Comments on "A New ML Based Interference Cancellation Technique for Layered Space-Time Codes"

Hufei Zhu, *Member, IEEE,* and Wen Chen, *Member, IEEE*

***Abstract*—In this comment, we justify that the computational complexity proposed in the paper "A New ML Based Interference Cancellation Technique for Layered Space-Time Codes" (*IEEE Trans. on Communications*, vol. 57, no. 4, pp. 930-936, 2009) is $O(N^3)$ rather than the claimed $O(N^2)$, where N is the number of receive antennas.**

***Index Terms*—Interference cancellation, layered space time codes, computational complexity.**

A MAXIMUM likelihood (ML) based interference cancellation (IC) detector was proposed in [1] for double space-time transmit diversity (DSTTD), which consists of two Alamouti's space-time block codes (STBC) units [2]. In many application areas of interest, the computational complexity of the detector in [1] can be less than that of the conventional minimum mean squared error (MMSE) IC detector for DSTTD [3]. However, the complexity claimed in [1] needs to be modified, as will be discussed in this comment.

Let N denote the number of receive antennas. In [1], the theoretical analysis gaves a complexity of $O(N^2)$ (i.e. $7N^2 + 62N - 103$ real multiplications and $12N^2 + 47N - 103$ real additions) [1, Table I], while numerical experiments were not carried out to verify the given complexity. In what follows, we show that the complexity is not $O(N^2)$, but $O(N^3)$, and then give the exact complexity that is verified by our numerical experiments.

Firstly, we show that a complexity of $O(N^3)$ is required to perform the orthonormalization process by equations (9), (13), (14) and (15) in [1]. Let $(\bullet)^T$ and $(\bullet)^H$ denote transpose and conjugate transpose of a vector, respectively. Equation (9) in [1] defines the basis vectors

$$\mathbf{v}_i = \begin{bmatrix} a_i & b_i & \mathbf{e}_i^T \end{bmatrix}^T, \tag{1}$$

where $i = 1, 2, \cdots, 2N-2$, and $\mathbf{e}_i$ is the $(2N-2) \times 1$ vector with the i^{th} element to be 1 and all others to be zero. Equation (13) in [1] utilizes $\mathbf{v}_1$ and $\mathbf{v}_2$, which is

$$\theta_1 = \mathbf{v}_1 / \|\mathbf{v}_1\|, \quad \theta_2 = \mathbf{v}_2 / \|\mathbf{v}_2\|. \tag{2}$$

Paper approved by D. L. Goeckel, the Editor for Space-Time and OFDM of the IEEE Communications Society. Manuscript received January 12, 2010; revised April 30, 2010.

H. Zhu is with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200240, P. R. China, and Huawei Technologies Co., Ltd., Shenzhen 518129, P. R. China (e-mail: zhuhufei@yahoo.com.cn).

W. Chen is with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200240, P.R. China, and SEU SKL for Mobile Communications (e-mail: wenchen@sjtu.edu.cn).

This work is supported by NSF China #60972031, by the SEU SKL project #W200907, by Huawei Funding #YJCB2009024WL and #YJCB2008048WL, by New Century Excellent Talent in Universities #NCET-06-0386, by National 973 project #2009CB824900, and by ISN project #ISN11-01.

Digital Object Identifier 10.1109/TCOMM.2010.083110.100001

Moreover, we represent equations (14) and (15) in [1] as

$$\begin{cases} \theta_{2n-1} = \left(\mathbf{v}_{2n-1} - \sum_{j=1}^{2n-2} c_{2n-1}^j \theta_j\right) / \|\cdots\|, & \text{(3a)} \\ \theta_{2n} = \left(\mathbf{v}_{2n} - \sum_{j=1}^{2n-2} c_{2n}^j \theta_j\right) / \|\cdots\|, & \text{(3b)} \end{cases}$$

where

$$\begin{cases} c_{2n-1}^j = \theta_j^H \mathbf{v}_{2n-1}, & \text{(4a)} \\ c_{2n}^j = \theta_j^H \mathbf{v}_{2n}, & \text{(4b)} \end{cases}$$

and $n = 2, 3, \cdots, N-1$. It can be seen that $\begin{bmatrix} \theta_{2n-1} & \theta_{2n} \end{bmatrix}$ consists of 2×2 Alamouti sub-blocks [4]. Thus we can obtain θ_{2n} from θ_{2n-1}, to avoid computing (3b) and (4b).

Let $\theta \sim \lfloor i, j, \cdots, k \rfloor$ denote that only the $i^{th}, j^{th}, \cdots, k^{th}$ entries in the vector θ are non-zero. From (1), we obtain

$$\mathbf{v}_i \sim \lfloor 1, 2, i+2 \rfloor, \tag{5}$$

where $i = 1, 2, \cdots, 2N-2$. From (2) and (5), we obtain

$$\theta_1 \sim \lfloor 1, 2, 3 \rfloor, \quad \theta_2 \sim \lfloor 1, 2, 4 \rfloor. \tag{6}$$

Let $n = 2$ in (3) to obtain

$$\theta_3 = \left(\mathbf{v}_3 - c_3^1\theta_1 - c_3^2\theta_2\right) / \|\cdots\| \sim \lfloor 1, 2, 3, 4, 5 \rfloor = \lfloor 1-5 \rfloor \tag{7}$$

and

$$\theta_4 = \left(\mathbf{v}_4 - c_4^1\theta_1 - c_4^2\theta_2\right) / \|\cdots\| \sim \lfloor 1-4, 6 \rfloor, \tag{8}$$

where (5) and (6) are utilized. From (6)–(8), it can be seen that for $n = 1, 2$, we have

$$\theta_{2n-1} \sim \lfloor 1-(2n+1) \rfloor, \quad \theta_{2n} \sim \lfloor 1-2n, 2n+2 \rfloor. \tag{9}$$

Assume for any n, θ_{2n-1} and θ_{2n} satisfy (9). This assumption will be verified in this paragraph. From (3), it can be seen that $\theta_{2(n+1)-1}$ includes the sum of θ_{2n-1}, θ_{2n} and $\mathbf{v}_{2(n+1)-1}$, while $\theta_{2(n+1)}$ includes the sum of θ_{2n-1}, θ_{2n} and $\mathbf{v}_{2(n+1)}$. From (5) and the assumption (9), we can conclude that $\theta_{2(n+1)-1}$ and $\theta_{2(n+1)}$ also satisfy (9). Then the assumption (9), which is valid for $n = 1, 2$, is still valid for all the subsequent $(n+1)$s where $n = 1, 2, \cdots, N-2$. Thus we have verified the assumption (9) for any n.

It can be seen from (9) that in (3a), $c_{2n-1}^j\theta_j$ requires more than j multiplications, while $\sum_{j=1}^{2n-2} c_{2n-1}^j\theta_j$ requires more than $\sum_{j=1}^{2n-2} j \approx 2n^2$ multiplications. Then totally it requires more than $\sum_{n=2}^{N-1} 2n^2 \approx \frac{2}{3}N^3$ multiplications to compute (3a) for $n = 2, 3, \cdots, N-1$. Thus we have shown that the actual complexities of the detector in [1] should be at least $O(N^3)$.

The dominant computations of the ML based IC detector [1] come from equations (9), (11), (13), (14), (23), (25) and

TABLE I
THE COMPUTATIONAL COMPLEXITIES OF THE EQUATIONS IN [1]

Equation Number	Complex Multiplications	Complex Additions	Real Multiplications	Real Additions
(9) and (11)	4(N-1)	2(N-1)	4(N-1)+4	3
(13)			9	4
(14)	$\frac{2}{3}N(N-1)(N-2)$	$\frac{2}{3}N(N-1)(N-2)$	$(6N+5)(N-2)$	$2(N+1)(N-2)$
(23)	$2(N-1)N$	$2(N-1)N$	$4(N-1)$	
(25)	$2(N-1)N$	$2(N-1)N$	$4(N-1)$	
(28)	$4N$	$4N$		
Sum	$\frac{2}{3}N^3+2N^2+\frac{16}{3}N-4$	$\frac{2}{3}N^3+2N^2+\frac{10}{3}N-2$	$6N^2+5N-9$	$2N^2-2N+3$

TABLE II
COMPLEXITY COMPARISON

	The ML based IC detector for DSTTD [1]			The MMSE IC detector for DSTTD [3]		
N	**Real Mult.**	**Real Add.**	**Total Flops**	**Real Mult.**	**Real Add.**	**Total Flops**
2	105	83	188	128	135	263
3	252	199	451	360	369	729
4	475	383	858	768	770	1538
5	790	651	1441	1400	1380	2780
6	1213	1019	2232	2304	2241	4545
7	1760	1503	3263	3528	3395	6923
8	2447	2119	4566	5120	4884	10004

(28) in [1], of which the complexities are listed in Table I. One complex multiplication takes four real multiplications and two real additions, while one complex addition needs two real additions. Therefore, it can be seen from Table I that the complexities of the detector are equivalent to

$$\frac{8}{3}N^3+14N^2+\frac{79}{3}N-25 \tag{10}$$

real multiplications and

$$\frac{8}{3}N^3+10N^2+\frac{46}{3}N-9 \tag{11}$$

real additions. The total complexity is the sum of real multiplications and additions [1], which is

$$\frac{16}{3}N^3+24N^2+\frac{125}{3}N-34 \tag{12}$$

floating-point operations (flops). We also carried out numerical experiments to count the flops required by the detector in [1]. The results of our numerical experiments are identical to those computed by (12), i.e., our numerical experiments have accurately verified (12).

Table I in [1] compared the complexities of the ML based IC detector for DSTTD in [1] and the conventional MMSE IC detector for DSTTD in [3]. From (10), (11) and (12), it can be seen that Table I in [1] should be modified to Table II in this comment, where the total complexity of the MMSE IC detector in [3] is

$$15N^3+\frac{73}{2}N^2-\frac{3}{2}N \tag{13}$$

flops [1]. From Table II, it can be seen that the complexity of the detector proposed in [1] is about 2.2 times smaller than that of the MMSE IC detector [3] when the number of receive antennas is 8.

REFERENCES

[1] S. Jung and J. Lee, "A new ML based interference cancellation technique for layered space-time codes," *IEEE Trans. Commun.*, vol. 57, no. 4, pp. 930-936, 2009.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, 1998.

[3] A. F. Naguib, N. Seshadri, and A. R. Calderbank, "Applications of space-time block codes and interference suppression for high capacity and high data rate wireless systems," in *Proc. 32nd Asilomar Conference on Signals, Systems & Computers*, vol. 2, pp. 1803-1810, Nov. 1998.

[4] A. H. Sayed, W. M., Younis, and A. Tarighat, "An invariant matrix structure in multiantenna communications," *IEEE Signal Processing Lett.*, vol. 12, no. 11, pp. 749-752, 2005.