

QoS-Based Source and Relay Secure Optimization Design with Presence of Channel Uncertainty

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Abstract—In this letter, we study relay-aided networks with presence of single eavesdropper. We provide joint beamforming design of the source and relay that can minimize the overall power consumption while satisfying our predefined quality-of-service (QoS) requirements. Additionally, we investigate the case that the channel between relay and eavesdropper suffers from channel uncertainty. Finally, simulation results are provided to verify the effectiveness of our algorithm.

Index Terms—QoS, security, channel uncertainty, beamforming.

I. INTRODUCTION

RECENTLY, research concerning secrecy capacity has captured considerable attentions, though initial concept of secure communication can be dated back to the 1970s [1]. Traditional high layer encryption-based method can hardly be applied in certain circumstances, e.g., wireless local area network (WLAN) or Ad hoc networks. Due to the fact that users' random accessing and leaving are difficult to predict in WLAN scenario, establishing an appropriate high layer protocol is not an easy task. Additionally, in Ad hoc networks a complete transmission might take several hops and be relayed by other users. Consequently, how to guarantee secure communication has become a critical issue.

Roughly speaking, the research in this area can be classified into three categories. The first category falls into the artificial-noise based algorithm that relies on generating additional noise bringing more negative effect to the eavesdropper than to the legal user. In [2], the authors investigate a point-to-point system with the presence of an eavesdropper and it has been shown how secrecy can be achieved by adding artificial noise. The second category falls into beamforming based algorithm. For instance, a joint beamforming design of relay and source is proposed in [3] with the assumption that the relay also plays as an eavesdropper that tends to wiretap the user's message. The last category is a combination of the above two sorts. Specifically, in [4] the authors study a broadcast scenario by utilizing both the artificial noise and beamforming together and simulation results demonstrate that joint design can achieve better performance.

It should be noticed that all the above studies are based on the perfect channel state information (CSI) assumptions.

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Although the channel between relay and legal user can be obtained through uplink feedback, such assumption is not appropriate for the channel between eavesdropper and relay since eavesdropper usually behaves in passive manner. Therefore, it is more practical to consider the imperfect CSI cases. In [5], the authors investigate a multipoint-to-multipoint system under norm-bounded error model and propose precoding designs that maximize the users' signal-to-interference-plus-noise-ratio (SINR). Besides, relay-aided multiple source-destination pairs networks have been studied in [6], where all channels suffer from norm-bounded errors. The authors provide relay precoding strategy that can minimize the power consumption while maintaining quality-of-service (QoS) requirements. Moreover, in [7] the authors tend to maximize the legal user's SINR while constraining the eavesdropper's SINR below a threshold.

In this letter, we study relay-aided networks that beamforming technology is adopted at both source and relay. Different from other related works, we assume that the channel between relay and eavesdropper is not perfect, specifically, following the norm-bounded model. Our target is to minimize the sum power consumption of relay and source while satisfying the legal user's QoS requirement and maintaining the eavesdropper's signal-to-noise-ratio (SNR) below a threshold.

Notations: In this paper, we use bold uppercase and lowercase letters denote matrices and vectors, respectively; $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ to denote the conjugate, transpose and conjugate transpose of a matrix or a vector, respectively; \mathbf{I}_N is an $N \times N$ identity matrix; $\mathbb{E}(\cdot)$ denotes the statistical expectation; $Tr(\cdot)$ and $\Re\{\cdot\}$ are the trace of a matrix and the real part of a variable, respectively; $vec(\cdot)$ represents the matrix vectorization; \otimes denotes the Kronecker product; $\|\cdot\|$ denotes the Frobenius norm; \succeq represents the property of semidefinite.

II. SYSTEM MODEL

Throughout this letter, we assume that Bob, equipped with single antenna, is a legal subscriber of cellular networks. Meanwhile, there also exists a single-antenna eavesdropper wiretapping the transmitting data for Bob. Besides, it is supposed that direct communication between source and Bob is inapplicable due to the large-scale fading caused by long distance between them. Therefore, relay is introduced so as to help the transmission shown in Fig. 1. The source and relay are equipped with N antennas and M antennas, respectively. Moreover, two timeslots are needed to complete a transmission process. In the first timeslot, the source transmits message intended for Bob, which can be expressed as

$$\mathbf{s} = \mathbf{q}\mathbf{x}, \quad (1)$$

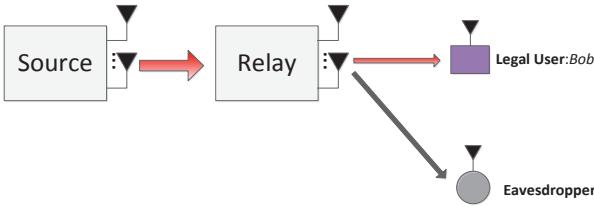


Fig. 1. Relay-aided networks with presence of single eavesdropper.

where $\mathbf{q} \in \mathbb{C}^{N \times 1}$ denotes the beamforming vector executed at source; x is the intended data for Bob which satisfies $\mathbb{E}\{xx^*\} = 1$. The signal received at relay can be written as

$$\mathbf{y}_r = \mathbf{H}\mathbf{s} + \mathbf{n}_r, \quad (2)$$

where $\mathbf{H} \in \mathbb{C}^{M \times N}$ represents the channel between relay and source; $\mathbf{n}_r \in \mathbb{C}^{M \times 1}$ is the additive Gaussian noise which satisfies $\mathbb{E}\{\mathbf{n}_r\mathbf{n}_r^H\} = \sigma_r^2 \mathbf{I}_M$. Afterwards, the received data at relay will be multiplied by precoding matrix $\mathbf{W} \in \mathbb{C}^{M \times M}$,

$$\mathbf{x}_r = \mathbf{W}\mathbf{y}_r = \mathbf{W}\mathbf{H}\mathbf{q}\mathbf{x} + \mathbf{W}\mathbf{n}_r. \quad (3)$$

In the second timeslot, relay will broadcast the signal \mathbf{x}_r . The received signal at Bob can be expressed as

$$y_b = \mathbf{g}_b \mathbf{x}_r + n_b = \mathbf{g}_b \mathbf{W}\mathbf{H}\mathbf{q}\mathbf{x} + \mathbf{g}_b \mathbf{W}\mathbf{n}_r + n_b, \quad (4)$$

where $\mathbf{g}_b \in \mathbb{C}^{1 \times M}$ is the channel between relay and Bob which can be acquired by the feedback information from Bob and n_b is the additive Gaussian noise at Bob satisfying $\mathbb{E}\{n_b n_b^*\} = \sigma_b^2$. In this letter, we assume that the channel knowledge about \mathbf{g}_b is perfect. However, the channel between relay and eavesdropper cannot be guaranteed to be perfect. In this letter, we will adopt a norm-bound error model where the norm of channel estimation error is inferior to a threshold. The channel between relay and eavesdropper can be presented as

$$\mathbf{g}_e = \bar{\mathbf{g}}_e + \Delta \mathbf{g}_e, \|\Delta \mathbf{g}_e\| \leq \varepsilon, \quad (5)$$

where $\bar{\mathbf{g}}_e \in \mathbb{C}^{1 \times M}$ is estimated channel between eavesdropper and relay and $\Delta \mathbf{g}_e \in \mathbb{C}^{1 \times M}$ is the channel estimation errors bounded by radius ε . Similarly, the received signal at eavesdropper is expressed as

$$y_e = \mathbf{g}_e \mathbf{x}_r + n_e = \mathbf{g}_e \mathbf{W}\mathbf{H}\mathbf{q}\mathbf{x} + \mathbf{g}_e \mathbf{W}\mathbf{n}_r + n_e, \quad (6)$$

where n_e is additive Gaussian noise satisfying $\mathbb{E}\{n_e n_e^*\} = \sigma_e^2$.

III. JOINT SOURCE AND RELAY BEAMFORMING DESIGN WITH PRESENCE OF CHANNEL UNCERTAINTY

In this letter, we aim to minimize the entire power consumption at source and relay while satisfying predefined QoS requirement for Bob and simultaneously constraining the SNR of eavesdropper below certain threshold, respectively. The SNR of Bob and eavesdropper can be expressed as

$$\text{SNR}_b = \frac{\mathbf{g}_b \mathbf{W}\mathbf{H}\mathbf{q}\mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \mathbf{g}_b^H}{\sigma_r^2 \mathbf{g}_b \mathbf{W}\mathbf{W}^H \mathbf{g}_b^H + \sigma_b^2}, \quad (7)$$

and

$$\text{SNR}_e = \frac{\mathbf{g}_e \mathbf{W}\mathbf{H}\mathbf{q}\mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \mathbf{g}_e^H}{\sigma_r^2 \mathbf{g}_e \mathbf{W}\mathbf{W}^H \mathbf{g}_e^H + \sigma_e^2}. \quad (8)$$

Hence, our optimization problem can be formulated as

$$\min_{\mathbf{q}, \mathbf{W}} \mathbb{E}(\|\mathbf{s}\|^2) + \mathbb{E}(\|\mathbf{x}_r\|^2), \quad (9a)$$

$$\text{s.t. } \text{SNR}_b \geq r_{th}^{(b)}, \quad (9b)$$

$$\text{SNR}_e \leq r_{th}^{(e)}, \quad (9c)$$

where $r_{th}^{(b)}$ and $r_{th}^{(e)}$ denote the predefined thresholds for Bob and eavesdropper, respectively.

Define $\mathbf{Q} = \mathbf{q}\mathbf{q}^H$, the base station power can be turned into

$$\mathbb{E}(\|\mathbf{s}\|^2) = \text{Tr}(\mathbf{Q}). \quad (10)$$

By introducing $\mathbf{w} = \text{vec}(\mathbf{W})$ and $\mathbf{Z} = \mathbf{w}\mathbf{w}^H$, and with the help of equalities $\text{Tr}(\mathbf{XYX}^H \mathbf{W}) = \text{vec}(\mathbf{X})^H (\mathbf{W}^T \otimes \mathbf{Y}) \text{vec}(\mathbf{X})$ and $\text{Tr}(\mathbf{AB}) = \text{Tr}(\mathbf{BA})$ [8], the relay's power can be transformed as

$$\begin{aligned} & \mathbb{E}(\|\mathbf{x}_r\|^2) \\ &= \text{Tr}(\mathbf{WHqq}^H \mathbf{H}^H \mathbf{W}^H + \sigma_r^2 \mathbf{WW}^H) \\ &= \text{Tr}(\mathbf{w}^H (\mathbf{I}_M \otimes (\mathbf{H}\mathbf{q}\mathbf{q}^H \mathbf{H}^H + \sigma_r^2 \mathbf{I}_M)) \mathbf{w}) \\ &= \text{Tr}(\mathbf{Z} (\mathbf{I}_M \otimes (\mathbf{HQH}^H + \sigma_r^2 \mathbf{I}_M))). \end{aligned} \quad (11)$$

Similarly, SNR of Bob can be rewritten as

$$\begin{aligned} \text{SNR}_b &= \frac{\mathbf{w}^H ((\mathbf{g}_b^H \mathbf{g}_b)^T \otimes (\mathbf{HQH}^H)) \mathbf{w}}{\mathbf{w}^H ((\mathbf{g}_b^H \mathbf{g}_b)^T \otimes (\sigma_r^2 \mathbf{I}_M)) \mathbf{w} + \sigma_b^2} \\ &= \frac{\text{Tr}(\mathbf{Z} ((\mathbf{g}_b^H \mathbf{g}_b)^T \otimes (\mathbf{HQH}^H)))}{\text{Tr}(\mathbf{Z} ((\mathbf{g}_b^H \mathbf{g}_b)^T \otimes (\sigma_r^2 \mathbf{I}_M))) + \sigma_b^2}. \end{aligned} \quad (12)$$

Nevertheless, the SNR of eavesdropper is difficult to handle due to the presence of channel uncertainty. Therefore, we resort to optimizing the worst case of eavesdropper's SNR. Here, we will separately find the upper bound of the numerator of SNR_e and lower bound of the denominator of SNR_e , respectively.

Before explicit computations of the lower and upper bounds, we will state the following two useful results [6] that will be utilized later. For the following two problems

$$\max_{\|\mathbf{x}\| \leq \delta} \mathcal{X}(\mathbf{x}) = \Re(\mathbf{x}^H \mathbf{y}), \quad (13)$$

$$\min_{\|\mathbf{x}\| \leq \delta} \mathcal{Y}(\mathbf{x}) = \Im(\mathbf{x}^H \mathbf{y}), \quad (14)$$

their solutions can be given by

$$\mathcal{X}((\delta/\|\mathbf{y}\|)\mathbf{y}) = \delta \|\mathbf{y}\|, \quad (15)$$

$$\mathcal{Y}(-(\delta/\|\mathbf{y}\|)\mathbf{y}) = -\delta \|\mathbf{y}\|. \quad (16)$$

Then, given $\mathbf{X}_1 \in \mathbb{C}^{N_1 \times N_2}$, $\mathbf{F} \in \mathbb{C}^{N_2 \times N_3}$, $\mathbf{X}_2 \in \mathbb{C}^{N_3 \times N_3}$ and $\mathbf{X}_3 \in \mathbb{C}^{N_2 \times N_4}$, the following equalities hold

$$\begin{aligned} & \|\mathbf{X}_1 \mathbf{F} \mathbf{X}_2 \mathbf{F}^H \mathbf{X}_3\| \\ & \stackrel{(a)}{=} \|\text{vec}(\mathbf{X}_1 \mathbf{F} \mathbf{X}_2 \mathbf{F}^H \mathbf{X}_3)\| \\ & \stackrel{(b)}{=} \|(\mathbf{X}_3^T \otimes \mathbf{X}_1) \text{vec}(\mathbf{F} \mathbf{X}_2 \mathbf{F}^H)\| \\ & \stackrel{(c)}{=} \|(\mathbf{X}_3^T \otimes \mathbf{X}_1)(\mathbf{F}^* \otimes \mathbf{F}) \text{vec}(\mathbf{X}_2)\| \\ & \stackrel{(d)}{=} \|(\text{vec}(\mathbf{X}_2)^T \otimes (\mathbf{X}_3^T \otimes \mathbf{X}_1)) \text{vec}(\mathbf{F}^* \otimes \mathbf{F})\|, \end{aligned} \quad (17)$$

where the equality (a) holds with the help of the equation $\|\mathbf{X}\| = \|\text{vec}(\mathbf{X})\|$; the equalities (b), (c) and (d) hold with the help of $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$ [8].

Furthermore, we define $\mathbf{f} = \text{vec}(\mathbf{F})$ and $\text{vec}(\mathbf{F}^* \otimes \mathbf{F}) = \mathbf{T}_f \text{vec}(\mathbf{f}\mathbf{f}^H)$, where $\mathbf{T}_f \in \mathbb{C}^{(N_2^2 N_3^2) \times (N_2^2 N_3^2)}$ is the transformation matrix formed by ones and zeros, which can be built by observing the relationship between $\text{vec}(\mathbf{F}^* \otimes \mathbf{F})$ and $\text{vec}(\mathbf{f}\mathbf{f}^H)$. Then, (17) can be transformed into

$$\begin{aligned} & \|\mathbf{X}_1 \mathbf{F} \mathbf{X}_2 \mathbf{F}^H \mathbf{X}_3\| \\ &= \|(\text{vec}(\mathbf{X}_2)^T \otimes (\mathbf{X}_3^T \otimes \mathbf{X}_1)) \mathbf{T}_f \text{vec}(\mathbf{f}\mathbf{f}^H)\|. \end{aligned} \quad (18)$$

Inserting (5) into the numerator of eavesdropper's SNR (8) and omitting the terms involving second order channel uncertainties, the upper bound of SNR_e 's numerator can be written as

$$\begin{aligned} & \mathbf{g}_e \mathbf{W} \mathbf{H} \mathbf{q} \mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \mathbf{g}_e^H \\ &= \bar{\mathbf{g}}_e \mathbf{W} \mathbf{H} \mathbf{q} \mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \bar{\mathbf{g}}_e^H + 2\Re\{\Delta \mathbf{g}_e \mathbf{W} \mathbf{H} \mathbf{q} \mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \bar{\mathbf{g}}_e^H\} \\ &\leq \bar{\mathbf{g}}_e \mathbf{W} \mathbf{H} \mathbf{q} \mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \bar{\mathbf{g}}_e^H + 2\varepsilon \|\mathbf{W} \mathbf{H} \mathbf{q} \mathbf{q}^H \mathbf{H}^H \mathbf{W}^H \bar{\mathbf{g}}_e^H\| \\ &= \text{Tr}(\mathbf{Z}((\bar{\mathbf{g}}_e^H \bar{\mathbf{g}}_e)^T \otimes (\mathbf{H} \mathbf{Q} \mathbf{H}^H))) + 2\varepsilon \|(\text{vec}(\mathbf{H} \mathbf{Q} \mathbf{H}^H)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f \text{vec}(\mathbf{Z})\|, \end{aligned} \quad (19)$$

where the inequality holds by using (15). Similarly, the lower bound of SNR_e 's denominator can be expressed as

$$\begin{aligned} & \sigma_r^2 \mathbf{g}_e \mathbf{W} \mathbf{W}^H \mathbf{g}_e^H + \sigma_e^2 \\ &= \sigma_r^2 \bar{\mathbf{g}}_e \mathbf{W} \mathbf{W}^H \bar{\mathbf{g}}_e^H + 2\Re\{\Delta \mathbf{g}_e \mathbf{W} \mathbf{W}^H \bar{\mathbf{g}}_e^H\} + \sigma_e^2 \\ &\geq \sigma_r^2 \bar{\mathbf{g}}_e \mathbf{W} \mathbf{W}^H \bar{\mathbf{g}}_e^H - 2\varepsilon \|\mathbf{W} \mathbf{W}^H \bar{\mathbf{g}}_e^H\| + \sigma_e^2 \\ &= \text{Tr}(\mathbf{Z}((\bar{\mathbf{g}}_e^H \bar{\mathbf{g}}_e)^T \otimes (\sigma_r^2 \mathbf{I}_M))) - 2\varepsilon \|(\text{vec}(\mathbf{I}_M)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f \text{vec}(\mathbf{Z})\|, \end{aligned} \quad (20)$$

where the inequality holds by using (16). Thus, the optimization problem (9) can be reformulated as

$$\begin{aligned} & \min_{\mathbf{Q}, \mathbf{Z}} \text{Tr}(\mathbf{Q}) + \text{Tr}(\mathbf{Z}(\mathbf{I}_M \otimes (\mathbf{H} \mathbf{Q} \mathbf{H}^H + \sigma_r^2 \mathbf{I}_M))), \\ & \text{s.t. } \text{Tr}(\mathbf{Z} \mathbf{A}) \geq r_{th}^{(b)} \sigma_b^2, \\ & \quad \text{Tr}(\mathbf{Z} \mathbf{B}) \geq 2\varepsilon \|(\text{vec}(\mathbf{H} \mathbf{Q} \mathbf{H}^H)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f \\ & \quad \text{vec}(\mathbf{Z})\| + 2r_{th}^{(e)} \varepsilon \|(\text{vec}(\mathbf{I}_M)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f \text{vec}(\mathbf{Z})\|, \\ & \quad \text{rank}(\mathbf{Q}) = 1, \text{rank}(\mathbf{Z}) = 1, \end{aligned} \quad (21)$$

where

$$\mathbf{A} = ((\mathbf{g}_b^H \mathbf{g}_b)^T \otimes (\mathbf{H} \mathbf{Q} \mathbf{H}^H)) - r_{th}^{(b)} ((\mathbf{g}_b^H \mathbf{g}_b)^T \otimes (\sigma_r^2 \mathbf{I}_M)), \quad (22)$$

$$\mathbf{B} = r_{th}^{(e)} ((\bar{\mathbf{g}}_e^H \bar{\mathbf{g}}_e)^T \otimes (\sigma_r^2 \mathbf{I}_M)) - ((\bar{\mathbf{g}}_e^H \bar{\mathbf{g}}_e)^T \otimes (\mathbf{H} \mathbf{Q} \mathbf{H}^H)). \quad (23)$$

However, the optimizing problem (21) is non-convex due to the rank constraints. Therefore, we resort to semidefinite relaxation technique that firstly neglects these rank constraints, and the optimization problem turns to be

$$\min_{\mathbf{Q}, \mathbf{Z}} \text{Tr}(\mathbf{Q}) + \text{Tr}(\mathbf{Z}(\mathbf{I}_M \otimes (\mathbf{H} \mathbf{Q} \mathbf{H}^H + \sigma_r^2 \mathbf{I}_M))), \quad (24a)$$

$$\text{s.t. } \text{Tr}(\mathbf{Z} \mathbf{A}) \geq r_{th}^{(b)} \sigma_b^2, \quad (24b)$$

$$\begin{aligned} & \text{Tr}(\mathbf{Z} \mathbf{B}) \geq 2\varepsilon \|(\text{vec}(\mathbf{H} \mathbf{Q} \mathbf{H}^H)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f \\ & \quad \text{vec}(\mathbf{Z})\| + 2r_{th}^{(e)} \varepsilon \|(\text{vec}(\mathbf{I}_M)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f \text{vec}(\mathbf{Z})\|. \end{aligned} \quad (24c)$$

Additionally, the above problem is still non-convex for both \mathbf{Q} and \mathbf{Z} due to the bilinear properties [9]. Nevertheless, with fixed \mathbf{Z} , the problem is convex for \mathbf{Q} . Similarly, with fixed \mathbf{Q} ,

the problem is convex for \mathbf{Z} . Therefore, we can use iterative algorithm to solve the optimization problem (24), which is stated in Algorithm 1. To solve problem (24) we used CVX, a

Algorithm 1 Joint beamforming design of source and relay.

- 1: Initialization:
Initialize the matrix $\mathbf{Q}^{(0)} = \frac{1}{N} \mathbf{P}_s$, $\xi^{(0)} = 10^3$, $\eta = 10^{-3}$, $n = 1$, $N_{max} = 30$.
 - 2: Iteration:
a) Compute $\mathbf{Z}^{(n)}$ by solving the problem (24) with fixed values of $\mathbf{Q}^{(n-1)}$.
b) Compute $\mathbf{Q}^{(n)}$ by solving the problem (24) with fixed value of $\mathbf{Z}^{(n)}$.
c) Record the power solution of problem (24) as $\xi^{(n)}$.
 - 3: Termination:
The algorithm terminates either when $\xi^{(n)}$ converges, i.e., $|\frac{\xi^{(n)} - \xi^{(n-1)}}{\xi^{(n)}}| \leq \eta$, or when $n \geq N_{max}$, where η is a predefined threshold and N_{max} is the maximum iteration number.
Output $\mathbf{Z}^{opt} = \mathbf{Z}^{(n)}$, $\mathbf{Q}^{opt} = \mathbf{Q}^{(n)}$.
Else, $n = n + 1$, and go to step 2.
-

package for specifying and solving convex programs [10]. Let us denote \mathbf{Q}^{opt} and \mathbf{Z}^{opt} as the solution obtained from CVX. If $\text{rank}(\mathbf{Q}^{opt}) = 1$ and $\text{rank}(\mathbf{Z}^{opt}) = 1$, then we can use eigenvalue decomposition to obtain the solutions of \mathbf{q} and \mathbf{w} ; Otherwise, randomization technique can be applied to obtain the solutions of \mathbf{q} and \mathbf{w} [11]. Specifically, we generate a set of random dual vectors which conform the Gaussian distribution, i.e., $\tilde{\mathbf{q}} \sim \mathcal{N}(0, \mathbf{Q}^{opt})$ and $\tilde{\mathbf{w}} \sim \mathcal{N}(0, \mathbf{Z}^{opt})$. Among these dual vectors, there might exist the pairs that violate the constraints of (24). Accordingly, we use α and β as the scale factors and denote $\hat{\mathbf{w}} = \alpha \tilde{\mathbf{w}}$ and $\hat{\mathbf{q}} = \beta \tilde{\mathbf{q}}$ as the new candidate pair. The values of α and β could be obtained by setting the constraints of (24) to equalities as shown in (25). Finally, the candidate pair that can achieve the minimum value of objective function (24a) can be viewed as a quasi-optimal solution. The randomization technique applied is summarized in Algorithm 2.

IV. SIMULATION RESULTS

Numerical results are demonstrated in this section so as to verify the effectiveness of our proposed method. Without loss of generality, we set $\sigma_r^2 = \sigma_b^2 = \sigma_e^2 = 1$ and $M = N = 4$. The simulation results are averaged over 1000 channel realizations.

Firstly, we investigate the power consumption versus different thresholds of (24) in Fig. 2. The non-robust precoding scheme corresponds to the case of setting $\varepsilon = 0$ in (24). From Fig. 2, we can observe that with fixed $r_{th}^{(e)}$ and $r_{th}^{(b)}$, the robust precoding scheme will always consume more power than the non-robust precoding scheme, which is reasonable since the worst-case is considered in our robust scheme. Similar performance can also be seen in [6]. Besides, for both of the robust beamforming scheme and the non-robust beamforming scheme, as the thresholds become tighter, more power consumption is expected which is in consistent with our analysis. However, such comparison cannot show the

$$\alpha = \sqrt{\left(\frac{r_{th}^{(b)} \sigma_b^2}{Tr(\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H \mathbf{A})} \right)}, \beta = \sqrt{\left(\frac{Tr(\alpha^2 \tilde{\mathbf{w}}\tilde{\mathbf{w}}^H \mathbf{B}) - 2r_{th}^{(e)} \varepsilon \| (vec(\mathbf{I}_M)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f vec(\alpha^2 \tilde{\mathbf{w}}\tilde{\mathbf{w}}^H) \|}{2\varepsilon \| (vec(\mathbf{H}\tilde{\mathbf{q}}\tilde{\mathbf{q}}^H \mathbf{H}^H)^T \otimes (\bar{\mathbf{g}}_e^* \otimes \mathbf{I}_M)) \mathbf{T}_f vec(\alpha^2 \tilde{\mathbf{w}}\tilde{\mathbf{w}}^H) \|} \right)} \quad (25)$$

Algorithm 2 Randomization technique for obtaining the source and relay precoders.

1: Initialization:

Generate a set of K random pairs of dual vectors $[\tilde{\mathbf{q}}^{(k)}, \tilde{\mathbf{w}}^{(k)}]$ which conform the Gaussian distribution $\tilde{\mathbf{q}}^{(k)} \sim \mathcal{N}(0, \mathbf{Q}^{opt})$. and $\tilde{\mathbf{w}}^{(k)} \sim \mathcal{N}(0, \mathbf{Z}^{opt})$, $k = 1, 2, \dots, K$. Set $i=0$.

2: Computation:

- a) $i = i + 1$. If the i -th pair $[\tilde{\mathbf{q}}^{(i)}, \tilde{\mathbf{w}}^{(i)}]$ does not violate the constraints of (24), then we compute (24a) and record the value as $\text{OPT}_{value}^{(i)}$.
- b) Otherwise, we compute the values of α and β by using (25), and compute $\hat{\mathbf{w}} = \alpha \tilde{\mathbf{w}}$ and $\hat{\mathbf{q}} = \beta \tilde{\mathbf{q}}$. Then, we use $[\hat{\mathbf{q}}^{(i)}, \hat{\mathbf{w}}^{(i)}]$ as the new candidate pair to calculate (24a) and record the value as $\text{OPT}_{value}^{(i)}$.
- c) If $i \neq K$, go to sub-step a).

3: Output:

Among all the values of $\text{OPT}_{value}^{(i)}$, $i = 1, 2, \dots, K$, we choose the smallest one and output its corresponding candidate pair vectors as the quasi-optimal solutions.

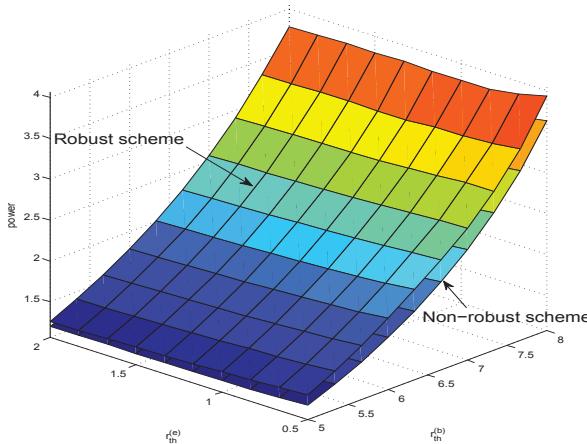


Fig. 2. Power consumption versus distinct values of $r_{th}^{(e)}$ and $r_{th}^{(b)}$, $\varepsilon = 0.01$.

actual performance of robust precoding scheme. The actual performance will be illustrated in Fig. 3.

Then, we examine distribution of the eavesdropper's SNR with distinct values of ε and $r_{th}^{(e)}$. With fixed ε and $r_{th}^{(e)}$, we can observe that for the non-robust precoding scheme almost half of eavesdropper's SNRs will be larger than the preset thresholds. Oppositely, the majority of our robust scheme's SNRs will be less than these thresholds. Additionally, since our designed beamforming vector is to constrain SNR of eavesdropper for the worst-case channel error which is unique, it cannot be guaranteed that our designed beamforming vector is also effective to other channel errors. Thus, that is why there are still SNRs larger than the thresholds for robust precoding.

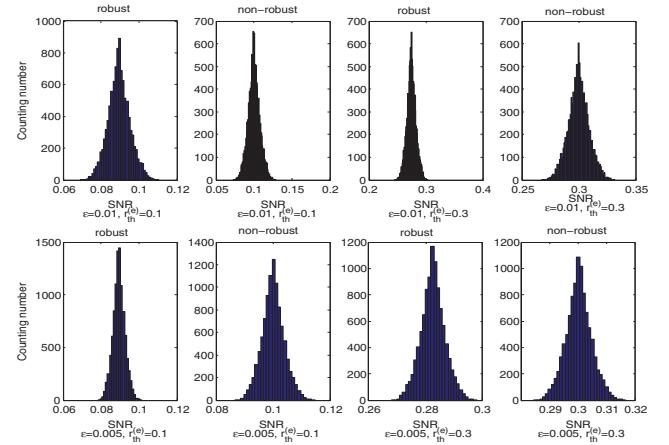


Fig. 3. Distribution of eavesdropper's SNR with distinct values of ε and $r_{th}^{(e)}$.

V. CONCLUSION

This letter proposes a source and relay secure optimization design with presence of channel uncertainty. It aims at minimizing the sum power consumption of source and relay while satisfying certain prefixed QoS requirements. Finally, simulation results verify the effectiveness of our algorithm compared with non-robust precoding scheme.

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