

Linear Transceiver and Receiver Design Methods for Multiuser MIMO Channels[†]

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Abstract—Multiuser multiple-input multiple-output (MU-MIMO) systems has attracted considerable interests due to the promising potentials. In this paper, we are trying to make some connections between linear transmitter/beamforming and receiver design in MU-MIMO systems with code-division multiple-access (CDMA) signature and receiver design methods. Some methods discussed previously in optimization of CDMA signatures and receivers may be applied to the multiuser MIMO systems, to design the linear beamforming and receiving vectors. With a general multiuser MIMO system model, we present three different design methods based on three different criteria: (i) Minimizing the effective TSC among all users; (ii) Minimizing the total mean square error (MSE); (iii) Maximizing the sum capacity of the system. We derive in details the design methods under those different design criteria. Simulation results show that different methods demonstrate differently under those criteria.

Index Terms—multiple-input multiple-output, multiuser, linear receiver, fading channels, beamforming, code-division multiplexing.

I. INTRODUCTION

Current information-theoretic interest in multi-input multi-output (MIMO) [1]-[2] communications has shifted, in part, away from point-to-point links into multiuser links [3]-[4], where several co-channel users with arrays attempt to communicate with each other or with some central base station. The works of [5]-[8] has shown that many of the advantages of using multiple antennas in a single-user scenario also translate to large gains in multiuser scenarios. Zero-forcing block diagonalization (ZFBD) beamforming method for multiuser MIMO downlink is presented in [9]. Designing transmit beamforming vectors based on maximizing the signal-to-leakage-and-noise ratio (SLNR) for all users is proposed in [10]. The theoretical studies and transceiver structure designs for uplink multiuser MIMO communication systems are discussed in [11]-[12].

There is also a large body of works on code-division multiple-access (CDMA) systems. Searching for optimal signature sets has been always with great attention for code-division multiplexing. In the theoretical context of complex/real-valued signature sets, the early work of Welch [13] on total-squared-correlation (TSC) bounds was followed

up by direct minimum-TSC design proposals [14]-[15] and iterative distributed optimization algorithms [16]-[17]. Signature sets that maximize user capacity are sought in [18]-[19]. Minimum-mean-square-error (MMSE) minimization is used for the design of signature sets for multiuser systems over multipath channels in [20]. Algorithms for group transmitter-receiver adaptation in the presence of multipath are presented for different scenarios in [21].

In this paper, we are trying to make some connections between linear transmitter/beamforming and receiver design in MU-MIMO systems with code-division multiple-access (CDMA) signature and receiver design methods. In other words, some methods discussed in optimization of CDMA signatures and receivers design [20]-[21], can be applied to the multiuser MIMO systems, to design the linear beamforming/transmitting vectors and receiving vectors. With a general multiuser MIMO system model, we present three different design methods based on three different criteria:

- Method 1: Minimizing the effective TSC among all users;
- Method 2: Minimizing the total mean square error (MSE);
- Method 3: Maximizing the sum capacity of the system.

We derive in details the design methods under those different design criteria. Simulation results show that different methods demonstrate differently under those criteria.

The notations used in this paper are as follows. $\{\cdot\}^T$ and $\{\cdot\}^H$ denotes the transpose and Hermitian transpose operation, $Tr(\cdot)$ denotes the trace of a matrix. \mathbb{C}^n denotes the n dimensional complex field. \mathbf{I}_n denotes the identity matrix of size $n \times n$. Assume that the log operation is with respect to base 2. We use boldface lowercase letters to denote column vectors and boldface uppercase letters to denote matrices.

The rest of this paper is organized as follows. The multiuser MIMO system model with linear transmitting and receiving vectors is given in Section II. Method 1 aiming to minimize the effective TSC with matched filter is presented in Section III. Method 2 aiming to minimize the total MSE with MMSE filter is discussed in Section IV. Method 3 aiming to maximize the sum capacity is shown in Section V. Section VI is dedicated to experimental studies. A few concluding remarks are drawn in Section VII.

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II. SYSTEM MODEL

Consider a typical multiuser MIMO multiple access channels, where K users, each equipped with n_T antennas, communicating simultaneously to a base station (BS) with n_R antennas. Assuming a flat fading channel and the channel coefficients between user i and the BS are collected in an $n_R \times n_T$ matrix \mathbf{H}_i , $i = 1, \dots, K$. The system model of multiuser MIMO uplink channels is shown in Fig. 1. We assume perfect uplink synchronization and CSI knowledge at transmitters for research convenience.

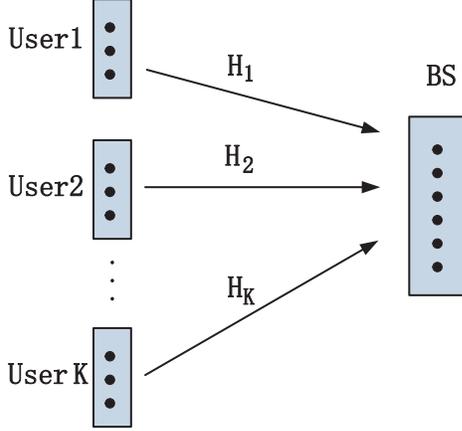


Fig. 1. System model

In the end of one transmission realization, the received signal vector \mathbf{r} at the BS is

$$\mathbf{r} = \sum_{i=1}^K \sqrt{p_i} \mathbf{H}_i \mathbf{s}_i + \mathbf{n}, \quad (1)$$

where $\mathbf{s}_i \in \mathbb{C}^{n_T}$ is the signal vector for user i ; p_i is the transmission power constraint for user i ; and \mathbf{n} represents additive noise vector satisfying $\mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$. All elements of channel matrix \mathbf{H}_i are generated i.i.d. according to a normal distribution $\mathcal{CN}(0, 1)$.

We consider the multiuser MIMO system with linear pre- and post-processing performed at the transmitter and receiver. Denote the transmit beamforming vector for user k as

$$\mathbf{w}_{T_k} \in \mathbb{C}^{n_T}, \quad \mathbf{w}_{T_k}^H \mathbf{w}_{T_k} = 1, \quad (2)$$

and the linear receive combining vector for user k as

$$\mathbf{w}_{R_k} \in \mathbb{C}^{n_R}, \quad \mathbf{w}_{R_k}^H \mathbf{w}_{R_k} = 1. \quad (3)$$

Then, at the transmitter, we have

$$\mathbf{s}_k = \mathbf{w}_{T_k} x_k, \quad (4)$$

where x_k is the transmission data for user k . The received signal vector at the BS becomes

$$\mathbf{r} = \sum_{i=1}^K \sqrt{p_i} \mathbf{H}_i \mathbf{w}_{T_i} x_i + \mathbf{n}, \quad (5)$$

and the processed signal for user k at the BS can be written as

$$y_k = \mathbf{w}_{R_k}^H \sum_{i=1}^K \sqrt{p_i} \mathbf{H}_i \mathbf{w}_{T_i} x_i + \mathbf{w}_{R_k}^H \mathbf{n} \quad (6)$$

$$= \sqrt{p_k} \mathbf{w}_{R_k}^H \mathbf{H}_k \mathbf{w}_{T_k} x_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} \mathbf{w}_{R_k}^H \mathbf{H}_i \mathbf{w}_{T_i} x_i + \mathbf{w}_{R_k}^H \mathbf{n}. \quad (7)$$

We are interested in design the linear beamforming vectors \mathbf{w}_{T_k} at the transmitters and also the receiving vectors \mathbf{w}_{R_k} , $k = 1, \dots, K$, in the following sections, we will discuss different design methods based on different design criteria.

III. METHOD 1: MINIMIZING EFFECTIVE TOTAL SQUARE CORRELATION WITH MATCHED FILTER

For the first design method in this section, we assume normalized matched filter (MF) receivers, i.e., the receiver filter of user k is

$$\mathbf{w}_{R_k}^{OPT1} = \frac{\mathbf{H}_k \mathbf{w}_{T_k}}{\|\mathbf{H}_k \mathbf{w}_{T_k}\|}. \quad (8)$$

In this case, the received signal at the output of receiver filter in (7) can be written as

$$y_k = \frac{(\mathbf{H}_k \mathbf{w}_{T_k})^H \mathbf{H}_k \mathbf{w}_{T_k} \sqrt{p_k} x_k}{\|\mathbf{H}_k \mathbf{w}_{T_k}\|} + \sum_{i=1, i \neq k}^K \frac{(\mathbf{H}_k \mathbf{w}_{T_k})^H \mathbf{H}_i \mathbf{w}_{T_i} \sqrt{p_i} x_i}{\|\mathbf{H}_k \mathbf{w}_{T_k}\|} + \mathbf{w}_{R_k}^H \mathbf{n}. \quad (9)$$

The received power of user k through its receiver is equivalent to

$$\begin{aligned} \bar{p}_k &\triangleq \left(\frac{(\mathbf{H}_k \mathbf{w}_{T_k})^H \mathbf{H}_k \mathbf{w}_{T_k} \sqrt{p_k}}{\|\mathbf{H}_k \mathbf{w}_{T_k}\|} \right)^2 \\ &= p_k (\mathbf{H}_k \mathbf{w}_{T_k})^H \mathbf{H}_k \mathbf{w}_{T_k}, \end{aligned} \quad (10)$$

and the interference power from user i to user k is equivalent to

$$\begin{aligned} I_{ki} &\triangleq \left(\frac{(\mathbf{H}_k \mathbf{w}_{T_k})^H \mathbf{H}_i \mathbf{w}_{T_i} \sqrt{p_i}}{\|\mathbf{H}_k \mathbf{w}_{T_k}\|} \right)^2 \\ &= \underbrace{p_i (\mathbf{H}_i \mathbf{w}_{T_i})^H \mathbf{H}_i \mathbf{w}_{T_i}}_{\bar{p}_i} \underbrace{\frac{|(\mathbf{H}_k \mathbf{w}_{T_k})^H \mathbf{H}_i \mathbf{w}_{T_i}|^2}{\|\mathbf{H}_k \mathbf{w}_{T_k}\|^2 \|\mathbf{H}_i \mathbf{w}_{T_i}\|^2}}_{\triangleq A_{ki}} \\ &= \bar{p}_i A_{ki}. \end{aligned} \quad (11)$$

Note that $A_{ii} = 1$.

Hence, the output signal to interference plus noise ratio (SINR) of receiver filter \mathbf{w}_{R_k} in (8) can be calculated as

$$\begin{aligned} SINR_k &= \frac{\bar{p}_k}{\sum_{i=1, i \neq k}^K I_{ki} + \sigma^2} \\ &= \frac{\bar{p}_k}{\sum_{i=1, i \neq k}^K \bar{p}_i A_{ki} + \sigma^2}. \end{aligned} \quad (12)$$

If we define the equivalent unit-energy transmitting vector for user i through channel \mathbf{H}_i as

$$\bar{\mathbf{w}}_{Ti} \triangleq \frac{\mathbf{H}_i \mathbf{w}_{Ti}}{\|\mathbf{H}_i \mathbf{w}_{Ti}\|}, \quad (13)$$

then the corresponding equivalent system of (1) is

$$\mathbf{r} = \sum_{i=1}^K \sqrt{p_i} \bar{\mathbf{w}}_{Ti} x_i + \mathbf{n}. \quad (14)$$

We define the effective total square correlation (TSC) of the equivalent beamforming vector set

$$[\bar{\mathbf{w}}_{T1}, \bar{\mathbf{w}}_{T2}, \dots, \bar{\mathbf{w}}_{TK}], \quad (15)$$

as follows,

$$\begin{aligned} TSC_{eff} &\triangleq \sum_{i,j=1}^K |\bar{\mathbf{w}}_{Ti}^H \bar{\mathbf{w}}_{Tj}|^2 \\ &= \sum_{i,j=1}^K \frac{|(\mathbf{H}_i \mathbf{w}_{Ti})^H \mathbf{H}_j \mathbf{w}_{Tj}|^2}{\|\mathbf{H}_i \mathbf{w}_{Ti}\|^2 \|\mathbf{H}_j \mathbf{w}_{Tj}\|^2} \\ &= \sum_{i,j=1}^K A_{ij}. \end{aligned} \quad (16)$$

Originally, TSC is defined for CDMA systems to evaluate the correlation between different signatures for multiple users. Here, the effective TSC is a description of correlation between equivalent transmitting vectors defined in (13) for all users.

We are seeking an algorithm that minimizes TSC_{eff} by updating one beamforming vector \mathbf{w}_{Tk} at a time. In order to do this, we first isolate the terms that depend on \mathbf{w}_{Tk} in TSC_{eff} as

$$\begin{aligned} &\arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} TSC_{eff} \\ &= \arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} \left(2 \sum_{j=1, j \neq k}^K |\bar{\mathbf{w}}_{Tk}^H \bar{\mathbf{w}}_{Tj}|^2 + \sum_{i \neq k, j \neq k} A_{ij} + K \right) \\ &= \arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} \sum_{j=1, j \neq k}^K |\bar{\mathbf{w}}_{Tk}^H \bar{\mathbf{w}}_{Tj}|^2 \\ &= \arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} \sum_{j=1, j \neq k}^K \frac{(\mathbf{H}_k \mathbf{w}_{Tk})^H \mathbf{H}_j \mathbf{w}_{Tj} (\mathbf{H}_j \mathbf{w}_{Tj})^H \mathbf{H}_k \mathbf{w}_{Tk}}{(\mathbf{H}_k \mathbf{w}_{Tk})^H \|\mathbf{H}_j \mathbf{w}_{Tj}\|^2 \mathbf{H}_k \mathbf{w}_{Tk}} \\ &= \arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} \frac{(\mathbf{H}_k \mathbf{w}_{Tk})^H \mathbf{F}_k \mathbf{H}_k \mathbf{w}_{Tk}}{(\mathbf{H}_k \mathbf{w}_{Tk})^H \mathbf{H}_k \mathbf{w}_{Tk}}, \end{aligned} \quad (17)$$

where

$$\mathbf{F}_k \triangleq \sum_{j=1, j \neq k}^K \frac{\mathbf{H}_j \mathbf{w}_{Tj} (\mathbf{H}_j \mathbf{w}_{Tj})^H}{\|\mathbf{H}_j \mathbf{w}_{Tj}\|^2}. \quad (18)$$

Therefore, as our first design criterion, to minimize the effective TSC (TSC_{eff}) regarding \mathbf{w}_{Tk} , it is equivalent to

$$\mathbf{w}_{Tk}^{OPT1} = \arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} TSC_{eff} \quad (19)$$

$$= \arg \min_{\mathbf{w}_{Tk}^H \mathbf{w}_{Tk}=1} \frac{\mathbf{w}_{Tk}^H \mathbf{H}_k^H \mathbf{F}_k \mathbf{H}_k \mathbf{w}_{Tk}}{\mathbf{w}_{Tk}^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_{Tk}}. \quad (20)$$

This is an generalized Rayleigh quotient problem that can be solved as

$$\mathbf{w}_{Tk}^{OPT1} = \frac{\mathbf{U}_1 \mathbf{D}_1^{-1/2} \mathbf{v}_1}{(\mathbf{v}_1^H \mathbf{D}_1^{-1} \mathbf{v}_1)^{1/2}}, \quad (21)$$

where \mathbf{U}_1 and \mathbf{D}_1 are the matrices with the eigenvectors and eigenvalues of the matrix $\mathbf{H}_k^H \mathbf{H}_k$, and \mathbf{v}_1 is the eigenvector of $\mathbf{D}_1^{-1/2} \mathbf{U}_1^H \mathbf{H}_k^H \mathbf{F}_k \mathbf{H}_k \mathbf{U}_1 \mathbf{D}_1^{-1/2}$ with minimum eigenvalue. Note that \mathbf{w}_{Tk} is scaled to have unit norm. When this update is applied to the transceiver beamforming vectors of all users sequentially, the TSC_{eff} decreases monotonically.

IV. METHOD 2: MINIMIZING THE TOTAL MSE WITH MMSE FILTER

In this section, we are considering to design the linear pre- and post-processing vectors under the criterion of minimizing the total system mean square error (MSE). First, we utilize the standard MMSE filter [22] for the k th user,

$$\begin{aligned} \mathbf{w}_{Rk}^{OPT2} &= \arg \min MSE_k \\ &= \arg \min E \left[|\mathbf{w}_{Rk}^H \mathbf{r} - x_k|^2 \right] \\ &= (\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}_k \mathbf{w}_{Tk}, \end{aligned} \quad (22)$$

where \mathbf{Z} is a $n_R \times n_R$ matrix with

$$\mathbf{Z} \triangleq \sum_{i=1}^K p_i \mathbf{H}_i \mathbf{w}_{Ti} \mathbf{w}_{Ti}^H \mathbf{H}_i^H. \quad (23)$$

The corresponding MSE for user k can be derived as

$$MSE_k = 1 - p_k \mathbf{w}_{Tk}^H \mathbf{H}_k^H (\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}_k \mathbf{w}_{Tk}. \quad (24)$$

Then, the total MSE for all user is

$$\begin{aligned} MMSE &\triangleq \sum_{k=1}^K MMSE_k \\ &= K - \sum_{k=1}^K p_k \mathbf{w}_{Tk}^H \mathbf{H}_k^H (\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}_k \mathbf{w}_{Tk} \\ &= K - \sum_{k=1}^K tr \left((\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} p_k \mathbf{H}_k \mathbf{w}_{Tk} \mathbf{w}_{Tk}^H \mathbf{H}_k^H \right) \\ &= K - tr \left((\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \sum_{k=1}^K p_k \mathbf{H}_k \mathbf{w}_{Tk} \mathbf{w}_{Tk}^H \mathbf{H}_k^H \right) \\ &= K - tr \left((\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \mathbf{Z} \right) \end{aligned} \quad (25)$$

We are trying to isolate the terms of MSE in (25) that depend on a particular beamforming vector \mathbf{w}_{Tk} and then look for ways of changing this vector to produce a smaller MSE. Exclude the item dependent on \mathbf{w}_{Tk} in \mathbf{Z} and define the matrix \mathbf{Z}_k which is independent of \mathbf{w}_{Tk} as

$$\begin{aligned} \mathbf{Z}_k &\triangleq \sum_{i=1, i \neq k}^K p_i \mathbf{H}_i \mathbf{w}_{Ti} \mathbf{w}_{Ti}^H \mathbf{H}_i^H + \sigma^2 \mathbf{I} \\ &= \mathbf{Z} + \sigma^2 \mathbf{I} - p_k \mathbf{H}_k \mathbf{w}_{Tk} \mathbf{w}_{Tk}^H \mathbf{H}_k^H \end{aligned} \quad (26)$$

Then, by matrix inverse lemma

$$\begin{aligned} (\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} &= (\mathbf{Z}_k + p_k \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H)^{-1} \\ &= \mathbf{Z}_k^{-1} - \frac{p_k \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1}}{1 + p_k \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k}}. \end{aligned} \quad (27)$$

Define a scalar x as

$$x \triangleq p_k \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k}. \quad (28)$$

Then with the knowledge of $Tr(\mathbf{X} + \mathbf{Y}) = Tr(\mathbf{X}) + Tr(\mathbf{Y})$ and $Tr(\mathbf{X}\mathbf{Y}) = Tr(\mathbf{Y}\mathbf{X})$, plug (26), (27) and (28) into equation (25), we will have the following result as shown in equation (29) of next page.

Therefore, as our second design criterion, to minimize total MSE regarding \mathbf{w}_{T_k} , it is equivalent to

$$\begin{aligned} \mathbf{w}_{T_k}^{OPT2} &= \arg \min_{\mathbf{w}_{T_k}^H \mathbf{w}_{T_k} = 1} MSE \\ &= \arg \max_{\mathbf{w}_{T_k}^H \mathbf{w}_{T_k} = 1} \frac{\mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-2} \mathbf{H}_k \mathbf{w}_{T_k}}{\mathbf{w}_{T_k}^H (\mathbf{I} + p_k \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k) \mathbf{w}_{T_k}}. \end{aligned} \quad (30)$$

This is again an generalized Rayleigh quotient problem that can be solved as

$$\mathbf{w}_{T_k}^{OPT2} = \frac{\mathbf{U}_2 \mathbf{D}_2^{-1/2} \mathbf{v}_2}{(\mathbf{v}_2^H \mathbf{D}_2^{-2} \mathbf{v}_2)^{1/2}}, \quad (31)$$

where \mathbf{U}_1 and \mathbf{D}_1 are the matrices with the eigenvectors and eigenvalues of the matrix $\mathbf{I} + p_k \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$, and \mathbf{v}_1 is the eigenvector of $\mathbf{D}_2^{-1/2} \mathbf{U}_2^H \mathbf{H}_k^H \mathbf{Z}_k^{-2} \mathbf{H}_k \mathbf{U}_2 \mathbf{D}_2^{-1/2}$ with maximum eigenvalue. The algorithm proceeds by repeating the above calculation for one user at a time, in a round-robin fashion; and MSE decreases monotonically.

V. METHOD 3: MAXIMIZING THE SUM CAPACITY

For the synchronous multiple access channel as (5), the information theoretic sum capacity of this channel is [23]

$$C_{sum} = \frac{1}{2} \log \det \left(\mathbf{I} + \frac{1}{\sigma^2} \mathbf{Z} \right), \quad (32)$$

with matrix \mathbf{Z} defined as equation (23).

Again we are trying to isolate the terms of C_{sum} in (32) that depend on a particular beamforming vector \mathbf{w}_{T_k} and then look for ways of changing this vector to produce a bigger sum capacity. Exclude the item dependent on \mathbf{w}_{T_k} in \mathbf{Z} and utilize the matrix \mathbf{Z}_k defined in (26) which is independent of \mathbf{w}_{T_k} , we have

$$\begin{aligned} C_{sum} &= \frac{1}{2} \log \det \left(\frac{\mathbf{Z}_k + p_k \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H}{\sigma^2} \right) \\ &= \frac{1}{2} \log \det \left(\frac{\mathbf{Z}_k}{\sigma^2} \right) + \frac{1}{2} \log \det (\mathbf{I} + p_k \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H) \\ &= \frac{1}{2} \log \det \left(\frac{\mathbf{Z}_k}{\sigma^2} \right) + \frac{1}{2} \log (1 + p_k \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k}) \end{aligned} \quad (33)$$

Therefore, as our third design criterion, to maximize the sum capacity C_{sum} regarding \mathbf{w}_{T_k} , it is equivalent to

$$\begin{aligned} \mathbf{w}_{T_k}^{OPT3} &= \arg \max_{\mathbf{w}_{T_k}^H \mathbf{w}_{T_k} = 1} C_{sum} \\ &= \arg \max_{\mathbf{w}_{T_k}^H \mathbf{w}_{T_k} = 1} p_k \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \\ &= \mathbf{v}_3 \end{aligned} \quad (34)$$

where \mathbf{v}_3 is the eigenvector of matrix $\mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k$ with the largest eigenvalue. When this update is repeated by users sequentially, C_{sum} increases monotonically.

We note that, although the three methods are optimal in design linear beamforming vector for one specific user under some criterion, in the multiuser optimization, the transmit beamforming vectors of different users' might be coupled with each other. We optimize the transmit beamforming vectors of all users under adaptation cycles. In each adaptation cycle, the transmitting/beamforming vectors will updated in a sequential user-after-user manner. Hence, the resulting multiuser beamforming vectors might only be a suboptimal solution.

VI. EXPERIMENTAL STUDIES

In this section, we present numerical results to evaluate the performance of the proposed three linear transceiver and receiver design methods under different criteria. First, we discuss the performance of three design methods with respect to effective TSC defined in (16), which describes the correlation between equivalent beamforming vectors. We set up the system model for simulation as two transmitting antennas for each user, four receiving antennas for the BS, and three users in total in the system, i.e., $n_T = 2$, $n_R = 4$, $K = 3$. The power constraint for each user is set up as $p_i \leq 15dB$. We average over 1000 randomly generated channel realizations and show the results in Fig. 2. In each adaptation cycle, the transmitting/beamforming vectors will updated in a sequential user-after-user manner. We draw the results in Fig. 2 with effective TSC of different methods versus the adaptation cycles. As we can see, Method 1, which is designed under the effective TSC minimization criterion, gives the best performance over the other two methods, as expected.

In Fig. 3, we show the performance of three design methods with respect to total MSE defined in (25) versus adaptation cycles, under the same simulation setup. We can see that under this criterion, Method 1, which gives the best performance under effective TSC criterion, demonstrates inferior result compared to the other two methods. Method 2, which is designed specially for minimizing the total MSE, gives the best performance. Method 3 performs comparably.

In Fig. 4, we show the performance of three design methods with respect to sum capacity defined in (32) versus adaptation cycles, under the same simulation setup. Method 3, which is designed particularly to maximizing the sum capacity, is superior to the other two methods, as expected.

$$\begin{aligned}
MMSE &= K - \text{tr} \left((\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \mathbf{Z} \right) \\
&= K - \text{tr} \left(\left(\mathbf{Z}_k^{-1} - \frac{p_k \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1}}{1+x} \right) (\mathbf{Z}_k + p_k \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H - \sigma^2 \mathbf{I}) \right) \\
&= K - \text{tr} \left(\mathbf{I} - \frac{p_k \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H}{1+x} + \mathbf{Z}_k^{-1} p_k \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H - \frac{p_k \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} p_k \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H}{1+x} \right. \\
&\quad \left. - \sigma^2 \mathbf{Z}_k^{-1} + \sigma^2 \frac{p_k \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k} \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1}}{1+x} \right) \\
&= K - n_R - \frac{x}{1+x} - x + \frac{x^2}{1+x} + \text{tr} (\sigma^2 \mathbf{Z}_k^{-1}) - \sigma^2 \frac{\mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-2} \mathbf{H}_k \mathbf{w}_{T_k}}{1+x} \\
&= K - n_R + \text{tr} (\sigma^2 \mathbf{Z}_k^{-1}) - \sigma^2 \frac{\mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-2} \mathbf{H}_k \mathbf{w}_{T_k}}{1 + p_k \mathbf{w}_{T_k}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \mathbf{H}_k \mathbf{w}_{T_k}}
\end{aligned} \tag{29}$$

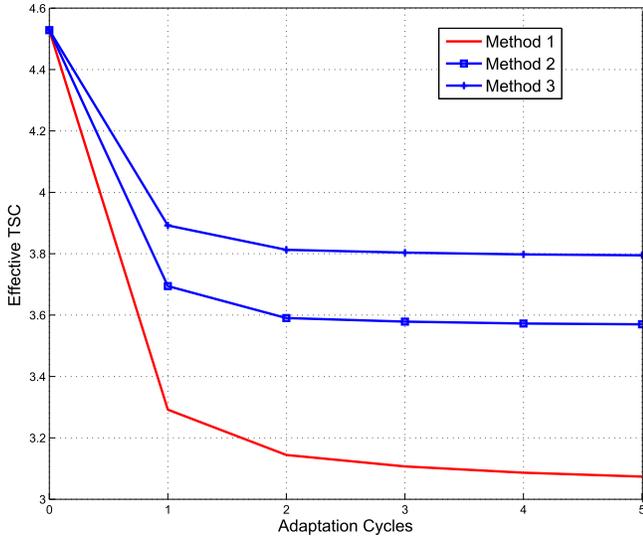


Fig. 2. Comparison of three methods regarding effective TSC

VII. CONCLUSIONS

In this paper, we discuss three design methods for linear transmitter/beamforming and receiver in multiuser MIMO communication systems. The three methods are under different optimization criteria: minimizing the effective TSC, minimizing the total MSE and maximizing the sum capacity. Under different design criterion, we give the detailed derivation of linear transmitter and receiver design methods. Simulation results show that different methods demonstrate differently under those criteria.

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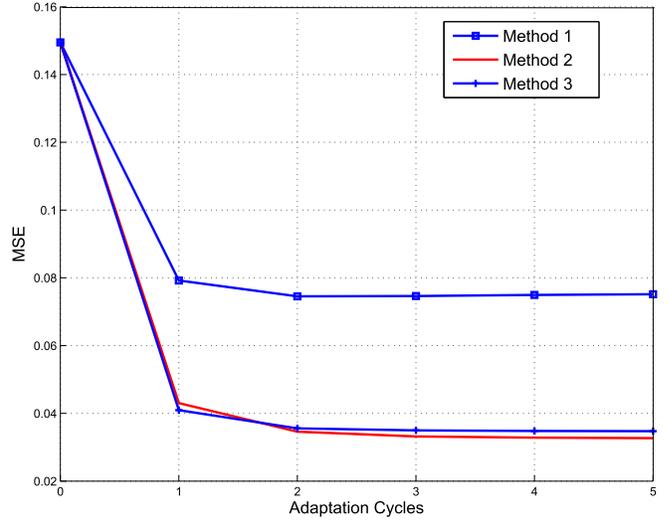


Fig. 3. Comparison of three methods regarding MSE

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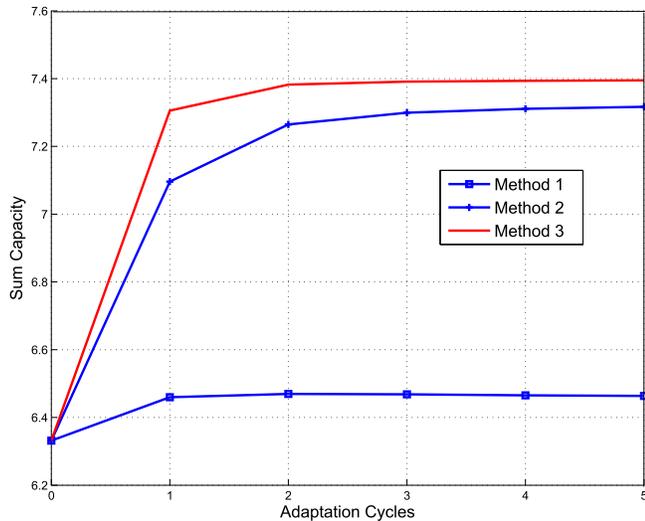


Fig. 4. Comparison of three methods regarding Sum Capacity

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