get a function w.r.t. $\mathbf{F}$. Then, the first-order necessary condition of $\mathcal{L}$ w.r.t. $\mathbf{F}^*$ yields

$$\mathbf{F} = \left(\sum_{k=1}^{K} \mathbf{\Theta}_k + \mu\mathbf{I}\right)^{-1} \left(\sum_{k=1}^{K} -\mathbf{\Delta}_k\right)(\mathbf{\Omega}+\mathbf{I})^{-1}, \quad (18)$$

where

$$\mathbf{\Omega} \triangleq \widehat{\mathbf{H}}_{rb}\mathbf{P}\mathbf{P}^\dagger\widehat{\mathbf{H}}_{rb}^\dagger,$$
$$\mathbf{\Theta}_k \triangleq \widehat{\mathbf{h}}_{kr}^\dagger a_{2k}^* a_{2k}\widehat{\mathbf{h}}_{kr},$$
$$\mathbf{\Delta}_k \triangleq \widehat{\mathbf{h}}_{kr}^\dagger a_{2k}^* a_{1k}\widehat{\mathbf{h}}_{kb}\mathbf{P}\mathbf{P}^\dagger\widehat{\mathbf{H}}_{rb}^\dagger - \widehat{\mathbf{h}}_{kr}^\dagger a_{2k}^*\mathbf{p}_k^\dagger\widehat{\mathbf{H}}_{rb}^\dagger.$$

μ is the Lagrangian multiplier which can also be solved by a 1-D search method since $\mathrm{Tr}(\mathbf{F}(\mu)(\mathbf{\Omega}+\mathbf{I})\mathbf{F}(\mu)^\dagger)$ is monotonically decreasing function of μ.

A. An Iterative Design Algorithm

In summary, an iterative design algorithm for PM $\mathbf{P}$ and BM $\mathbf{F}$ can be summarized as following algorithm diagram.

Algorithm 1 : A General Iterative Design Algorithm

1: **Initialize:** $c_k = P_b/K, \mathbf{P} = \sqrt{c_k}\mathbf{I}$, $\mathbf{F} = \rho\mathbf{I}$, $\mathbf{a}_k$ uses (12) with $\mathbf{P} = \sqrt{c_k}\mathbf{I}$ and $\mathbf{F} = \rho\mathbf{I}$, where ρ satisfies the power constraint, $k = 1, 2, \cdots, K$.
2: **Repeat:**
3: Update $\mathbf{p}_k$ using (15) for fixed $\mathbf{A}$ and $\mathbf{F}$,
4: Update the $\mathbf{c}$ by using the following steps: $c_i = c_i - \triangle$, and $c_j = c_j + \triangle$, where $i = \arg\min_{\{i=1,\ldots,K\}} \widehat{e}_i$, $j = \arg\max_{\{j=1,\ldots,K\}} \widehat{e}_j$, and $\triangle = (1 - \frac{K\widehat{e}_i}{\sum_{k=1}^{K}\widehat{e}_k})c_i$.
5: Update all $\mathbf{a}_k$ using (12) for fixed $\mathbf{P}$ and $\mathbf{F}$;
6: Update $\mathbf{F}$ using (18) for fixed $\mathbf{A}$ and $\mathbf{P}$;
7: **Until:** The termination criterion is satisfied.

Based on the steps (3)-(6), the largest $\widehat{e}_k$ is decreased in each iteration, while, there is a lower bounded for $\widehat{e}_k$ ($k = 1, \cdots, K$) under the power constraints (2a) and (2b). Hence, the proposed algorithm is convergent. Fig. 2 shows the convergence in simulation. One can also refer to the block coordinate descent algorithm in [14] for the convergence analysis. In addition, the computational complexity of this algorithm is $O(M^3)$, where M is the number of antenna at BS.

IV. Numerical Results

This section presents numerical results to evaluate the proposed scheme. For fair comparison, the other schemes for comparison are also considering the DLs contributions, which are:

1) SVD-RZF in [10],
2) RZF-ZF&RZF in [7],
3) ZF-ZF&ZF in [6].

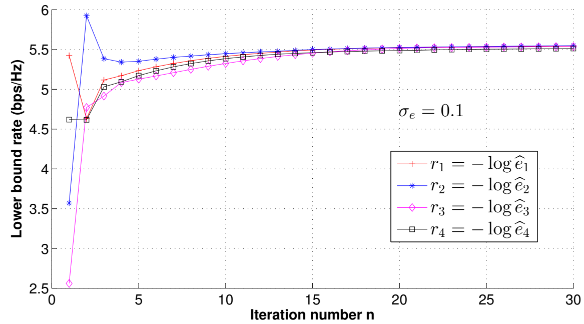

Fig. 2. Convergence property for 1 (randomly selected) channel realization with $P_b = P_r(\mathrm{SNR} = 28\mathrm{dB})$, where $M = 4, K = 4$, BS is at 0 point, relay is at 0.5 point, all users are at 1.0 point. $\sigma_{e,kb} = \sigma_{e,kr} = \sigma_{e,rb} = \sigma_e$.

All these schemes are adjusted to suitable for max-min achievable rate among all users case for fair comparison. The channel gains are set to be the combination of large scale fading and small scale fading, i.e., all channel vectors (or matrix) have i.i.d. $\mathcal{CN}(0, \frac{1}{\ell^\tau})$ entries, where ℓ is the distance between two nodes, and $\tau = 3$ is the path loss exponent. In these simulations, we consider that BS and relay are deployed in a line with users, where all the users are deployed at the same point.

Fig. 3 shows the achievable sum-rates and the achievable minimum rate among all users of the difference schemes. Fig. 4 shows the average sum-rate and the average minimum rate over 2000 random channel realizations versus the RS's position. From Fig. 3 and Fig. 4, we can see that the sum-rate and the minimum-rate of the proposed scheme is higher than those of the other linear schemes at all error factor regime and all RS's position. This is because that the DLs contributions of SVD-RZF scheme are approximately equal to zero, and the ZF-ZF&ZF and RZF-ZF&RZF schemes will amplify noise signal, especially at the case that the DLs gains are close to zero. However, the proposed scheme can better deal with the multi-user interferences and noise, and the relation between DLs gains and source-relay-users channel gains. It can be observed that the proposed scheme has better robustness.

V. Conclusion

In this paper, we propose a robust matrices design scheme for the MIMO relaying BC with DLs based on imperfect CSIT to maximize the minimum achievable rate among all users. The proposed scheme is robust to the imperfect CSIT and takes into account the effects of the DLs, relay links, interferences and noise. Numerical results show that the proposed robust scheme outperforms other linear schemes with or without considering DLs in design.

Acknowledgment

This work is supported by the Guangxi Natural Science Foundation# 2014GXNSFBA118276, by the University Science Research Project of Guangxi #YB2014016, by

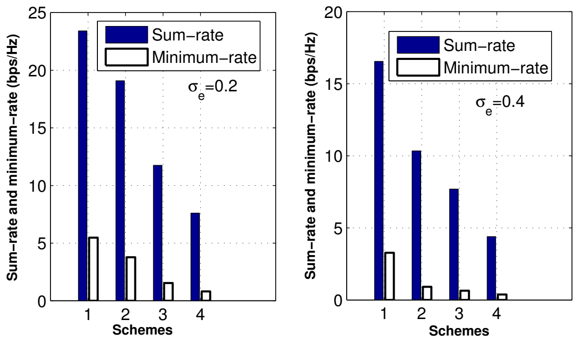

Fig. 3. The sum-rate and minimum rate of the difference schemes for 1 (randomly selected) channel realization with $\sigma_{e,kb} = \sigma_{e,kr} = \sigma_{e,rb} = \sigma_e = 0.2$ and $\sigma_e = 0.4$, where $P_b = P_r$ (SNR=28dB), $M = 4$, $K = 4$, BS is at 0 point, relay is at 0.5 point, and all users are at 1.0 point. (1 $\triangleq$ Proposed scheme, 2 $\triangleq$SVD-RZF, 3 $\triangleq$RZF-ZF&RZF, and 4 $\triangleq$ZF-ZF&ZF.)

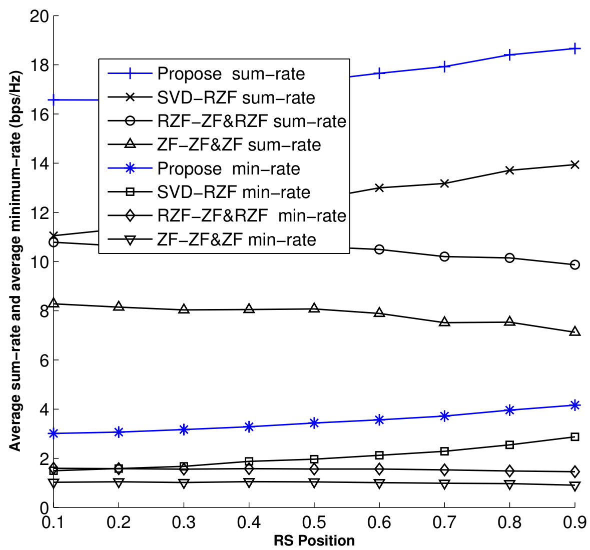

Fig. 4. Average sum-rate and minimum-rate versus RS's position (between 0 and 1.0), where BS is at 0 point and all users are at 1.0 point, $M = 4, K = 4$, $P_s = P_r$ (SNR = 28dB), and $\sigma_{e,kb} = \sigma_{e,kr} = \sigma_{e,rb} = 0.3$.

the National 973 Project #2012CB316106, by NSF China #61328101, by STCSM Science and Technology Innovation Program #13510711200.

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