

Capacity Analysis of Cognitive MISO Multicast Network

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Abstract—In this paper, we consider a scenario where a secondary base station (SBS) with K antennas utilizes the licensed spectrum of a single primary user (PU) to send the same information to N secondary users (SUs). The SBS selects only one antenna out of the K antennas by using maximum signal power to leak interference power ratio strategy (SLIR). Using asymptotic analysis, we first derive the average capacity of multicast transmission scheme with perfect channel state information from the SBS to the PU (interference CSI) available at the SBS. Then, we characterize its capacity loss in the case that perfect interference CSI is not always available at the SBS.

I. INTRODUCTION

In a spectrum sharing system, secondary users (SUs) are allowed to transmit information simultaneously to active primary users (PUs) only if the interference power to each PU can be guaranteed [1]–[3]. The maximum allowable interference power is called interference temperature [5], which guarantees the quality of service (QoS) of the PUs regardless of the SU's spectrum utilization. The main problem of spectrum sharing system can be considered as the co-channel interference.

Multiple antenna technology is one potential solution exploiting spatial diversity and multiplexing gain for wireless radio systems [6], [8]. Since the computational complexity of multiple antenna technology increases exponentially with the number of antennas, the antenna selection diversity has been adopted as an alternative for MIMO-based multiplexing, precoding and beam-forming. This is because that the selection diversity gain is achieved with a relatively lower feedback burden and simple antenna selection process [9], [10]. Using multiple-antenna technology in cognitive radio has been investigated in [4], [7].

At the same time, for a point-to-multipoint link, there has been an increasing interest in the applications of sending same information to multiple receivers. In the multicast scheme, the multicast gain resulting from the fact any information transmitted is overhead by all users, which is an important issue in wireless fading channel. In [11], the authors indicate that the throughput of the multicast transmission scheme is limited by the worst channel user, for which capacity approximates as one.

In this paper, we consider a scenario where a secondary base station (SBS) with K antennas utilizes the licensed spectrum of a single primary user (PU) to send the same information to N secondary users (SUs). We analyze the average capacity of multicast network in spectrum sharing

environment using antenna selection at the SBS. We derive the average capacity of the multicast transmission schemes when perfect interference CSI is available at the SBS, and show that it approximates as $N \log(1 + \Gamma(1 + \frac{1}{K}) b_N Q)$, where $\Gamma(\cdot)$ denotes Gamma function, $b_N = (1 - N^{-\frac{1}{K}})^{-1} - 1$, and Q is interference temperature. Furthermore, we characterize the average capacities of the scheme with imperfect interference CSI available at the SBS and shows that it approximates as $N \log(1 + \frac{\lambda}{1+\lambda} \Gamma(1 + \frac{1}{K}) b_N Q)$, where $1/\lambda$ denotes the mean of the channel estimation error.

II. SYSTEM MODEL AND PRELIMINARY

As shown in Fig. 1, a spectrum sharing multicast network in a single-cell system is considered where an SBS with K antennas utilizes a spectrum licensed to a single PU to transmit the same information to N SUs. The PU and SUs are equipped with single antenna, and the SBS selects one antenna out of the K antennas for transmission. The channel gains from the k -th antenna of the SBS to the PU and the j -th SU are denoted by α_k and β_{kj} , respectively, where $k \in \{1, 2, \dots, K\}$ and $j \in \{1, 2, \dots, N\}$. The α_k and β_{kj} are assumed to be independent and identically distributed (i.i.d.) exponential random fading. Assume that the SBS has perfect information of SU's channel gains β_{kj} all the time. Utilizing the feedback scheme, the SBS can obtain the interference CSI information through periodic sensing of pilot signal from the PU in the hypothesis of the channel reciprocity [12]. The SBS knows the interference channel α_k , and by which to compute the maximum allowable transmission power P_{\max} to satisfy the interference temperature constraint at the PU. In this paper, we consider the interference at the PU created by the SBS. When the k -th antenna is selected, the interference created by the SBS is defined as $I_{th} = P_{\max} \alpha_k \leq Q$. Hence, the maximum transmission power P_{\max} satisfying the condition is equal to $\frac{Q}{\alpha_k}$, and the received signal strength at the j -th SU is $P_j = \frac{\beta_{kj} Q}{\alpha_k}$.

We define the signal power to leak interference ratio (SLIR) for the k -th antenna as $\gamma_{kj} = \frac{\beta_{kj}}{\alpha_k}$, where $k \in \{1, \dots, K\}$. The SBS chooses the k -th antenna out of K antennas to send information by the maximum SLIR strategy, which can be expressed as $k = \arg \max_{k \in \{1, 2, \dots, K\}} \gamma_{kj}$. From the concept of SLIR, it is easy to understand that the interference channel

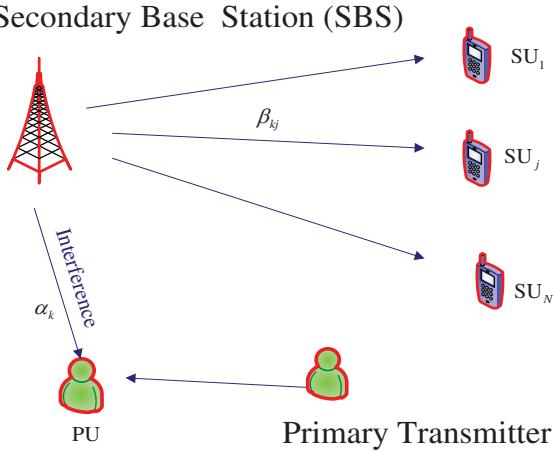


Fig. 1. The system model for the SU network coexisting with one PU

and SU's channel are both considered, and the interference channel is prior considered in contrast to SU's channel gain. This can ensure that the performance of the PU has little negative influence. Similar to that in [17], the interference from primary transmitters can be translated into the noise term under the hypothesis that interference from the primary transmitters follows a white Gaussian distribution, which can be justified by the Central Limit Theorem (CLT) if there are many primary transmitters. To simplify mathematical analysis, all channel gains are assumed to be the i.i.d. exponential random variables with unit mean and the variance of white Gaussian noise is assumed to be 1. Then, the probability distribution function (PDF) of α_k is denoted as $f_{\alpha_k}(x) = e^{-x}$, for $x > 0$. Furthermore, the PDF of γ_{kj} can be calculated as follows:

$$f_{\gamma_{kj}}(z) = \int_0^{\infty} e^{-(z+1)y} y dy = \frac{1}{(z+1)^2}, z > 0, \quad (1)$$

where z is the random variable representing $z = \frac{\beta_{kj}}{\alpha_k}$. The cumulative distribution function (CDF) of γ_{kj} is $F_{\gamma_{kj}}(z) = 1 - \frac{1}{z+1}$. When the SBS adopts the Maximum SLIR strategy, the CDF and PDF of the normalized received signal strength ρ_j at the j -th SU are, respectively,

$$F_{\rho_j}(\rho) = (1 - (\rho + 1)^{-1})^K, \quad (2)$$

$$f_{\rho_j}(\rho) = K(1 - (1 + \rho)^{-1})^{K-1}(1 + \rho)^{-2}, \quad (3)$$

where $\rho_j = \frac{P_j}{Q}$, $j \in \{1, \dots, N\}$.

III. ASYMPTOTIC CAPACITY WITH PERFECT INTERFERENCE CSI

In the following, we will analyze the average capacity of multicast network in spectrum sharing where the same information is transmitted to all SUs. In the multicast scheme, the SBS always transmits to all SUs at information rate decodable by the worst channel SU. This scheme maximally exploits the multicast gain by always transmitting to the SUs with the

least instantaneous SNR. Therefore, the average capacity of the multicast scheme is given by [11],

$$\begin{aligned} C_M &\triangleq NE[\ln(1 + \rho_{\min} Q)] \\ &= N \int_0^{\infty} \ln(1 + \rho Q) f_{\rho_{\min}}(\rho) d\rho, \end{aligned} \quad (4)$$

where $\rho_{\min} \triangleq \min_{1 \leq i \leq N} \rho_i$, whose PDF is given by

$$f_{\rho_{\min}}(\rho) = N f_{\rho_i}(\rho) (1 - F_{\rho_i}(\rho))^{N-1}. \quad (5)$$

However, a closed-form of (4) is not available. Even if the closed-form is obtained, it is difficult to fully understand the effects of main parameters such as K and N on the capacity by a numerical evaluation. Thus, we provide an asymptotic approach to understand the behavior of (4) for the large N scenario.

Theorem 1: The average capacity of the multicast scheme in spectrum sharing with perfect interference CSI approximates as

$$C_M = N \ln \left(1 + \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right). \quad (6)$$

Proof: Extreme value theory deals with asymptotic distributions of extreme values, such as minima. It can be used to analyze the performance of the above scheme approach. Using extreme value theory in [14], the distribution of the minimum received SNR satisfies

$$\Pr \left(\frac{\rho_{\min}}{b_N} \leq \rho \right) \sim W(\rho) \quad \text{as } N \rightarrow \infty, \quad (7)$$

where $W(\cdot)$ is a Weibull type distribution with CDF $W(\rho) = 1 - \exp(-\rho^\delta)$ for $\rho > 0$, and δ satisfies

$$\lim_{t \rightarrow -\infty} \frac{F_{\rho_j}(-\frac{1}{t\rho})}{F_{\rho_j}(-\frac{1}{t})} = \rho^{-\delta}. \quad (8)$$

Using L'Hospital rule, it is easy to show that $\delta = K$. The variable b_N satisfies $F(b_N) = \frac{1}{N}$. Consider the fact that (2) can be re-written as

$$(1 - (b_N + 1)^{-1})^K = \frac{1}{N}. \quad (9)$$

After some mathematical operations,

$$b_N = \left(1 - N^{-\frac{1}{K}} \right)^{-1} - 1. \quad (10)$$

This means that the distribution of $\frac{\rho_{\min}}{b_N}$ approaches to a Weibull random variable as N increases. In other words, for some constant $l > 0$,

$$\Pr \left(N^{\frac{1}{K}} \rho_{\min} \leq \rho \right) \sim \Pr(lW \leq \rho). \quad (11)$$

From the result in Theorem 2.1 of [15], it is concluded that

$$\frac{E[\rho_{\min}]}{b_N} \sim l E[W] = \Gamma \left(1 + \frac{1}{K} \right). \quad (12)$$

Therefore, we have $E[\rho_{\min}] = b_N \Gamma \left(1 + \frac{1}{K} \right)$. Using Jensen's inequality, the average capacity of the multicast scheme can be upper bounded as

$$C_M = NE[\ln(1 + \rho_{\min} Q)] \leq N \ln(1 + E[\rho_{\min}] Q). \quad (13)$$

On the other hand, we lower bound the average capacity as

$$\begin{aligned} C_M &= N \int_0^\infty \ln(1 + \rho Q) dF_{\min}(\rho) \\ &\geq N \int_{b_N \Gamma(1+1/K)}^\infty \ln(1 + \rho Q) dF_{\rho_{\min}}(\rho). \end{aligned} \quad (14)$$

Due to $\Gamma(1 + \frac{1}{K}) \leq 1$ for $K \in \mathbb{N}$, we have

$$\begin{aligned} C_M &\geq N \int_{b_N \Gamma(1+\frac{1}{K})}^\infty \ln \left(1 + \Gamma \left(1 + \frac{1}{K} \right) \rho Q \right) dF_{\rho_{\min}}(\rho) \\ &\geq N \ln \left(1 + \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right) \\ &\times \left[1 - F_{\rho_{\min}} \left(b_N \Gamma \left(1 + \frac{1}{K} \right) \right) \right], \end{aligned} \quad (15)$$

where $F_{\rho_{\min}}(\rho) = 1 - (1 - F_{\rho_j}(\rho))^N$. Considering the fact that $F_{\rho_j}(\rho) = (1 - (\rho + 1)^{-1})^K$, we get

$$F_{\rho_{\min}}(\rho) = 1 - \left(1 - (1 - (\rho + 1)^{-1})^K \right)^N \approx 1. \quad (16)$$

Therefore,

$$C_M \geq N \ln \left(1 + \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right). \quad (17)$$

Combining this with the upper bound, we have

$$C_M = N \ln \left(1 + \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right). \quad (18)$$

■

Remark 1: From Theorem 1, we can see that the capacity property of the multicast transmission scheme in a spectrum sharing includes two cases: i) When $K = 1$, we can see that b_N is $\frac{1}{N}$ for the large N . Due to $\ln(1 + (c/x)) = c/x$ for large x and constant c , it is concluded that the capacity approximates as Q , which possesses capacity saturation in terms of N , i.e., the capacity is only relevant to Q . In this case, the capacity increases linearly as the interference temperature Q increases, which agrees with reality. However, no selection diversity gain is obtained. ii) When $K > 1$, $b_N \approx N^{-\frac{1}{K}}$ for large N . Therefore, the capacity approximates as $N^{1-\frac{1}{K}}Q$. In this case, the capacity does not bear saturation, which is relevant to N , K and Q . We also observe that the capacity increases linearly as N increases in the case of large K and fixed Q . Thus the selection gain is approximately $N^{1-\frac{1}{K}}$.

IV. ASYMPTOTIC CAPACITY WITH IMPERFECT INTERFERENCE CSI

Due to limited cooperation between the SBS and the PU, the accurate α_k is usually hard to be obtained at the SBS. We assume that the SBS performs minimum mean square error (MMSE) estimation. Assume that the interference channel estimate is modeled as $h_k = \hat{h}_k + \Delta h_k$ [13], where \hat{h}_k and Δh_k denote the perfect interference channel and the channel estimation error, respectively. Each element of Δh_k is assumed to be *i.i.d.* complex Gaussian distributed. Therefore, its envelope is rayleigh distributed. Furthermore, $\hat{h}_k, \Delta h_k$ are assumed

to be independent. By the property of MMSE estimation, \hat{h}_k and Δh_k are uncorrelated. We can get $\alpha_{h_k} = \alpha_{\hat{h}_k} + \alpha_{\Delta h_k}$, where $\alpha_{\hat{h}_k}$ and $\alpha_{\Delta h_k}$ denote the magnitude square of \hat{h}_k and Δh_k , respectively, which follow the exponential distribution. Assume that $\alpha_{\hat{h}_k}$ is normalized to have unit mean, and $\Delta \alpha_{h_k}$'s mean value is $1/\lambda$. After some calculation, For $\lambda \neq 1$, the PDF and CDF of α_{h_k} are

$$f_{\alpha_{h_k}}(x) = \frac{\lambda}{\lambda - 1} (e^{-x} - e^{-\lambda x}), \quad (19)$$

and

$$F_{\alpha_{h_k}}(x) = \frac{\lambda}{\lambda - 1} \left(1 - \frac{1}{\lambda} - e^{-x} + \frac{1}{\lambda} e^{-\lambda x} \right), \quad (20)$$

respectively. Using a similar argument in Section III, Eq. (2), and Eq. (3) are found as follows, respectively

$$F_{\rho_j}(\rho) = \left(1 - \frac{\lambda}{(\rho + \lambda)(\rho + 1)} \right)^K, \quad (21)$$

$$\begin{aligned} f_{\rho_j}(\rho) &= \frac{K\lambda}{\lambda - 1} \left(\frac{1}{(\rho + 1)^2} - \frac{1}{(\rho + \lambda)^2} \right) \\ &\times \left(1 - \frac{\lambda}{(\rho + \lambda)(\rho + 1)} \right)^{K-1}. \end{aligned} \quad (22)$$

In the following, we asymptotically analyze the capacity to understand the effects of channel estimation error in spectrum sharing environments. We can get the asymptotic capacity of the multicast scheme using extreme value theory.

Theorem 2: The average capacity of the multicast scheme under imperfect interference CSI available at the SBS approximates as

$$C_{ME} = N \ln \left(1 + \frac{\lambda}{1 + \lambda} \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right). \quad (23)$$

Proof: According to the extreme value theory, the distribution of the minimum received SNR satisfies

$$\Pr \left(\frac{\rho_{\min}}{b'_N} \leq \rho \right) \sim W(\rho) \quad \text{as } N \rightarrow \infty, \quad (24)$$

where $W(\cdot)$ is a Weibull type distribution and the variable b'_N satisfies $F(b'_N) = \frac{1}{N}$. From Eq. (21), we can get

$$\frac{\lambda}{(b'_N + \lambda)(b'_N + 1)} = 1 - N^{-\frac{1}{K}}. \quad (25)$$

Taking the logarithm on both sides of (25), we have

$$\ln \lambda - \ln \left[b'^2_N + (1 + \lambda) b'_N + \lambda \right] = \ln \left(1 - N^{-\frac{1}{K}} \right), \quad (26)$$

From (25), b'_N approaches to zero as N goes to infinity. Since b'^2_N is far less than b'_N , the terms $\ln(\lambda)$ and $\ln[(1 + \lambda) b'_N + \lambda]$ in the left hand side of (26) become dominant. Therefore, we have

$$\frac{\lambda}{(1 + \lambda) b'_N + \lambda} = 1 - N^{-\frac{1}{K}}. \quad (27)$$

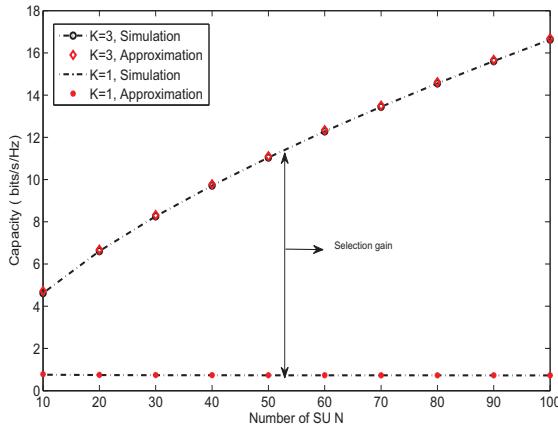


Fig. 2. Average capacity of the multicast scheme in spectrum sharing versus the number N of SU

After some mathematical operation,

$$b'_N = \frac{\lambda}{1+\lambda} \left(\left(1 - N^{-\frac{1}{K}} \right)^{-1} - 1 \right). \quad (28)$$

According to the similar argument in Theorem 1, we have

$$C_{ME} = N \ln \left(1 + \frac{\lambda}{1+\lambda} \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right). \quad (29)$$

Fig. 3. Average capacity of the multicast scheme in spectrum sharing versus the number N of SU with imperfect interference CSI available at the SBS

available at the SBS. It shows that the channel estimation error will result in capacity loss. This is because that the channel estimation error will leak interference power levels at the PU. The capacity loss will decrease as λ increases, since larger λ means smaller channel estimation error. We further see that the capacity loss approaches to zero when λ goes to large and the approximated expression of the capacity in the case of large λ well agrees with the simulation results even if the number of SU is small.

VI. CONCLUSION

Using the extreme value theory, we first analyze the asymptotic capacity for the MISO multicast network with perfect interference CSI in spectrum sharing. The asymptotic capacity of the multicast transmission scheme approximates as $N \ln \left(1 + \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right)$. Then, we investigate the capacity loss when perfect interference CSI is not available at the SBS, and found that it approximates as $N \ln \left(1 + \frac{\lambda}{1+\lambda} \Gamma \left(1 + \frac{1}{K} \right) b_N Q \right)$.

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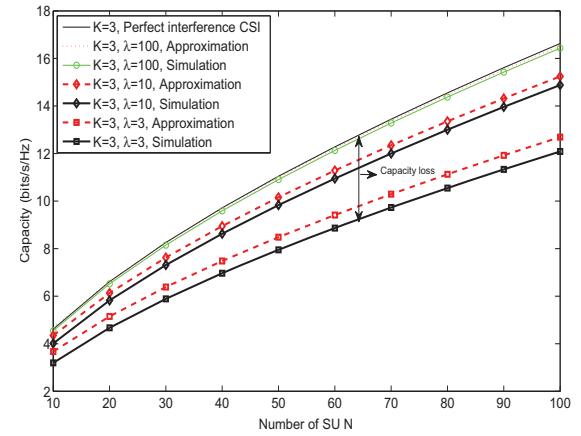
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V. NUMERICAL RESULTS

We present simulation results to validate our theoretical claims. These results are obtained through Monte-Carlo simulations. The SBS chooses one antenna out of K antennas by using maximize SLIR. The predetermined interference threshold $Q = -3$ dB (i.e. $Q=0.5$ Watt). Fig. 2 shows the capacity of the multicast transmission scheme in spectrum sharing versus the number N of SU when perfect interference CSI is available at the SBS. When $K = 1$, the capacity scales as 0.5 nat, which is irrelevant to N . When $K = 3$, the capacity scales as $N^{\frac{2}{3}}Q$ nat. As an overall observation, the selection gain increases as N increases, and the approximated expression of the capacity well agrees with the simulation results even if the number of SU is small.

Fig. 3 shows the capacity of the multicast transmission scheme in spectrum sharing versus the number N of SU for different λ when imperfect interference channel CSI is



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