

Network Coding in Wireless Cooperative Networks with Multiple Antenna Relays[†]

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Abstract—Network coding is a technique in cooperative networks by allowing intermediate nodes to mix messages from multiple sources. In this paper, we consider network coding application in wireless multiple access relay channels with and without direct links. The relays are equipped with multiple antennas for both system models. We investigate several different network coding techniques applicable for each system model, including analog network coding (ANC), space-time analog network coding (STANC), physical-layer network coding (PLNC), space-time decode-and-forward (STDF), etc. We describe in details those different schemes in two system models with transmission time slots constraints, and compare the error rate performance. Interestingly, simulation studies show that those schemes with network coding have not better performance than the schemes with decode-and-forward (DF).

Index Terms—multiple access relay channel, network coding, cooperative, space-time coding.

I. INTRODUCTION

In the past decade, network coding (NC) [1] has rapidly emerged as a major research area in electrical engineering and computer science. Originally designed for wired networks, network coding is a generalized routing approach that breaks the traditional assumption of simply forwarding data, and allows intermediate nodes to send out functions of their received packets, by which the multicast capacity given by the max-flow min-cut theorem can be achieved. Subsequent works of [2]-[4] made the important observation that, for multicasting, intermediate nodes can simply send out a linear combination of their received packets. Linear network coding with random coefficients is considered in [5].

In order to address the broadcast nature of wireless transmission, physical layer network coding (PLNC) [6] was proposed to embrace interference in wireless networks in which intermediate nodes attempt to decode the modulo-two sum (XOR) of the transmitted messages. Compute-and-forward network coding, based on the linear structure of lattice codes, is proposed in [7]-[8] and subsequent works follow in [9]-[10]. Analog network coding (ANC) is presented in [11] where relays simply amplify-and-forward received mixed signals. Several other network coding realizations in wireless networks are discussed in [12]-[14].

Regarding network coding in wireless multiple access relay channels, throughput analysis is given in [15] under collision

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model. Complex field network coding is presented in [16]. Analog network coding mappings in multiple access relay channels with direct links and without direct links is discussed in [17]. Multiple-access relay channels with compute-and-forward relays are studied in [18]-[19]. Regarding network coding with MIMO space time coding technique, Alamouti scheme [20] is applied to decode-and-forward (DF) network coding for two-way relay channels with multiple antenna relay in [21]-[22]. Space-time analog network coding (S-TANC), which combines analog network coding with space-time coding techniques for multi-way relaying system has been proposed in [23].

In this paper, we consider network coding application in wireless multiple access relay channels. Two system model is considered:

- (i) System model A: Multiple access relay channel without direct links;
- (ii) System model B: Multiple access relay channel with direct links.

The relays are equipped with multiple antennas for both system models. We investigate several different network coding techniques applicable for each system model, including analog network coding (ANC), space-time analog network coding (STANC), physical-layer network coding (PLNC), space-time decode-and-forward (STDF), etc. We describe in details those different schemes in two system models with transmission time slots constraints, and compare the error rate performance.

Interestingly, we find that under three time slots constraint, space-time decode-and-forward (STDF) gives superior performance than decode-and-forward (DF), analog network coding (ANC) and space-time analog network coding (S-TANC) schemes for system model A; while under two time slots constraint, the simply decode-and-forward (DF) scheme outperforms the direct transmission, physical layer network coding (PLNC) and analog network coding (ANC) schemes for system model B.

The rest of this paper is organized as follows. Section II presents four different schemes under three time slots constraint for system A and Section III presents four different schemes under two time slots constraint for system B. Simulation studies is given in Section IV. A few concluding remarks are drawn in Section VI.

II. SYSTEM MODEL A

Consider cooperative system model A: multiple access relay channels without direct links. Two sources $\mathcal{S}_1, \mathcal{S}_2$ communicate with destination \mathcal{D} via a relay \mathcal{R} without direct links from sources to destination, as shown in Fig. 1. We assume the sources $\mathcal{S}_1, \mathcal{S}_2$ and the destination \mathcal{D} are equipped with single antenna, while relay \mathcal{R} is equipped with two antennas.

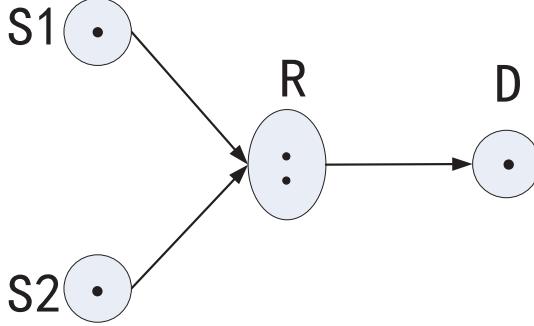


Fig. 1. System model A without direct links

The information transmission is performed in two phases with three time slots in total. In the first phase two source nodes transmit simultaneously to relay \mathcal{R} in one time slot; while in the second phase relay \mathcal{R} transmits to destination \mathcal{D} in the remaining two time slots.

The received signal at relay \mathcal{R} at the end of first phase is

$$\mathbf{y}_R = \begin{bmatrix} y_{R1} \\ y_{R2} \end{bmatrix} = \sqrt{E_x} \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \mathbf{n}_R, \quad (1)$$

where $\mathbf{y}_R = [y_{R1}, y_{R2}]^T$ is the received vector at relay with two antennas; x_i is the transmit data symbol¹ from node \mathcal{S}_i which has been normalized to $E\{|x_i|^2\} = 1$; E_x is the power constraint for data symbol transmission. The transmitting data vector of two sources is denoted as

$$\mathbf{x} = [x_1, x_2]^T, \quad (2)$$

and $\mathbf{x} \in \Omega_{\mathbf{x}}$, where $\Omega_{\mathbf{x}}$ is the data vector alphabet set; $\mathbf{n}_R = [n_{R1}, n_{R2}]^T$ is the additive Gaussian noise vector at relay; h_{ri} is the channel coefficient between source node \mathcal{S}_i and relay antenna r and we define

$$\mathbf{h}_r \triangleq [h_{r1}, h_{r2}]^T. \quad (3)$$

All channel coefficients and additive noise elements are generated i.i.d. according to a normal distribution $\mathcal{CN}(0, 1)$.

In the second phase, relay \mathcal{R} will transmit to the destination \mathcal{D} according to different schemes. As no direct links exist in system model A, physical layer network coding (PLNC) cannot be applied in system model A directly. Note that all schemes take three time slots for one transmission realization.

¹For source \mathcal{S}_i , x_i is the transmitted symbol after modulation based on the transmitted bit b_i .

A. Model A Scheme 1: Decode-and-Forward (DF)

With this scheme, after receiving \mathbf{y}_R in (1), relay \mathcal{R} will first decode for two sources

$$\hat{\mathbf{x}}_R = \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_R - \sqrt{E_x} \mathbf{H} \mathbf{x}\|^2. \quad (4)$$

Then, relay \mathcal{R} will transmit \hat{x}_{R1} and \hat{x}_{R2} in two time slots as follows,

$$\begin{aligned} \mathbf{y}_D &= \sqrt{E_R} \begin{bmatrix} \hat{x}_{R1} & 0 \\ 0 & \hat{x}_{R2} \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} + \mathbf{n}_D \\ &= \underbrace{\sqrt{E_R} \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}}_{\mathbf{G}_1} \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix} + \mathbf{n}_D, \end{aligned} \quad (5)$$

which is equivalent to

$$\mathbf{y}_D = \sqrt{E_R} \mathbf{G}_1 \hat{\mathbf{x}}_R + \mathbf{n}_D, \quad (6)$$

where g_r , $r = 1, 2$ is the channel coefficient between relay antenna r and destination \mathcal{D} .

The decoding procedure at destination \mathcal{D} will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \sqrt{E_R} \mathbf{G}_1 \mathbf{x}\|^2. \quad (7)$$

B. Model A Scheme 2: Space Time Decode-and-Forward (STDF)

In this scheme, relay \mathcal{R} will first decode for two sources the same way as scheme 1, equation (4), then transmit \hat{x}_{R1} and \hat{x}_{R2} according to Alamouti space time coding [20].

In the second time slot, the relay will transmit $[\hat{x}_{R1}, \hat{x}_{R2}]^T$ and in the third time slot, the relay will transmit $[-\hat{x}_{R1}^*, \hat{x}_{R2}^*]^T$. Denote the corresponding received signals at destination \mathcal{D} in the second phase (with two time slots) as y_{D1} and y_{D2} , then

$$[y_{D1}, y_{D2}] = \sqrt{E_R} [g_1, g_2] \begin{bmatrix} \hat{x}_{R1} & -\hat{x}_{R2}^* \\ \hat{x}_{R2} & \hat{x}_{R1}^* \end{bmatrix} + [n_{D1}, n_{D2}]. \quad (8)$$

After receiving signals from relay \mathcal{R} in the second phase, destination \mathcal{D} arranges the received signals into a vector $\mathbf{y}_D = [y_{D1}, -y_{D2}^*]^T$, which can be rewritten as

$$\begin{aligned} \mathbf{y}_D &= \begin{bmatrix} y_{D1} \\ -y_{D2}^* \end{bmatrix} \\ &= \sqrt{E_R} \underbrace{\begin{bmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{bmatrix}}_{\mathbf{G}_2} \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix} + \mathbf{n}_D \\ &= \sqrt{E_R} \mathbf{G}_2 \hat{\mathbf{x}}_R + \mathbf{n}_D, \end{aligned} \quad (9)$$

where $\mathbf{n}_D = [n_{D1}, -n_{D2}^*]^T$.

The decoding procedure at destination \mathcal{D} will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \sqrt{E_R} \mathbf{G}_2 \mathbf{x}\|^2. \quad (10)$$

C. Model A Scheme 3: Analog Network Coding (ANC)

In this scheme, relay \mathcal{R} will utilize analog network coding to process the received signals. First, after receiving \mathbf{y}_R of (1), relay \mathcal{R} constructs the following signal vector $\mathbf{t} = [t_1, t_2]^T$ based on the received signals on each antenna

$$\mathbf{t} = \begin{bmatrix} \beta_1 y_{R1} \\ \beta_2 y_{R2} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}}_{\mathbf{B}} (\sqrt{E_x} \mathbf{Hx} + \mathbf{n}_R), \quad (11)$$

where $\beta_r, r = 1, 2$ is the scaling factor to meet the per-antenna power constraint P_R at relay \mathcal{R} given by

$$\beta_r = \sqrt{\frac{1}{E\{|y_{Rr}|^2\}}} = \sqrt{\frac{1}{E_x \|\mathbf{h}_r\|^2 + 1}}. \quad (12)$$

Then, relay \mathcal{R} will transmit t_1 and t_2 in two time slots as follows,

$$\begin{aligned} \mathbf{y}_D &= \sqrt{E_R} \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} + \mathbf{n}_D \\ &= \underbrace{\sqrt{E_R} \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}}_{\mathbf{G}_1} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + \mathbf{n}_D \\ &= \sqrt{E_R} \sqrt{E_x} \mathbf{G}_1 \mathbf{B} \mathbf{H} \mathbf{x} + \sqrt{E_R} \mathbf{G}_1 \mathbf{B} \mathbf{n}_R + \mathbf{n}_D \end{aligned} \quad (13)$$

The decoding procedure at destination \mathcal{D} will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \sqrt{E_R} \sqrt{E_x} \mathbf{G}_1 \mathbf{B} \mathbf{H} \mathbf{x}\|^2. \quad (14)$$

D. Model A Scheme 4: Space Time Analog Network Coding (STANC)

In this scheme, we consider combine analog network coding with Alamouti space time coding. After constructing $\mathbf{t} = [t_1, t_2]^T$ as equation (11), relay \mathcal{R} will transmit $[t_1, t_2]^T$ in the second time slot and $[-t_2^*, t_1^*]^T$ in the third time slot. Denote the corresponding received signals at destination \mathcal{D} in the second phase (with two time slots) as y_{D1} and y_{D2} , then

$$[y_{D1}, y_{D2}] = \sqrt{E_R} [g_1, g_2] \begin{bmatrix} t_1 & -t_2^* \\ t_2 & t_1^* \end{bmatrix} + [n_{D1}, n_{D2}], \quad (15)$$

where $g_r, r = 1, 2$ is the channel coefficient between relay antenna r and destination \mathcal{D} .

After receiving signals from relay \mathcal{R} in the second phase, destination \mathcal{D} arranges the received signals into a vector $\mathbf{y}_D = [y_{D1}, -y_{D2}^*]^T$, which can be rewritten as

$$\begin{aligned} \mathbf{y}_D &= \begin{bmatrix} y_{D1} \\ -y_{D2}^* \end{bmatrix} \\ &= \sqrt{E_R} \underbrace{\begin{bmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{bmatrix}}_{\mathbf{G}_2} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + \mathbf{n}_D \\ &= \sqrt{E_R} \sqrt{E_x} \mathbf{G}_2 \mathbf{B} \mathbf{H} \mathbf{x} + \sqrt{E_R} \mathbf{G}_2 \mathbf{B} \mathbf{n}_R + \mathbf{n}_D \end{aligned} \quad (16)$$

where $\mathbf{n}_D = [n_{D1}, -n_{D2}^*]^T$.

The decoding procedure at destination \mathcal{D} will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \sqrt{E_R} \sqrt{E_x} \mathbf{G}_2 \mathbf{B} \mathbf{H} \mathbf{x}\|^2. \quad (17)$$

The comparison of four schemes for system model A in three slots are shown in Table 1.

Table 1: Different Schemes for System Model A

Model A	Time Slot 1	Time Slot 2	Time Slot 3
DF	$S1 : x_1$ $S2 : x_2$	$R : \begin{bmatrix} \hat{x}_{R1} \\ 0 \end{bmatrix}$	$R : \begin{bmatrix} 0 \\ \hat{x}_{R2} \end{bmatrix}$
STDF	$S1 : x_1$ $S2 : x_2$	$R : \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix}$	$R : \begin{bmatrix} -\hat{x}_{R2}^* \\ \hat{x}_{R1}^* \end{bmatrix}$
ANC	$S1 : x_1$ $S2 : x_2$	$R : \begin{bmatrix} t_1 \\ 0 \end{bmatrix}$	$R : \begin{bmatrix} 0 \\ t_2 \end{bmatrix}$
STANC	$S1 : x_1$ $S2 : x_2$	$R : \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$	$R : \begin{bmatrix} -t_2^* \\ t_1^* \end{bmatrix}$

III. SYSTEM MODEL B

Consider cooperative system model B: multiple access relay channels with direct links. Two sources $\mathcal{S}_1, \mathcal{S}_2$ communicate with destination \mathcal{D} via relay \mathcal{R} with direct links from sources to destination, as shown in Fig. 2. We also assume the sources $\mathcal{S}_1, \mathcal{S}_2$ and the destination \mathcal{D} are equipped with single antenna, while relay \mathcal{R} is equipped with two antennas.

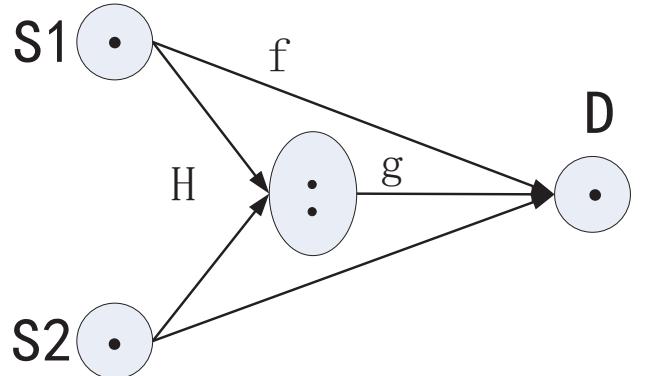


Fig. 2. System model B with direct links

One realization of the information transmission is performed in two time slots. We will describe in details the different four possible schemes, which all take two time slots for one transmission realization. Since we constrain the transmission is performed within two time slots, space-time coding cannot be directly applied to system model B directly.

A. Model B Scheme 1: Direct transmission

In this scheme, we assume the relay will keep silent during all transmission realization and the sources will communicate to the destination one by one directly. \mathcal{S}_1 will transmit in the first time slot; \mathcal{S}_2 will transmit in the second time slot.

$$\begin{aligned} y_{D1} &= \sqrt{E_x} f_1 x_1 + n_{D1}, \\ y_{D2} &= \sqrt{E_x} f_2 x_2 + n_{D2}, \end{aligned}$$

which can be combined to $\mathbf{y}_D = [y_{D1}, y_{D2}]^T$ as

$$\mathbf{y}_D = \sqrt{E_x} \underbrace{\begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{D1} \\ n_{D2} \end{bmatrix}}_{\mathbf{n}_D}, \quad (18)$$

or equivalently

$$\mathbf{y}_D = \sqrt{E_x} \mathbf{F} \mathbf{x} + \mathbf{n}_D. \quad (19)$$

f_i is the direct link channel coefficient between source \mathcal{S}_i to destination \mathcal{D} ; n_{Di} is the additive Gaussian noise at the i -th time slot. All channel coefficients are generated i.i.d. according to a normal distribution $\mathcal{N}(0, 1)$.

Hence the decoding procedure at destination \mathcal{D} will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \sqrt{E_x} \mathbf{F} \mathbf{x}\|^2. \quad (20)$$

B. Model B Scheme 2: Decode-and-Forward (DF)

In this scheme, two sources will transmit to relay \mathcal{R} and destination \mathcal{D} simultaneously in the first time slot, while in the second time slot relay \mathcal{R} will transmit the decoded signals to destination \mathcal{D} .

The received signal at destination \mathcal{D} at the end of first time slot is

$$y_{D1} = \sqrt{E_x} f_1 x_1 + \sqrt{E_x} f_2 x_2 + n_{D1}. \quad (21)$$

The received vector at relay \mathcal{R} with two antennas at the end of first time slot is

$$\mathbf{y}_R = \sqrt{E_x} \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{R1} \\ n_{R2} \end{bmatrix}, \quad (22)$$

where h_{ri} is the channel coefficient between source node \mathcal{S}_i and relay antenna r ; n_{Ri} , $i = 1, 2$ is the additive Gaussian noise. Let

$$\mathbf{H} \triangleq [\mathbf{h}_1, \mathbf{h}_2]^T, \quad (23)$$

in other words, \mathbf{h}_i^T is the i -th row vector of matrix \mathbf{H} .

After receiving signals as (22), the relay will first decode for two sources

$$\hat{\mathbf{x}}_R = \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_R - \sqrt{E_x} \mathbf{H} \mathbf{x}\|^2. \quad (24)$$

Then, with two antennas and power constraint E_R , relay \mathcal{R} will transmit $[\hat{x}_{R1}, \hat{x}_{R2}]^T$. The received signal at destination \mathcal{D} in the second time slot is,

$$y_{D2} = \sqrt{E_R} g_1 \hat{x}_{R1} + \sqrt{E_R} g_2 \hat{x}_{R2} + n_{D2}. \quad (25)$$

where g_r , $r = 1, 2$, is the channel coefficient between relay antenna r and destination \mathcal{D} .

Recall the received signals in the first time slot (21) and in the second time slot (25) at destination \mathcal{D} , we have

$$\begin{cases} y_{D1} = \sqrt{E_x} [f_1, f_2] \mathbf{x} + n_{D1}, \\ y_{D2} = \sqrt{E_R} [g_1, g_2] \hat{\mathbf{x}}_R + n_{D2}. \end{cases} \quad (26)$$

If we construct the matrix \mathbf{A}_1 as

$$\mathbf{A}_1 \triangleq \begin{bmatrix} \sqrt{E_x} f_1 & \sqrt{E_x} f_2 \\ \sqrt{E_R} g_1 & \sqrt{E_R} g_2 \end{bmatrix}, \quad (27)$$

then the decoding procedure will be

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \mathbf{A}_1 \mathbf{x}\|^2. \quad (28)$$

C. Model B Scheme 3: Physical Layer Network Coding (PLNC)

In this scheme, two sources will also transmit to relay \mathcal{R} and destination \mathcal{D} simultaneously in the first time slot. Relay \mathcal{R} will decode for the two sources at the end of first time slot. The procedure is the same as (21)-(24). Then relay \mathcal{R} will calculate $\hat{x}_{R1} \oplus \hat{x}_{R2}$ ². For BPSK modulation, we will have the following relationship.

Table 2: $x_1 \oplus x_2$ for BPSK modulation

x_1	x_2	$x_1 + x_2$	$x_1 \oplus x_2$	$x_1 * x_2$
-1	-1	-2	-1	1
-1	1	0	1	-1
1	-1	0	1	-1
1	1	2	-1	1

It is easily to conclude that

$$\hat{x}_{R1} \oplus \hat{x}_{R2} = -\hat{x}_{R1} * \hat{x}_{R2}. \quad (29)$$

According to digital network coding strategy, with two antennas and power constraint E_R , the relay will transmit $[\hat{x}_{R1} \oplus \hat{x}_{R2}, \hat{x}_{R1} \oplus \hat{x}_{R2}]^T$ in the second time slot,

$$\begin{aligned} y_{D2} &= \sqrt{E_R} [g_1, g_2] \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R1} \oplus \hat{x}_{R2} \end{bmatrix} + n_{D2} \\ &= \sqrt{E_R} (g_1 + g_2)(-\hat{x}_{R1} * \hat{x}_{R2}) + n_{D2}. \end{aligned} \quad (30)$$

²The relay actually first demodulates \hat{x}_{Ri} to information bit \hat{b}_{Ri} , then calculate $\hat{b}_{R1} \oplus \hat{b}_{R2}$, and finally modulates them again. We simply denote the modulated $\hat{b}_{R1} \oplus \hat{b}_{R2}$ as $\hat{x}_{R1} \oplus \hat{x}_{R2}$.

Recall the received signals in the first phase (21) and in the second phase (30) at destination \mathcal{D} , we have

$$\begin{cases} y_{D1} = \sqrt{E_x}f_1x_1 + \sqrt{E_x}f_2x_2 + n_{D1} \\ y_{D2} = \sqrt{E_R}(g_1 + g_2)(-\hat{x}_{R1} * \hat{x}_{R2}) + n_{D2}, \end{cases} \quad (31)$$

and will decode $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2]^T$ as

$$\hat{\mathbf{x}} = \arg \min_{x_1, x_2} \|y_{D1} - \sqrt{E_x}f_1x_1 - \sqrt{E_x}f_2x_2\|^2 + \|y_{D2} + \sqrt{E_R}(g_1 + g_2)(x_1 * x_2)\|^2. \quad (32)$$

D. Model B Scheme 4: Analog Network Coding (ANC)

In this scheme, relay \mathcal{R} will utilize analog network coding to process the received signals. First, after receiving \mathbf{y}_R of (22), relay \mathcal{R} constructs the signal vector $\mathbf{t} = [t_1, t_2]^T$ as follows,

$$\mathbf{t} = \begin{bmatrix} \beta_1 y_{R1} \\ \beta_2 y_{R2} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}}_{\mathbf{B}} (\sqrt{E_x} \mathbf{H} \mathbf{x} + \mathbf{n}_R), \quad (33)$$

where $\beta_r, r = 1, 2$ is the scaling factor at relay \mathcal{R} given by

$$\beta_r = \sqrt{\frac{1}{E\{|y_{Rr}|^2\}}} = \sqrt{\frac{1}{E_x \|\mathbf{h}_r\|^2 + 1}}. \quad (34)$$

Then, relay \mathcal{R} will transmit t_1, t_2 in the second time slot as follows,

$$\begin{aligned} y_{D2} &= \sqrt{E_R}[g_1, g_2] \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + n_{D2} \\ &= \sqrt{E_R} \sqrt{E_x} \mathbf{g}^T \mathbf{B} \mathbf{H} \mathbf{x} + \sqrt{E_R} \mathbf{g}^T \mathbf{B} \mathbf{n}_R + n_{D2}, \end{aligned} \quad (35)$$

where

$$\mathbf{g} \triangleq [g_1, g_2]^T, \quad (36)$$

is the channel vector between relay \mathcal{R} with two antennas to destination \mathcal{D} .

Combining the received signals in the first time slot (21) and in the second time slot (35) at destination \mathcal{D} , with $\mathbf{f} \triangleq [f_1, f_2]^T$, we have

$$\mathbf{y}_D = \underbrace{\sqrt{E_x} \begin{bmatrix} \mathbf{f}^T \\ \sqrt{E_R} \mathbf{g}^T \mathbf{B} \mathbf{H} \end{bmatrix}}_{\mathbf{A}_2} \mathbf{x} + \underbrace{\begin{bmatrix} n_{D1} \\ \sqrt{E_R} \mathbf{g}^T \mathbf{B} \mathbf{n}_R + n_{D2} \end{bmatrix}}_{\mathbf{z}}. \quad (37)$$

which can be decoded as

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \|\mathbf{y}_D - \mathbf{A}_2 \mathbf{x}\|^2. \quad (38)$$

The comparison of four schemes for system model B in three slots are shown in Table 3.

Table 3: Different Schemes for System Model B

Model B	Time Slot 1	Time Slot 2
Direct	$S1 : x_1$	$S2 : x_2$
DF	$S1 : x_1$	$R : \begin{bmatrix} \hat{x}_{R1} \\ \hat{x}_{R2} \end{bmatrix}$
PLNC	$S1 : x_1$	$R : \begin{bmatrix} \hat{x}_{R1} \oplus \hat{x}_{R2} \\ \hat{x}_{R1} \oplus \hat{x}_{R2} \end{bmatrix}$
ANC	$S1 : x_1$	$R : \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$

IV. SIMULATION STUDIES

In this section, we present numerical results to evaluate the performance of all possible schemes for system model A and system model B. Let $E_x = E_R$, i.e., the transmission power constraint at sources and relay are equivalent. With the average of 100000 randomly generated channel realizations, we show in Fig. 3 the error rate comparisons of four possible schemes for system model A (multiple access relay channel without direct links). The error rate is for the transmission signal vector \mathbf{x} defined in (2). All schemes are under three time slots constraint.

We can observe that space time coding technique improves the performance, i.e., STDF scheme outperforms DF scheme and STANC scheme outperforms ANC scheme. Also, the STDF scheme gives the best performance, which means, if the relay has the ability to decode, then, using STDF is a better choice among other schemes.

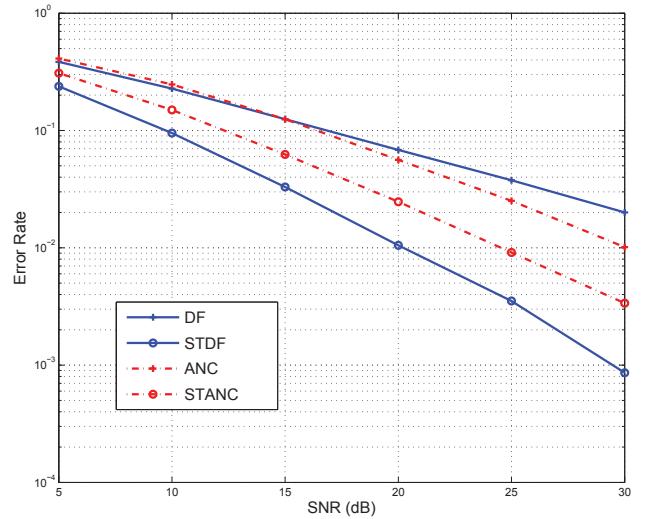


Fig. 3. Comparison of four schemes for system model A

Then, we investigate the performance of all possible schemes for system model B (multiple access relay channel with direct links) with error rate comparisons given in Fig. 4.

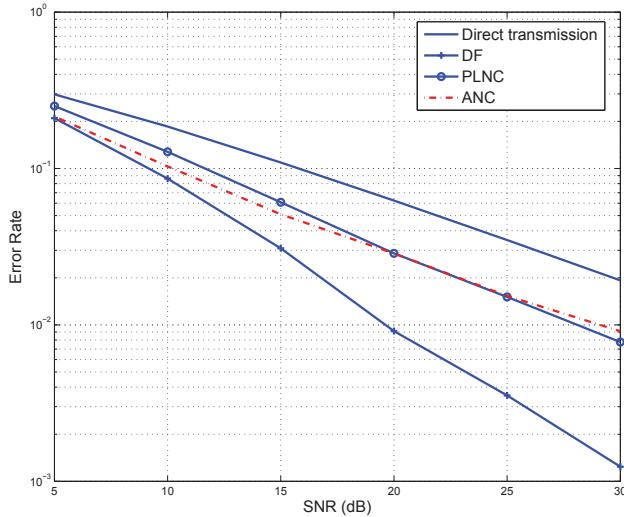


Fig. 4. Comparison of four schemes for system model B

All schemes are under two time slots constraint. Interestingly, we find that the schemes of PLNC and ANC both give inferior performance to the simply DF scheme, which means, when relay is equipped with multiple antennas, simply transmit decoded messages onto different antennas outperforms mixing signals as in PLNC and ANC schemes.

V. CONCLUSIONS

In this paper, we consider two wireless cooperative system models: system model A as multiple access relay channel without direct links; system model B as multiple access relay channel with direct links. The relays are equipped with multiple antennas for both models. For each system model, we consider four possible transmission schemes to possibly combining network coding and space-time coding techniques. Interestingly, we find that under three time slots constraint, space-time decode-and-forward (STDF) gives superior performance than decode-and-forward (DF), analog network coding (ANC) and space-time analog network coding (STANC) schemes for system model A; while under two time slots constraint, the simply decode-and-forward (DF) scheme outperforms the direct transmission, physical layer network coding (PLNC) and analog network coding (ANC) schemes for system model B.

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