Polar Coding for Noncoherent Block Fading Channels

Mengfan Zheng, Wen Chen
Shanghai Institute of Advanced Communications and Data Sciences,
Dept. of Electronic Engineering, Shanghai Jiao Tong University,
Shanghai, P. R. China
Emails: {zhengmengfan, wenchen}@sjtu.edu.cn

Cong Ling
Dept. of Electrical and Electronic Engineering,
Imperial College London,
United Kingdom
Email: c.ling@imperial.ac.uk

***Abstract*—In this paper, we consider the problem of polar coding for block fading channels without any instantaneous channel state information (CSI). We first decompose a block fading channel of T_c symbols per coherent interval into T_c binary-input sub-channels in a mutual-information-preserving way, and then design a multilevel (MLC) polar coding scheme for them. The proposed scheme achieves the ergodic mutual information of binary-input block fading channels with only channel distribution information (CDI). Simulations results are presented to compare the performance of the proposed MLC scheme and polar codes designed for i.i.d. fading channels with interleaving, which can provide some guidance for practical designs.**

I. Introduction

The fading channel is a widely adopted time-varying model for real-world wireless communications. In this model, the channel gain changes over time satisfying a certain distribution, called the channel distribution information (CDI). In a block fading channel model, the channel gain is assumed to be constant over a fixed time interval T_c, known as the coherent time, and change to a new independent value afterwards. In many of today's communication systems, channel estimation is performed in the first place to obtain the instantaneous channel state information (CSI), and then data transmission follows. However, in many scenarios, the coherence time is very short (e.g., only a few symbol intervals). In this case, channel estimation may significantly lower the overall data rate. Besides, the estimation precision is quite limited. Consequently, communication without instantaneous CSI (or noncoherent communication) is preferable.

Polar codes are the first family of codes that provably achieves the capacity of any binary-input discrete memoryless channels (B-DMC) with low encoding and decoding complexity [1]. There have been studies on polar coding for fading channels under various CSI assumptions. In [2], polar coding for quasi-static fading channels with two states was studied. Polar coding for block fading channels with full CSI and i.i.d. fading channels with CDI was considered in [3]. For block fading binary symmetric and additive exponential noise channels with CSI at the receiver (CSI-R), a hierarchical polar coding scheme was proposed in [4], which achieves capacity, but only works for block fading channels with finite states. A simple method for construction of polar codes for Rayleigh fading channel was presented in [5]. Polar codes and polar lattices for i.i.d. fading channels with CSI-R were constructed in [6], which achieve the ergodic capacity through single-stage polarization. An adaptive polar coding scheme for block fading channels with CSI at the transmitter (CSI-T) was proposed in [7], [8], which can provide better performance than conventional polar BICM schemes and LDPC codes. All of the aforementioned polar coding schemes for block fading channels require either some CSI or very large coherent time. As far as we know, polar coding for block fading channels with only CDI and small/medium coherent time has not been investigated in literature yet.

In this paper, we aim to design mutual-information-achieving polar codes for binary-input block fading channels with only CDI. By viewing the transmitted symbols in one coherent block as a supersymbol, we can decompose a block fading channel of coherent time T_c into T_c parallel sub-channels. The input of the jth ($j \in [T_c]$) sub-channel is the jth input bit in each coherent block, while the outputs of the jth sub-channel are the outputs of each coherent block together with the previous $j-1$ input bits. It can be shown that such a decomposition preserves the mutual information of the block fading channel. Thus, to achieve the ergodic mutual information, one simply needs to design a symmetric-capacity-achieving polar code for each sub-channel. Such an approach is also known as multilevel coding (MLC) [9], [10], which has been studied in the area of polar coded modulation [11]. We compare the performance of our proposed MLC scheme with interleaved polar codes designed for i.i.d. fading channels (which we refer to as the BICM scheme) by simulations. Although the MLC scheme can achieve a higher rate asymptotically, simulation result shows that it requires a very large code-length to outperform the BICM scheme. This can provide some guidance for practical polar code designs for noncoherent block fading channels – the MLC scheme is more suitable for delay-tolerant scenarios, while the BICM scheme can provide better performance at short and medium code-lengths.

The rest of this paper is organized as follows. Section

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II provides some related knowledge on polar codes. In Section III we introduce the block fading channel model and describe the main idea of our scheme. Details of our proposed scheme are presented in Section IV. Section V presents some simulation results to show the performance of our proposed scheme. Section VI concludes this paper with some discussions.

II. Preliminaries on Polar Codes

For $N = 2^n$ with n being an arbitrary integer, let $X^{1:N}$ be N consecutive channel inputs to a B-DMC $W(Y|X)$, and $U^{1:N} = X^{1:N}\mathbf{G}_N$, where $\mathbf{G}_N = \mathbf{B}_N\mathbf{F}^{\otimes n}$ is the generator matrix of polar codes, with $\mathbf{B}_N$ being the bit-reversal matrix and $\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. It is shown that as N goes to infinity, $U^{1:N}$ polarizes in the sense that conditioned on $Y^{1:N}$, U^i ($i \in [N]$) becomes either almost independent of $U^{1:i-1}$ and uniformly distributed, or almost determined by $U^{1:i-1}$ [12]. Based on this phenomenon, for $\delta_N = 2^{-N^\beta}$ with $\beta \in (0, 1/2)$, we define the *reliable bit set* as

$$\mathcal{I} = \{i \in [N] : Z(U^i|Y^{1:N}, U^{1:i-1}) \leq \delta_N\}, \qquad (1)$$

where

$$Z(X|Y) = 2\sum_{y\in\mathcal{Y}} P_Y(y)\sqrt{P_{X|Y}(0|y)P_{X|Y}(1|y)}, \qquad (2)$$

is the Bhattacharyya parameter of a random variable pair (X, Y) with X being binary and Y being defined on an arbitrary discrete alphabet. It is shown that

$$\lim_{N\to\infty}\frac{1}{N}|\mathcal{I}| = I(X;Y), \qquad (3)$$

where $I(X;Y)$ is the symmetric capacity of W.

To construct a polar code for W, define $\mathcal{F} = \mathcal{I}^C$, where $\mathcal{I}^C$ denotes the complement set of $\mathcal{I}$. Assign $\{u^i\}_{i\in\mathcal{I}}$ with information bits, and $\{u^i\}_{i\in\mathcal{F}}$ with some fixed value known by both sides, known as *frozen bits*. Then codewords are generated by $x^{1:N} = u^{1:N}\mathbf{G}_N$ since $\mathbf{G}_N = \mathbf{G}_N^{-1}$.

Upon receiving $y^{1:N}$, the receiver uses a successive cancellation (SC) decoder to decode:

$$\bar{u}^i = \begin{cases} u^i, & \text{if } i \in \mathcal{F} \\ \arg\max_{u\in\{0,1\}} P_{U^i|Y^{1:N}U^{1:i-1}}(u|y^{1:N}, u^{1:i-1}), & \\ & \text{if } i \in \mathcal{I} \end{cases}. \qquad (4)$$

The block error probability of such a scheme can be upper bounded by

$$P_e \leq \sum_{i\in\mathcal{I}} Z(U^i|Y^{1:N}, U^{1:i-1}) = O(2^{-N^\beta}). \qquad (5)$$

III. Problem Statement

A block fading channel with coherent interval T_c is defined as follows. At time interval i ($i = 1, 2, ...$), the channel is modeled as

$$\mathbf{y}^i = h^i\mathbf{x}^i + \mathbf{w}^i, \qquad (6)$$

where $h^i \in \mathbb{R}$ is the channel gain at time interval i, $\mathbf{x}^i = [x_1^i, x_2^i, ..., x_{T_c}^i]^{\mathrm{T}} \in \{-1, 1\}^{T_c}$ is the binary input signal after BPSK modulation, $\mathbf{y}^i = [y_1^i, y_2^i, ..., y_{T_c}^i]^{\mathrm{T}} \in \mathbb{R}^{T_c}$ is the channel output, and $\mathbf{w}^i = [w_1^i, w_2^i, ..., w_{T_c}^i]^{\mathrm{T}} \in \mathbb{R}^{T_c}$ is the white Gaussian noise, with $w_j^i \sim \mathcal{N}(0, \sigma^2)$ for every $j \in [T_c]$.

We study the case when both the transmitter and the receiver only have the CDI of the channel. Consider a series of transmissions over N fading blocks. In this paper, we call the N consecutive coded blocks a *frame*. Denote $\mathbf{X} = [\mathbf{x}^1, ..., \mathbf{x}^N]$ and $\mathbf{Y} = [\mathbf{y}^1, ..., \mathbf{y}^N]$, and let $\mathbf{z}_j$ ($j \in [T_c]$) denote the jth row vector of $\mathbf{X}$. Then the mutual information of a transmission frame can be expanded as

$$I(\mathbf{X};\mathbf{Y}) = \sum_{j=1}^{T_c} I(\mathbf{z}_j;\mathbf{Y}|\mathbf{z}_{1:j-1}), \qquad (7)$$

where $\mathbf{z}_{1:j-1}$ is short for $\{\mathbf{z}_1, ..., \mathbf{z}_{j-1}\}$. Similar abbreviations will be used throughout this paper. Note that

$$\begin{aligned} \lim_{N\to\infty}\frac{1}{N}I(\mathbf{z}_j;\mathbf{Y}|\mathbf{z}_{1:j-1}) &= I(X_j;Y_{1:T_c}|X_{1:j-1}) \\ &= I(X_j;Y_{1:T_c}, X_{1:j-1}), \end{aligned}$$

which is the symmetric capacity of a binary-input channel

$$W^{(j)}(\mathbf{y}, x_{1:j-1}|x_j) : \{-1, 1\} \to \mathbb{R}^{T_c} \times \{-1, 1\}^{j-1}, \qquad (8)$$

with transition probability density function (PDF)

$$p(\mathbf{y}, x_{1:j-1}|x_j) = \sum_{x_{j+1:T_c}} p(x_{1:j-1}, x_{j+1:T_c})p(\mathbf{y}|\mathbf{x}), \qquad (9)$$

where $p(\mathbf{y}|\mathbf{x})$ is the joint transition PDF of a coherent block without instantaneous CSI.

Based on the expansion of (7), we can use an MLC-based approach to design polar codes for block fading channels with only CDI. The encoding of a frame consists of T_c component polar codes, designed for each of the T_c sub-channels respectively. When an encoded frame is generated, the sender transmits it block by block. Having received a signal frame, the receiver uses a multistage decoder to decode the component polar codes one by one. At stage j ($j \in [T_c]$), it decodes the jth sub-channel based on the received frame together with the estimates of previous stages. If the component polar codes are symmetric-capacity-achieving, the ergodic mutual information of the binary-input block fading channel under the CDI assumption is also achievable with this scheme.

As an example, we assume h follows the Rayleigh distribution with PDF

$$f(h) = \frac{h}{\sigma_h^2}e^{-\frac{h^2}{2\sigma_h^2}}. \qquad (10)$$

Then

$$p(\mathbf{y}|\mathbf{x}) = \int_0^\infty \Big(\prod_{j=1}^{T_c}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(y_j-hx_j)^2}{2\sigma^2}}\Big)\frac{h}{\sigma_h^2}e^{-\frac{h^2}{2\sigma_h^2}}\,\mathrm{d}h. \qquad (11)$$

Fig. 1 shows a comparison of achievable rates of binary-input AWGN channel, binary-input Rayleigh fading channel with CSI-R, and binary-input block Rayleigh fading channels of different coherent time with only CDI. We can see that as the coherent time increases, the achievable rate with only can be seen as a realization of the joint channel estimation and data transmission paradigm in noncoherent communications.

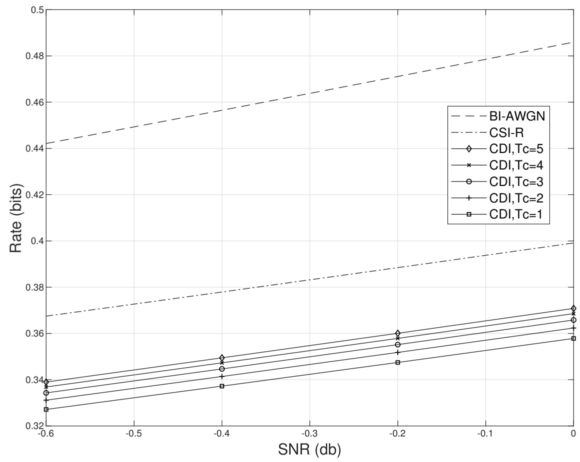

Fig. 1. Achievable rates of binary-input block Rayleigh fading channels.

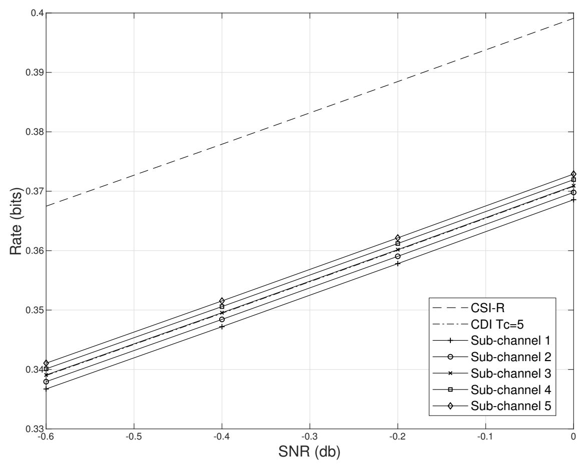

Fig. 2. Symmetric capacity of sub-channels of a block Rayleigh fading channel with coherent time $T_c = 5$.

CDI gets closer and closer to that with CSI-R. It has been shown for several cases (e.g., [13], [14]) that the noncoherent capacity of a block fading channel will approach the coherent capacity as $T_c \to \infty$. In the considered SNR region (-0.6 db to 0 db), a binary-input block Rayleigh fading channel with 5 symbols per coherent interval has a performance gain about 0.25 db over the i.i.d. fading channel under the CDI assumption, and a performance loss about 0.5 db compared with the CSI-R curve.

Fig. 2 shows the symmetric capacity of the five sub-channels of a block Rayleigh fading channel of coherent time $T_c = 5$, compared with that of the original channel and the CSI-R rate. We can see that the achievable rate of a sub-channel increases with its index. This can be intuitively explained as follows. After decoding a sub-channel, the decoder gains more knowledge about the CSI (although not explicitly shown), and the achievable rates of the following sub-channels become larger. Thus, our proposed CDI scheme can be seen as a realization of the joint channel estimation and data transmission paradigm in noncoherent communications.

IV. Details of the Proposed Scheme

For the jth ($j \in [T_c]$) sub-channel $W^{(j)}(\mathbf{y}, x_{1:j-1}|x_j)$, let $\mathbf{u}_j = \mathbf{z}_j \mathbf{G}_N$. Define the reliable bit set by

$$\mathcal{I}_j \triangleq \{i \in [N] : Z(U_j^i | Y_{1:T_c}^{1:N}, U_{1:j-1}^{1:N}, U_j^{1:i-1}) \leq \delta_N\}, \quad (12)$$

with $U_{1:0}^{1:N} = \emptyset$, and the frozen bit set by $\mathcal{F}_j \triangleq \mathcal{I}_j^C$. The multilevel encoding procedure goes as follows.

- For the jth ($j \in [T_c]$) sub-channel, insert information bits to $\{u_j^i\}_{i \in \mathcal{I}_j}$ and frozen bits to $\{u_j^i\}_{i \in \mathcal{F}_j^c}$.
- Compute $\mathbf{z}_j = \mathbf{u}_j \mathbf{G}_N$ for each $j \in [T_c]$ and generate the final coded frame by $\mathbf{X} = (\mathbf{z}_1; ...; \mathbf{z}_{T_c})$.
- The sender transmits $\mathbf{X}$ column by column.

Having received $\mathbf{Y}$, the receiver performs multistage decoding as follows. In the jth ($1 \leq j \leq T_c$) stage, the decoder decodes $\mathbf{u}_j$ with the aid of the estimates in previous stages:

$$\bar{u}_j^i = \begin{cases} u_j^i, & \text{if } i \in \mathcal{F}_j \\ \arg\max_{u \in \{0,1\}} P_{U_j^i | Y_{1:T_c}^{1:N} U_{1:j-1}^{1:N} U_j^{1:i-1}}(u | y_{1:T_c}^{1:N}, \\ \quad \bar{u}_{1:j-1}^{1:N}, u_j^{1:i-1}), & \text{if } i \in \mathcal{I}_j \end{cases} \quad (13)$$

where $\bar{u}_{1:0}^{1:N} = \emptyset$.

The block error probability of the jth component polar code provided that the previous component codes are correctly decoded can be upper bounded by

$$P_e^{(j)} \leq \sum_{i \in \mathcal{I}_j} Z(U_j^i | Y_{1:T_c}^{1:N}, U_{1:j-1}^{1:N}, U_j^{1:i-1}) = O(2^{-N^\beta}) \quad (14)$$

according to the definition of the information bit set. Thus, the overall frame error probability can be upper bounded by

$$P_e \leq \sum_{j=1}^{T_c} P_e^{(j)} = T_c O(2^{-N^\beta}). \quad (15)$$

The asymptotic rate of the jth ($j \in [T_c]$) component polar code is

$$\lim_{N \to \infty} R_j = \lim_{N \to \infty} \frac{|\mathcal{I}_j|}{N} = I(X_j; Y_{1:T_c} | X_{1:j-1}). \quad (16)$$

Thus, the asymptotic rate of the scheme is

$$\lim_{N \to \infty} R = \lim_{N \to \infty} \frac{1}{T_c} \sum_{j=1}^{T_c} R_j = \frac{1}{T_c} I(X_{1:T_c}; Y_{1:T_c}), \quad (17)$$

which equals the ergodic mutual information of the block fading channel.

From (15) and (17) we can claim that our proposed polar coding scheme achieves the ergodic mutual information of binary-input block fading channels with only CDI.

TABLE I
RATES OF SUB-CODES FOR THE BLER THRESHOLD OF 0.01.

N	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
64	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
128	0.2813	0.2813	0.2813	0.2813	0.2813	0.2813	0.2813	0.2813
256	0.2968	0.3008	0.3008	0.3047	0.3047	0.3047	0.3047	0.3047
512	0.3203	0.3223	0.3223	0.3242	0.3242	0.3242	0.3262	0.3262
1024	0.3428	0.3448	0.3457	0.3467	0.3486	0.3496	0.3496	0.3506

V. SIMULATION RESULTS

A. Code rates of sub-channels

We first use the Monte-Carlo approach to construct our polar codes for a Rayleigh block fading channel of coherent time $T_c = 8$. The design SNR is 0 dB, and the block error rate (BLER) threshold for determining the information bit set for each sub-channel is 0.01. The code rates of the 8 sub-channels for different code-lengths are shown in Table I. It can be seen that for short code-lengths, the rates of all sub-channels are the same due to the finite block-length effect. However, as the code-length increases, sub-channels with larger indices will have larger rates. This confirms the theoretical analysis in Section III that the symmetric capacity of a sub-channel increases with its index.

B. Error Performance

Next, we demonstrate the performance of our proposed MLC scheme by simulations. As a comparison, we also show the performance of polar codes designed for i.i.d. fading channels with and without random interleaving (respectively denoted by BICM and iid in Fig. 3 and Fig. 4). We still consider a Rayleigh block fading channel of coherent time $T_c = 8$. The code-length N of the sub-codes ranges from 64 to 1024, and the overall code rate is set to $3/8$. Details of the sub-code rates are listed in Table II. To be fair, the code-length of the i.i.d. fading polar codes are $T_c \times N$. For comparison purpose, we only adopt the traditional SC decoding algorithm in the simulations. The performance can be further improved by using more powerful decoders (such as the SC-list decoder [15], [16]).

Fig. 3 and Fig. 4 respectively show the frame error rate (FER) and bit error rate (BER) comparison. It can be seen that polar codes designed for i.i.d. fading channels with random interleaving give the best error performance, while those without interleaving perform quite bad. Theoretically speaking, the BICM scheme can only achieve the noncoherent capacity of the i.i.d fading channel, which is smaller than that of the block fading channel. The reason why the MLC scheme performs worse than the BICM scheme is that the considered code-lengths are quite short, thus the detrimental effect of the short code-length on the error performance prevails over the limited achievable rate increase. Observe that the performance gap between the MLC scheme and the BICM scheme diminishes as the code-length increases. We can expect that the gap will disappear at some code-length and the MLC scheme will outperform the BICM scheme thereafter. This result shows that the proposed scheme may be more suitable for delay-tolerant scenarios, since it requires a large code-length to offer good performance.

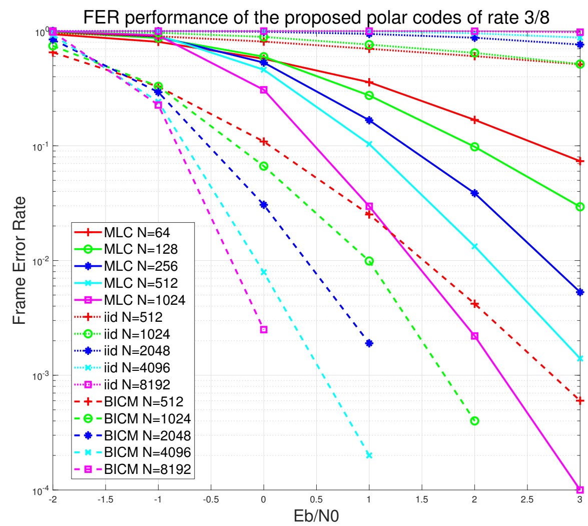

Fig. 3. FER performance comparison.

TABLE II
INFORMATION BIT SET SIZE OF THE SUB-CODES IN THE SIMULATION.

Sub-code-length	Size of the information bit sets
64	[24 24 24 24 24 24 24 24]
128	[48 48 48 48 48 48 48 48]
256	[94 94 95 96 96 97 98 98]
512	[189 190 191 192 192 193 194 195]
1024	[381 382 383 384 384 385 386 387]

VI. DISCUSSION

In this paper, we take a MLC approach to solve the problem of coding for noncoherent block fading channels. Another approach to deal with this problem is the multiple access channel (MAC) approach, which views a block fading channel of coherent time T_c as a T_c-user MAC. We will briefly explain the connection between this approach and ours. From the MAC perspective, code design will be based on MAC polar codes (e.g., [17]). Our proposed MLC-based scheme can be seen as a special case of the MAC-based scheme, i.e., it is equivalent to a MAC polar code designed

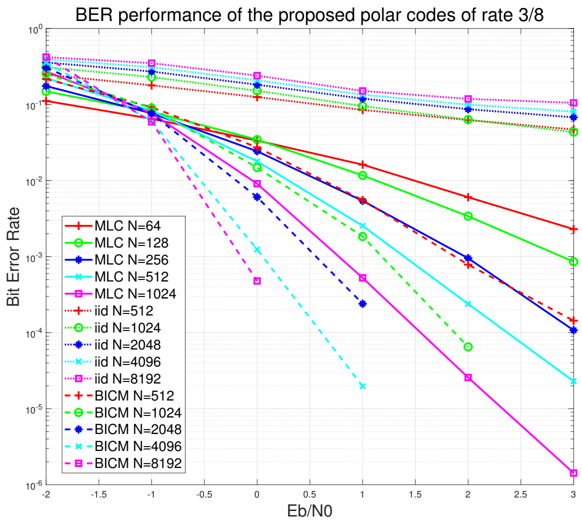

Fig. 4. BER performance comparison.

to achieve a corner point of the achievable rate region of the T_c-user MAC. By using other permutations for MAC polarization, one can allocate rates for different "users" more flexibly. However, although this connection is valid, the MAC approach looks unnecessarily complicated for our problem. In the end, it's the chain rule, not the MAC, that matters for our problem.

We have only presented a basic scheme for the noncoherent block fading channels in this paper. Further improvements can be made on it. For example, when the coherent time is moderately long so that the traditional paradigm of channel estimation before data transmission is not so efficient, while the MLC scheme is a bit too complicated (as it involves many stages of coding and the computation for the LLRs of the sub-channels becomes complex), we can design a hybrid scheme that takes advantage of both the MLC scheme and the BICM scheme. We simply need to partition the sub-channels into several groups and apply the BICM scheme inside a group and the MLC scheme across different groups. We can also design a joint channel estimation and data transmission scheme by applying the MLC scheme only for the first few sub-channels and using the decoding result to do channel estimation. Then we can apply the scheme of [6] together with interleaving for the rest sub-channels. We will leave these for future research.

References

[1] E. Arıkan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Transactions on Information Theory*, vol. 55, no. 7, pp. 3051–3073, 2009.

[2] J. J. Boutros and E. Biglieri, "Polarization of quasi-static fading channels," in *2013 IEEE International Symposium on Information Theory*, 2013, pp. 769–773.

[3] A. Bravo-Santos, "Polar codes for the Rayleigh fading channel," *IEEE Communications Letters*, vol. 17, no. 12, pp. 2352–2355, 2013.

[4] H. Si, O. O. Koyluoglu, and S. Vishwanath, "Polar coding for fading channels: Binary and exponential channel cases," *IEEE Transactions on Communications*, vol. 62, no. 8, pp. 2638–2650, 2014.

[5] P. Trifonov, "Design of polar codes for Rayleigh fading channel," in *2015 International Symposium on Wireless Communication Systems (ISWCS)*, 2015, pp. 331–335.

[6] L. Liu and C. Ling, "Polar codes and polar lattices for independent fading channels," *IEEE Transactions on Communications*, vol. PP, no. 99, pp. 1–1, 2016.

[7] S. Liu, Y. Hong, and E. Viterbo, "Polar codes for block fading channels," in *2017 IEEE Wireless Communications and Networking Conference Workshops (WCNCW)*, 2017, pp. 1–6.

[8] ——, "Adaptive polar coding with high order modulation for block fading channels," in *2017 IEEE International Conference on Communications Workshops (ICC Workshops)*, 2017, pp. 755–760.

[9] H. Imai and S. Hirakawa, "A new multilevel coding method using error-correcting codes," *IEEE Transactions on Information Theory*, vol. 23, no. 3, pp. 371–377, 1977.

[10] U. Wachsmann, R. F. H. Fischer, and J. B. Huber, "Multilevel codes: theoretical concepts and practical design rules," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1361–1391, 1999.

[11] M. Seidl, A. Schenk, C. Stierstorfer, and J. B. Huber, "Polar-coded modulation," *IEEE Transactions on Communications*, vol. 61, no. 10, pp. 4108–4119, 2013.

[12] E. Arıkan, "Source polarization," in *2010 IEEE International Symposium on Information Theory*, 2010, pp. 899–903.

[13] M. Peleg and S. Shamai, "On the capacity of the blockwise incoherent MPSK channel," *IEEE Transactions on Communications*, vol. 46, no. 5, pp. 603–609, 1998.

[14] Y. Liang and V. V. Veeravalli, "Capacity of noncoherent time-selective Rayleigh-fading channels," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3095–3110, 2004.

[15] K. Chen, K. Niu, and J. Lin, "Improved successive cancellation decoding of polar codes," *IEEE Transactions on Communications*, vol. 61, no. 8, pp. 3100–3107, 2013.

[16] I. Tal and A. Vardy, "List decoding of polar codes," *IEEE Transactions on Information Theory*, vol. 61, no. 5, pp. 2213–2226, May 2015.

[17] E. Arıkan, "Polar coding for the Slepian-Wolf problem based on monotone chain rules," in *2012 IEEE International Symposium on Information Theory Proceedings (ISIT)*, July 2012, pp. 566–570.