Resource Allocation in Relay-Aided OFDM Cognitive Radio Networks

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Abstract—In this paper, we investigate the resource allocation problem in a single-user relay-aided cognitive radio (CR) underlay network. Both the CR network and the primary network operate under the orthogonal frequency-division multiplexing (OFDM) scheme. Different from the conventional resource allocation problem, the relay node here is capable of performing subcarrier permutation over two hops such that the signal received over a particular subcarrier is forwarded via a different subcarrier. The objective is to maximize the throughput of the CR network subject to a limited power budget at the secondary source and the relay node and to interference constraints at the primary receiver. Optimization is performed under a unified framework where power allocation at the source node, power allocation at the relay node, and subcarrier pairing at the two hops are jointly optimized. The joint resource-allocation scheme yields an asymptotically optimal solution. We further design a suboptimal algorithm that sacrifices little on performance but could significantly reduce computational complexity. Finally, numerical examples are provided to corroborate the proposed studies.

Index Terms—Cognitive radio (CR), orthogonal frequency-division multiplexing (OFDM), relay network, resource allocation, subcarrier pairing.

I. INTRODUCTION

COGNITIVE radio (CR) is a promising solution to the problem of spectrum scarcity [1], [2], in which an unlicensed secondary user (SU) is allowed to opportunistically access the spectrum of a licensed primary user (PU) when the latter is idle or to share the spectrum simultaneously subject to certain interference limitations [3]–[5]. In the meantime, orthogonal frequency-division multiplexing (OFDM) has shown its great potential to combat intersymbol interference and enhance transmission efficiency [6]. Thus, the combination of OFDM and CR becomes an important technology for future-generation wireless systems [7]–[15].

In [7], the power allocation problem in OFDM-based CR networks under the overlay transmission protocol was studied. This work was later extended to the relay-aided transmission scenario in [8], where a suboptimal algorithm that optimizes source and relay power allocation in an alternating fashion was proposed. However, neither [7] nor [8] considered the battery limitations of transmitting nodes, i.e., maximum power constraints are missing. The problem of throughput maximization in multicarrier CR systems under transmit power and interference constraints for the underlay CR scheme was investigated in [9] and was later extended to an *amplify-and-forward* (AF)-relay-aided transmission in [10]. Although the study in [9] is based on the sum power constraint, a more natural choice should be the individual battery power constraint for each transmitting node. Cooperative relaying in OFDM-based CR networks was explored in [11], where the optimal strategy of spectrum allocation and relay assignment was proposed. Recently, a joint algorithm for spectrum sensing adaptation and resource allocation optimization has been proposed in [12]. Furthermore, power allocation in multiple-relay scenarios, relay selection schemes under cooperative transmission, and optimal route selection in multihop CR networks were studied in [13]–[15], respectively.

In a multihop network, the channel gains over different hops are mutually independent. Hence, the subcarriers that experience deep fading over one hop may not experience deep fading over the other hops. This brings a new degree of freedom in resource allocation, which is to properly match the subcarrier of different hops, also known as subcarrier pairing [16]–[18]. The concept of subcarrier pairing was first introduced in [16], where Herdin showed that the system throughput can be enhanced if the subcarriers on the two hops are coupled in the order of their channel magnitude.[1] Later, in [17] and [18], joint power allocation and the subcarrier pairing problem in AF and *decode-and-forward* relay networks, respectively, were considered, whereas that in the multiuser two-way relay scenario was exploited in [19]. In CR networks, however, subcarrier pairing faces a new challenge and is a nontrivial task. For example, a subcarrier with the highest channel gain over the first hop may also have the strongest interference to the PUs, and pairing this subcarrier with the strongest subcarrier over the second hop may result in strong interference with the PU. Thus, a more considerate

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[1]An equal power-allocation strategy is adopted at the source and relay nodes.
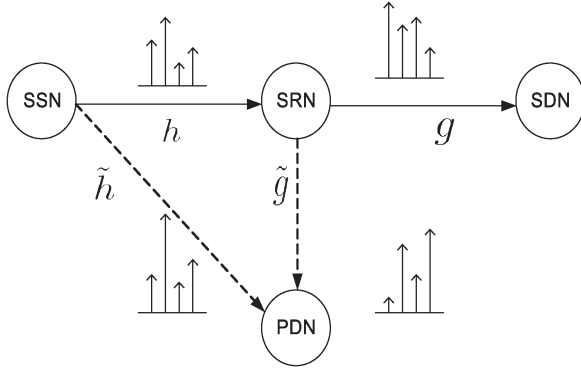
Fig. 1. Relay-enhanced CR network model.

subcarrier pairing strategy is required, i.e., jointly embracing the throughput maximization of the SU and the interference mitigation to the PU. To the best of the authors' knowledge, subcarrier pairing in AF-relay-enhanced CR networks has yet to be explored.

In this paper, we consider an AF-relay-aided CR network that is operated with the OFDM scheme and the underlay spectrum sharing strategy. We seek for optimal resource allocation at the source and the relay, including power allocation over different subcarriers at secondary source/relay nodes and subcarrier pairing at the secondary relay station. We also propose a suboptimal low-complexity algorithm to trade performance for computational complexity. The simulation results show that the suboptimal scheme yields very close performance to the joint optimization algorithm.

The remainder of this paper is organized as follows: In Section II, we present the system model of the relay-aided CR transmission and formulate the joint resource allocation problem. In Section III, we develop a joint optimization algorithm from the dual composition method. In Section IV, we design a low-complexity resource-allocation scheme. Numerical results are presented in Section V to corroborate the proposed studies, and conclusions are made in Section VI.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model

Consider a relay-aided CR network where a secondary source node (SSN) communicates with a secondary destination node (SDN) with the help of a secondary relay node (SRN), as shown in Fig. 1. Underlay CR transmission is adopted where the unlicensed SU simultaneously shares the whole spectrum with the licensed PU. Similar to [7]–[15], we assume that the instantaneous channel coefficients for all links are known *a prior* while such knowledge can be obtained from channel estimation techniques [20], [21] and feedback approaches [22], [23]. More practical considerations, e.g., using statistical channel knowledge or the erroneous channel knowledge [24], are out of the scope of this work. The SSN and SDN are located far from each other such that the direct communication link is missing [17]. On the other hand, the primary destination node (PDN) is located close enough to both the SSN and the SRN such that it experiences interference during both transmission phases (see Fig. 1). OFDM transmission with K subcarriers is

applied for the CR network. Unlike the work in [10], where Shamim *et al.* assumed that the signal received at the SRN over a particular subcarrier is forwarded to the destination over the same subcarrier, we adopt a tone permutation policy such that the signal received on subcarrier k over the SSN-to-SRN link is forwarded via subcarrier j over the SRN-to-SDN link, where j is not necessarily equal to k.

For the SSN-to-SRN link, we denote the channel coefficient over the kth subcarrier as h_k, the power allocation as p_k, and the transmitted symbol as x_k. The signal received at the SRN over the kth subcarrier is

$$y_k^{\text{SRN}} = \sqrt{p_k}h_k x_k + z_k \tag{1}$$

where z_k is additive white Gaussian noise (AWGN) with variance σ_k^2. The signal received over the jth subcarrier at the SDN is then

$$y_j^{\text{SDN}} = \sqrt{q_j}\eta_k(g_j h_k \sqrt{p_k}x_k + g_j z_k) + w_j \tag{2}$$

where $\eta_k \triangleq (\sqrt{p_k|h_k|^2 + \sigma_k^2})^{-1}$, q_j is the transmit power allocated to the jth subcarrier, g_j denotes the corresponding channel coefficient between the SRN and the SDN, and w_j represents AWGN at the SDN with variance σ_j^2.

The throughput over subcarrier pair (k, j) at a high signal-to-noise ratio (SNR) can be approximated by [17], [25]

$$c_{(k,j)} \approx \frac{1}{2}\log_2\left(1 + \frac{\left(\sqrt{p_k}|h_k|\sqrt{q_j}|g_j|\right)^2}{\sigma_j^2\left(\sqrt{p_k}|h_k|\right)^2 + \sigma_k^2\left(\sqrt{q_j}|g_j|\right)^2}\right) \tag{3}$$

where factor 1/2 is due to the two time slots used in a complete transmission. In particular, [25] demonstrated that the resource allocation obtained from (3) yields very close results to the actual throughput even in the low-SNR scenario.

B. Constraint Definitions

Since each subcarrier over hop-1 can be paired with one and only one subcarrier in hop-2, we define $\pi_{(k,j)} \in \{0, 1\}$ as the binary variable such that

$$\pi_{(k,j)} = \begin{cases} 1, & \text{if the } k\text{th subcarrier of the first hop is paired} \\ & \text{with the } j\text{th subcarrier of the second hop} \\ 0, & \text{otherwise.} \end{cases}$$

With the given definition, the subcarrier pairing constraint can be mathematically expressed as

$$\sum_{k=1}^{K}\pi_{(k,j)} = 1, \quad \forall j \qquad \sum_{j=1}^{K}\pi_{(k,j)} = 1, \quad \forall k. \tag{4}$$

Under the proposed transmission protocol, the SSN and the SRN cause interference to the PDN in the first and second time slots, respectively. To protect the PDN from excessive interference, power allocation at the SSN and the SRN must satisfy

$$\sum_{k=1}^{K}p_k|\tilde{h}_k|^2 \leq I_{\text{th}}, \qquad \sum_{j=1}^{K}q_j|\tilde{g}_j|^2 \leq I_{\text{th}} \tag{5}$$

where $\tilde{h}_k$ and $\tilde{g}_j$ denote the channel coefficients of SSN-to-PDN and SRN-to-PDN links, respectively, whereas I_{th} denotes the maximum allowable interference threshold.[2] If the PU is more sensitive to interference, a per-subcarrier-based interference constraint can be imposed as

$$p_k|\tilde{h}_k|^2 \leq I_{\text{th}}^k, \quad \forall k \qquad q_j|\tilde{g}_j|^2 \leq I_{\text{th}}^j, \quad \forall j. \qquad (6)$$

Following [17], we assume two types of power constraints:

I) separate transmit power constraints at the SSN and the SRN such that the summation of power allocated over all subcarriers is within the maximum available power budget of the corresponding node, i.e.,

$$\sum_{k=1}^{K} p_k \leq P_{\text{SSN}}, \qquad \sum_{j=1}^{K} q_j \leq P_{\text{SRN}}; \qquad (7)$$

II) a sum power constraint over the SSN and the SRN, i.e.,

$$\sum_{k=1}^{K} p_k + \sum_{j=1}^{K} q_j \leq P_T. \qquad (8)$$

C. Problem Formulation

Our target is to jointly optimize power allocation at the SSN, power allocation at the SRN, and subcarrier pairing at the two hops such that the overall system throughput of the SU is maximized subject to the maximum power constraints of the battery and the allowable interference constraints of the PU.

Remark: The difference from the power allocation used in a conventional relay network is illustrated here. In conventional relay networks, a better power-allocation scheme is to set $p_k > p_{k'}$ and $q_j > q_{j'}$ whenever $|h_k|^2 > |h_{k'}|^2$ and $|g_j|^2 > |g_{j'}|^2$.[3] However, in CR-based relay networks, for $|h_k|^2 > |h_{k'}|^2$ and $|g_j|^2 > |g_{j'}|^2$ but $|\tilde{h}_k|^2 > |\tilde{h}_{k'}|^2$ and $|\tilde{g}_j|^2 > |\tilde{g}_{j'}|^2$, we may still need $p_k < p_{k'}$ and $q_j < q_{j'}$.

Mathematically, we should optimize over variables $\boldsymbol{\pi} = \{\pi_{(k,j)} \in \{0,1\}\}$, $\mathbf{p} = \{p_k \geq 0\}$, and $\mathbf{q} = \{q_j \geq 0\}$ for all $k = \{1, \ldots, K\}$, $j = \{1, \ldots, K\}$. The following four problems are formulated:

P1: optimization under the separate power constraint and the sum interference constraint, i.e.,

$$\max_{\boldsymbol{\pi},\mathbf{p},\mathbf{q}}, \quad \sum_{k=1}^{K}\sum_{j=1}^{K} \pi_{(k,j)}c_{(k,j)}, \qquad \text{s.t.} \quad (4),(5),(7); \qquad (9)$$

P2: optimization under the separate power constraint and the per-subcarrier interference constraint, i.e.,

$$\max_{\boldsymbol{\pi},\mathbf{p},\mathbf{q}} \quad \sum_{k=1}^{K}\sum_{j=1}^{K} \pi_{(k,j)}c_{(k,j)}, \qquad \text{s.t.} \quad (4),(6),(7); \qquad (10)$$

P3: optimization under the sum power constraint and the sum interference constraint, i.e.,

$$\max_{\boldsymbol{\pi},\mathbf{p},\mathbf{q}} \quad \sum_{k=1}^{K}\sum_{j=1}^{K} \pi_{(k,j)}c_{(k,j)}, \qquad \text{s.t.} \quad (4),(5),(8); \qquad (11)$$

P4: optimization under the sum power constraint and the per-subcarrier interference constraint, i.e.,

$$\max_{\boldsymbol{\pi},\mathbf{p},\mathbf{q}} \quad \sum_{k=1}^{K}\sum_{j=1}^{K} \pi_{(k,j)}c_{(k,j)}, \qquad \text{s.t.} \quad (4),(6),(8). \qquad (12)$$

In the following, we shall discuss the solution to problem P1 in detail, whereas solutions to P2, P3, and P4 follow similar steps and are summarized in the corresponding sections.

III. Joint Resource-Allocation Scheme

Due to subcarrier pairing constraint (4), problem (9) is classified as mixed-integer programming, which is, in general, difficult to solve. Under the time-sharing condition [26], the duality gap of (9) is proved to be asymptotically zero for sufficiently large K, and a rigorous investigation can be found in [27]. Since the time-sharing condition is readily satisfied in our case, we can solve the dual problem of (9), which is defined as [28]

$$\min_{\lambda \geq 0, \mu \geq 0, \tilde{\lambda} \geq 0, \tilde{\mu} \geq 0} \quad D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu}) \qquad (13)$$

where λ, μ, $\tilde{\lambda}$, and $\tilde{\mu}$ are dual variables associated with the constraints in (5) and (7), and dual function $D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu})$ is given by

$$D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu}) = \max_{\mathbf{p},\mathbf{q},\boldsymbol{\pi}} L(\mathbf{p},\mathbf{q},\boldsymbol{\pi},\lambda,\mu,\tilde{\lambda},\tilde{\mu}), \quad \text{s.t.} \quad (4). \qquad (14)$$

The Lagrangian in (14) is defined as

$$L(\mathbf{p},\mathbf{q},\boldsymbol{\pi},\lambda,\mu,\tilde{\lambda},\tilde{\mu})$$
$$= \sum_{k=1}^{K}\sum_{j=1}^{K} \pi_{(k,j)}c_{(k,j)} + \lambda\left(P_{\text{SSN}} - \sum_{k=1}^{K} p_k\right)$$
$$+ \mu\left(P_{\text{SRN}} - \sum_{j=1}^{K} q_j\right) + \tilde{\lambda}\left(I_{\text{th}} - \sum_{k=1}^{K} p_k|\tilde{h}_k|^2\right)$$
$$+ \tilde{\mu}\left(I_{\text{th}} - \sum_{j=1}^{K} q_j|\tilde{g}_j|^2\right). \qquad (15)$$

A. Solving the Dual Function

To solve the dual problem in (13), we need to first find the solution of (14). A Lagrange dual decomposition approach is used to obtain the optimum dual function for the given dual variables λ, μ, $\tilde{\lambda}$, and $\tilde{\mu}$.

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[2]The interference threshold is adopted based on quality-of-service requirements of primary users.

[3]More power is allocated to the stronger subcarriers while less power is allocated to the weaker subcarriers.

From subcarrier pairing constraint (4), we have $\sum_{k=1}^{K} d_k = \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} d_k$, $\sum_{j=1}^{K} e_j = \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} e_j$, where $d_k \in \{p_k, p_k |\tilde{h}_k|^2\}$ and $e_j \in \{q_j, q_j |\tilde{g}_j|^2\}$. Using these observations, the objective in (14) can be restated as in (16), shown at the bottom of the page.

Thus, (14) is decomposed into the following two subproblems.

1) Subproblem 1: For any valid subcarrier pair (k, j), i.e., $\pi_{k,j} = 1$, the optimal power allocation p_k^* and q_j^* could be found from

$$\max_{p_k, q_j} \quad c_{(k,j)} - p_k \left(\lambda + \tilde{\lambda} |\tilde{h}_k|^2 \right) - q_j \left(\mu + \tilde{\mu} |\tilde{g}_j|^2 \right)$$

$$\text{s.t.} \quad p_k \geq 0, \; q_j \geq 0. \tag{17}$$

Since (17) is a standard convex problem, the optimal solution can be obtained from the Karush–Kuhn–Tucker (KKT) conditions as

$$p_k^* = \frac{1}{\left(1 + \sqrt{\frac{\alpha_{(k,j)}}{\beta_{(k,j)}}}\right)} \left(\frac{1}{\lambda + \tilde{\lambda} |\tilde{h}_k|^2} - \frac{\left(\sqrt{\alpha_{(k,j)}} + \sqrt{\beta_{(k,j)}}\right)^2}{a_k \beta_{(k,j)}} \right)^+ \tag{18}$$

$$q_j^* = \frac{1}{\left(1 + \sqrt{\frac{\beta_{(k,j)}}{\alpha_{(k,j)}}}\right)} \left(\frac{1}{\mu + \tilde{\mu} |\tilde{g}_j|^2} - \frac{\left(\sqrt{\alpha_{(k,j)}} + \sqrt{\beta_{(k,j)}}\right)^2}{\alpha_{(k,j)} b_j} \right)^+ \tag{19}$$

where $a_k \triangleq |h_k|^2 / \sigma_k^2$, $b_j \triangleq |g_j|^2 / \sigma_j^2$, $\alpha_{(k,j)} \triangleq a_k (\mu + \tilde{\mu} |\tilde{g}_j|^2)$, $\beta_{(k,j)} \triangleq (\lambda + \tilde{\lambda} |\tilde{h}_k|^2) b_j$, and $(v)^+ \triangleq \max(0, v)$. Detailed derivation steps are provided in the Appendix.

Remark: Different from the conventional water-filling solution where power depends only on the dual variables (water-level) that are associated with the power constraint (i.e., λ, μ), the water level here also depends on the dual variables that are associated with the PU's interference constraints (i.e., $\tilde{\lambda}, \tilde{\mu}$). Further, (18) and (19) show that the power at the kth subcarrier of the first hop and the jth subcarrier of the second hop also depends on the corresponding interference channel (i.e., $\tilde{h}_k$ and $\tilde{g}_j$, respectively), resulting in a different water level for each subcarrier. Moreover, the power allocation over the kth subcarrier at the SSN also depends on the jth subcarrier's interference channel of the SRN-to-PDN link. Similarly, the power allocation over the jth subcarrier at the SRN also depends on the kth subcarrier's interference channel of the SSN-to-PDN link.

Substituting (18) and (19) into (16), the dual function becomes

$$D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu}) = \max_{\boldsymbol{\pi}} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} F_{(k,j)}(\lambda, \mu, \tilde{\lambda}, \tilde{\mu})$$

$$+ \lambda P_{\text{SSN}} + \mu P_{\text{SRN}} + I_{\text{th}}(\tilde{\lambda} + \tilde{\mu})$$

$$\text{s.t.} \quad (4) \tag{20}$$

where function $F_{(k,j)}$ is obtained by using the values of p_k^* and q_j^* into the objective of (17) and is expressed as (21), shown at the bottom of the page, with $x_{(k,j)} \triangleq (\lambda + \tilde{\lambda} |\tilde{h}_k|^2)(\sqrt{\alpha_{(k,j)}} + \sqrt{\beta_{(k,j)}})^2$ and $y_{(k,j)} \triangleq (\mu + \tilde{\mu} |\tilde{g}_j|^2)(\sqrt{\alpha_{(k,j)}} + \sqrt{\beta_{(k,j)}})^2$. Note that $F_{(k,j)}$ is independent of power variables p_k and q_j and is defined for given values of the dual variables.

2) Subproblem 2: For known power allocation, subcarrier pairing can be obtained from

$$\max_{\boldsymbol{\pi}} \quad \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} F_{(k,j)}(\lambda, \mu, \tilde{\lambda}, \tilde{\mu})$$

$$\text{s.t.} \quad (4). \tag{22}$$

$$D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu}) = \max_{\mathbf{p}, \mathbf{q}, \boldsymbol{\pi}} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \left(c_{(k,j)} - p_k \left(\lambda + \tilde{\lambda} |\tilde{h}_k|^2 \right) - q_j \left(\mu + \tilde{\mu} |\tilde{g}_j|^2 \right) \right) + \lambda P_{\text{SSN}} + \mu P_{\text{SRN}} + I_{\text{th}}(\tilde{\lambda} + \tilde{\mu})$$

$$= \max_{\boldsymbol{\pi}} \left\{ \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \cdot \max_{p_k, q_j} \left(c_{(k,j)} - p_k \left(\lambda + \tilde{\lambda} |\tilde{h}_k|^2 \right) - q_j \left(\mu + \tilde{\mu} |\tilde{g}_j|^2 \right) \right) \right\}$$

$$+ \lambda P_{\text{SSN}} + \mu P_{\text{SRN}} + I_{\text{th}}(\tilde{\lambda} + \tilde{\mu}) \tag{16}$$

$$F_{(k,j)} = \log_2 \left(\frac{a_k b_j \alpha_{(k,j)} \beta_{(k,j)} - a_k \alpha_{(k,j)} y_{(k,j)} - b_j \beta_{(k,j)} x_{(k,j)} + x_{(k,j)} y_{(k,j)}}{\left(1 + \sqrt{\frac{\beta_{(k,j)}}{\alpha_{(k,j)}}}\right) \left(\alpha_{(k,j)}^2 \beta_{(k,j)} - \frac{\alpha_{(k,j)}^2 x_{(k,j)}}{a_k}\right) + \left(1 + \sqrt{\frac{\alpha_{(k,j)}}{\beta_{(k,j)}}}\right) \left(\alpha_{(k,j)} \beta_{(k,j)}^2 - \frac{\beta_{(k,j)}^2 y_{(k,j)}}{b_j}\right)} \right)$$

$$- \left(\frac{\left(a_k \beta_{(k,j)} - x_{(k,j)}\right) \left(b_j \alpha_{(k,j)} + b_j \sqrt{\alpha_{(k,j)} \beta_{(k,j)}}\right) + \left(b_j \alpha_{(k,j)} - y_{(k,j)}\right) \left(a_k \beta_{(k,j)} + a_k \sqrt{\alpha_{(k,j)} \beta_{(k,j)}}\right)}{a_k b_j \left(\beta_{(k,j)} + \sqrt{\alpha_{(k,j)} \beta_{(k,j)}}\right) \left(\alpha_{(k,j)} + \sqrt{\alpha_{(k,j)} \beta_{(k,j)}}\right)} \right) \tag{21}$$

We define a $K \times K$ profit matrix $\boldsymbol{F}$ such that

$$\boldsymbol{F} = \begin{bmatrix} F_{(1,1)} & F_{(1,2)} & \cdots & F_{(1,K)} \\ F_{(2,1)} & F_{(2,2)} & \cdots & F_{(2,K)} \\ \vdots & & \vdots & \vdots \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ F_{(K-1,1)} & F_{(K-1,2)} & \cdots & F_{(K-1,K)} \\ F_{(K,1)} & F_{(K,2)} & \cdots & F_{(K,K)} \end{bmatrix} \quad (23)$$

where the row and column indexes denote different customers and different items to sell, respectively, whereas the value of each entry can be deemed as profit. Problem (22) is equivalent to maximizing the sum profit by an optimal selling strategy that can only sell one item to one customer. This kind of linear assignment problem can be efficiently solved using the standard Hungarian algorithm with complexity $O(K^3)$ [29] and yields optimum subcarrier pairing $\boldsymbol{\pi}^*$.

B. Solving the Dual Problem With Subgradient Method

Substituting $\mathbf{p}^*$, $\mathbf{q}^*$, and $\boldsymbol{\pi}^*$ into (15) and then the result back into (14), we obtain the optimal dual function for the given values of the dual variables. The optimal dual variables can be obtained from dual problem (13) using the subgradient method [30]. For any initial values $\lambda^{(0)}$, $\mu^{(0)}$, $\tilde{\lambda}^{(0)}$, and $\tilde{\mu}^{(0)}$, the dual variables at the $(i+1)$th iteration could be updated as

$$\lambda^{(i+1)} = \left(\lambda^{(i)} - \delta^{(i)} \Delta^{(i)}(\lambda) \right)^+$$

$$\mu^{(i+1)} = \left(\mu^{(i)} - \delta^{(i)} \Delta^{(i)}(\mu) \right)^+ \quad (24)$$

$$\tilde{\lambda}^{(i+1)} = \left(\tilde{\lambda}^{(i)} - \delta^{(i)} \Delta^{(i)}(\tilde{\lambda}) \right)^+$$

$$\tilde{\mu}^{(i+1)} = \left(\tilde{\mu}^{(i)} - \delta^{(i)} \Delta^{(i)}(\tilde{\mu}) \right)^+ \quad (25)$$

where $\delta^{(i)}$ is the appropriate step size of the ith iteration. Moreover, $\Delta(\lambda)$, $\Delta(\mu)$, $\Delta(\tilde{\lambda})$, and $\Delta(\tilde{\mu})$ are the subgradients of $D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu})$ whose expressions are given by

$$\Delta(\lambda) = P_{\text{SSN}} - \sum_{k=1}^{K} p_k^* \quad \Delta(\mu) = P_{\text{SRN}} - \sum_{j=1}^{K} q_j^* \quad (26)$$

$$\Delta(\tilde{\lambda}) = I_{\text{th}} - \sum_{k=1}^{K} |\tilde{h}_k|^2 p_k^* \quad \Delta(\tilde{\mu}) = I_{\text{th}} - \sum_{j=1}^{K} |\tilde{g}_j|^2 q_j^* \quad (27)$$

where p_k^* and q_j^* are the optimal power values obtained from (18) and (19) at λ, μ, $\tilde{\lambda}$, and $\tilde{\mu}$.[4]

Remark: The updates of the subgradients follow an interesting supply and demand strategy. For example, in $\Delta(\lambda)$, λ can be considered as the price for power at the source node. The price will be increased if the demand (the total allocated power) is higher than the available stock (the maximum available power). If the demand is less than the available power budget, the price is decreased to encourage the SSN to increase its transmit power.

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[4]Note that for each iteration, all variables $\boldsymbol{\pi}^*$, p_k^*, and q_j^* should be recomputed under $\lambda^{(i)}$, $\mu^{(i)}$, $\tilde{\lambda}^{(i)}$, and $\tilde{\mu}^{(i)}$. The iterations stop once a certain criterion is fulfilled.

C. Solving Problems P2, P3, and P4

Optimization problems (10)–(12) can be solved with similar steps as those in Section III-A and B. Some differences are described here.

In P2, there are $2(K+1)$ dual variables to be updated, i.e.,

$$\tilde{\lambda}_k^{(i+1)} = \left(\tilde{\lambda}_k^{(i)} - \delta^{(i)} \Delta^{(i)}(\tilde{\lambda}_k) \right)^+$$

$$\tilde{\mu}_j^{(i+1)} = \left(\tilde{\mu}_j^{(i)} - \delta^{(i)} \Delta^{(i)}(\tilde{\mu}_j) \right)^+ \quad (28)$$

where $\Delta(\tilde{\lambda}_k) \triangleq I_{\text{th}}^k - |\tilde{h}_k|^2 p_k^*$, $\Delta(\tilde{\mu}_j) \triangleq I_{\text{th}}^j - |\tilde{g}_j|^2 q_j^*$, and $\tilde{\lambda}_k$ and $\tilde{\mu}_j$ are the dual variables associated with the kth and the jth constraints in (6), respectively. The mathematical expressions for p_k^* and q_j^* are similar to those in (18) and (19) and could be obtained by replacing $\tilde{\lambda}$ and $\tilde{\mu}$ with $\tilde{\lambda}_k$ and $\tilde{\mu}_j$.

In P3, the separate power constraints in (7) are combined into a single total power constraint. Let ξ be the dual variable associated with the total power constraint in (8). It should be updated from

$$\xi^{(i+1)} = \left(\xi^{(i)} - \delta^{(i)} \left(P_T - \left(\sum_{k=1}^{K} p_k^* + \sum_{j=1}^{K} q_j^* \right) \right) \right)^+ \quad (29)$$

where p_k^* and q_j^* are obtained by substituting $\lambda = \xi$ and $\mu = \xi$ into (18) and (19).

In P4, both (28) and (29) should be taken into consideration.

IV. Suboptimal Algorithm

The joint subcarrier pairing and power loading scheme in the previous section provides a near-optimal solution when the number of subcarriers is large. However, computational complexity also increases when the number of subcarriers increases. Here, we propose a suboptimal approach that trades performance for lower complexity by separating power allocation from subcarrier pairing.

We solve optimization (9) using a stepwise approach. In the first step of the following algorithm, we find subcarrier pairing for an anticipated power allocation, and then, in the second step, power allocation is done for the obtained pairing.

1) Step 1: It is desirable to allocate more power to good channels (i.e., h_k and g_k) to enhance the SU's throughput. Meanwhile, it is also desirable to allocate less power to the strong interference channels ($\tilde{h}_k$ and $\tilde{g}_j$) to reduce interference to PUs. Motivated by these, let us first define the ratio between the actual channel gain to the interference channel gain as

$$H_k = \frac{I_{\text{th}} |h_k|^2}{|\tilde{h}_k|^2}, \quad \forall k, \qquad G_j = \frac{I_{\text{th}} |g_j|^2}{|\tilde{g}_j|^2}, \quad \forall j.$$

Then, if $H_k > H_{k'}$, we should set $p_k > p_{k'}$. Similarly, for $G_j > G_{j'}$, we should set $q_j > q_{j'}$. We then propose to pair the carriers in the sorted order of H_k and G_j, i.e., the carrier with the strongest H_k is paired with that with the strongest G_j.

2) Step 2: Next, we seek for power allocation given the subcarrier pairing obtained in step 1. Assume that subcarrier

k is paired with subcarrier $m(k)$, where $m(\cdot)$ denotes the corresponding mapping function. The optimization in (9) is then

$$\max_{\mathbf{p},\mathbf{q}} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\left(\sqrt{p_k}|h_k| \sqrt{q_{m(k)}} |g_{m(k)}| \right)^2}{\sigma_k^2 \left(\sqrt{p_k}|h_k| \right)^2 + \sigma_{m(k)}^2 \left(\sqrt{q_{m(k)}} |g_{m(k)}| \right)^2} \right)$$

$$\text{s.t.} \sum_{k=1}^{K} p_k |\tilde{h}_k|^2 \leq I_{\text{th}}, \sum_{k=1}^{K} q_{m(k)} \left| \tilde{g}_{m(k)} \right|^2 \leq I_{\text{th}},$$

$$\sum_{k=1}^{K} p_k \leq P_{\text{SSN}}, \sum_{k=1}^{K} q_{m(k)} \leq P_{\text{SRN}} \qquad (30)$$

which can be solved through the dual decomposition technique, similarly as in Section III. Hence, (16) becomes

$$D(\lambda, \mu, \tilde{\lambda}, \tilde{\mu}) = \max_{\mathbf{p},\mathbf{q}} \sum_{k=1}^{K} \left(\log_2(1 + \text{SNR}_k) - p_k \left(\lambda + \tilde{\lambda}|\tilde{h}_k|^2 \right) \right.$$

$$\left. - q_{m(k)} \left(\mu + \tilde{\mu} \left| \tilde{g}_{m(k)} \right|^2 \right) \right)$$

$$+ \lambda P_{\text{SSN}} + \mu P_{\text{SRN}} + I_{\text{th}} \left(\tilde{\lambda} + \tilde{\mu} \right) \qquad (31)$$

where $\text{SNR}_k = (\sqrt{p_k}|h_k| \sqrt{q_{m(k)}}|g_{m(k)}|)^2 / \sigma_k^2 (\sqrt{p_k}|h_k|)^2 + \sigma_{m(k)}^2 (\sqrt{q_{m(k)}}|g_{m(k)}|)^2$. The given problem could be decomposed into K subproblems, and then, the KKT conditions can be applied for each. Optimum power variables p_k^* and $q_{m(k)}^*$ could be found from (18) and (19) by replacing j with $m(k)$. Similarly, the dual variable updates are obtained from (24) and (25).

This algorithm could also be extended to P2, P3, and P4. If different subcarriers have different interference requirements, then H_k and G_j can be redefined as

$$H_k = \frac{I_{\text{th}}^k |h_k|^2}{|\tilde{h}_k|^2}, \quad \forall k, \qquad G_j = \frac{I_{\text{th}}^j |g_j|^2}{|\tilde{g}_j|^2}, \quad \forall j.$$

V. SIMULATION RESULTS AND DISCUSSIONS

This section provides numerical examples to demonstrate the performance of the developed schemes. We assume six-tap channels taken from independent and identically distributed Gaussian random variables for all links and choose $K = 32$. Without loss of generality, we assume that $P_{\text{SSN}} = P_{\text{SRN}} = P_{\text{max}}$. Further, noise variances at the SRN and the SDN are set as $\sigma_k^2 = \sigma_j^2, \forall k, j$. The figure of merit is taken as the per-tone rate of the SU, i.e., the sum rate divided by K. The value of the step size for subgradient updates is taken as $\delta = 1 \times 10^{-3}$. We compare the following four different algorithms.

- JntSol: The joint optimization solution presented in Section III.
- PowSol: The algorithm where only power allocation is performed while subcarrier pairing is not considered. The problem is equivalent to the optimization in (30) with $m(k) = k, \forall k$, and the solution follows the same procedure described in the second step of the suboptimal algorithm.
- LCA: The solution that corresponds to the low-complexity suboptimal algorithm presented in Section IV.

- SUB: A suboptimal algorithm based on the equal-power-allocation scheme developed in [9] for the single-hop system, where, at each node, power is equally distributed among the carriers subject to the condition that the interference constraint is satisfied. Specifically, for our system, the power for P1, P2, P3, and P4 is $v_i = \min(P_{\text{max}}/K, I_{\text{th}}/\sum_{i=1}^{K} \tilde{f}_i)$, $v_i = \min(P_{\text{max}}/K, \min_i(I_{\text{th}}^i/\tilde{f}_i))$, $v_i = \min(P_{\text{max}}/2K, I_{\text{th}}/\sum_{i=1}^{K} \tilde{f}_i)$, and $v_i = \min(P_{\text{max}}/2K, \min_i(I_{\text{th}}^i/\tilde{f}_i))$, respectively, with $v_i \in \{p_i, q_i\}$, $\tilde{f}_i \in \{|\tilde{h}_i|^2, |\tilde{g}_i|^2\}$, $i \in \{k, j\}$.

A. Complexity and Convergence

We first look into the complexities involved in the proposed schemes and then show the convergence behavior of different algorithms.

Algorithm 1: Joint optimization scheme
1) Initialize dual variables and step size.
2) Obtain the power allocation and find $F_{(k,j)}$ for all k, j such that
    for $k = 1$ to K
    for $j = 1$ to K
    a) Find p_k^* and q_j^* using (18) and (19), respectively.
    b) Find $F_{(k,j)}$ from (21)
    end for
    end for
3) Obtain $\mathbf{F}$ and obtain $\boldsymbol{\pi}^*$ from Hungarian method.
4) Update dual variables using (24) and (25).
5) Repeat step 2 to step 4 until convergence

Algorithm 2: Suboptimal scheme
1) Find the subcarrier pairing in the sorted order of H_k and G_j for all (k, j) and obtain the mapping function $m(.)$ accordingly.
2) Initialize dual variables and step size.
3) Obtain the power allocation such that
    for $k = 1$ to K
    Find p_k^* and $q_j^* = q_{m(k)}$ using (18) and (19), respectively.
    end for
4) Update dual variables using (24) and (25).
5) Repeat step 3 to step 4 until convergence

The different steps involved in the joint optimization scheme are summarized in Algorithm 1. In step 2, computing p_k and q_j for all k, j requires a complexity of $O(K^2)$ (multiplications and additions). Furthermore, obtaining $F_{(k,j)}$ involves $O(K^2)$ operations (log, multiplications, additions), where the subcarrier pairing in step 3 is found through the Hungarian algorithm, requiring a complexity of $O(K^3)$. If the objective of the dual function in (13) is minimized in I iterations, the total computational complexity of the joint resource-allocation algorithm in section becomes $O(IK^3)$.

The proposed suboptimal algorithm (LCA) is given in Algorithm 2, where the subcarrier pairing in step 1 involves a

TABLE I
COMPLEXITY COMPARISON OF DIFFERENT ALGORITHMS

Algorithm	Complexity	Running Time (Seconds)			
		P1	P2	P3	P4
JntSol	$O(IK^3)$	366.2310	948.64	87.1563	726.5781
PowSol	$O(I''K)$	1.2969	3.1563	0.0313	2.4688
LCA	$O(I'K)$	0.4375	3.1719	0.0156	1.5469
SUB	$O(K)$	0.0156	0.021	0.010	0.0182
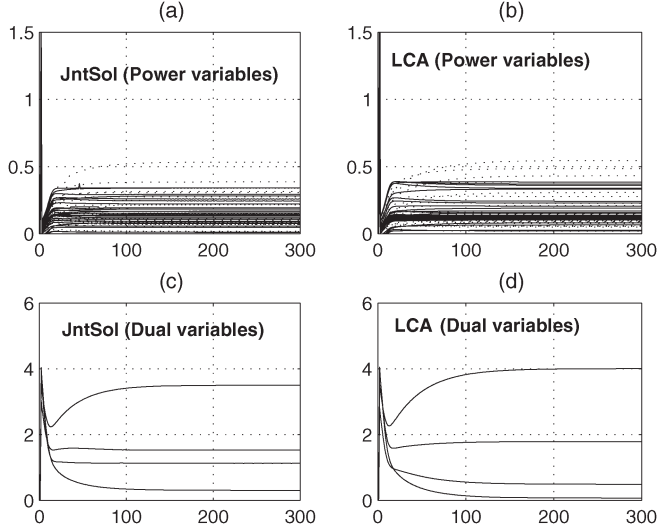
Fig. 2. Convergence for optimization P1. (a) JntSol: primary variables (p_k, q_j). (b) LCA: primary variables (p_k, q_j). (c) JntSol: dual variables $(\lambda, \mu, \tilde{\lambda}, \text{and } \tilde{\mu})$. (d) LCA: dual variables $(\lambda, \mu, \tilde{\lambda}, \text{and } \tilde{\mu})$.

maximum of $2K$ computations (divisions). Further, the power allocation in step 3 requires K operations (additions, multiplications), and thus, the complexity of solving (30) is $O(I'K)$, where I' is the number of iterations required in subgradient search. Without loss of generality, we can set I' close to I. Then, the overall complexity of the proposed suboptimal algorithm is much less than the complexity of the joint resource-allocation schemes.

Table I summarizes the complexities involved in each algorithm, where I'' represents the number of iterations needed for the convergence of PowSol. Moreover, the running times of different algorithms for a channel realization at $P_{\max} = 5$ reference.[5] The PU's interference thresholds are set as $= I_{\text{th}}^k = I_{\text{th}}^j = I_t h/K = 0.1$.

We look into the convergence behavior of two algorithms proposed in Sections III and IV in Figs. 2–4, where the x-axis represents the number of iterations. Fig. 2 shows the convergence of JntSol and LCA for problem P1. The convergence of the $2K$ power variables (solid lines are $p_k, \forall k$; dashed lines show $q_j, \forall j$) and the four dual variables $(\lambda, \mu, \tilde{\lambda}, \tilde{\mu})$ in JntSol are shown in Fig. 2(a) and (c), whereas Fig. 2(b) and (d) shows the convergence of the primal $(p_k, q_j, \forall k, j)$ and dual variables $(\lambda, \mu, \tilde{\lambda}, \tilde{\mu})$ for LCA, respectively. The total power at the secondary transmitting nodes is taken as $P_{\max} = 5$, and the maximum allowable interference power at the PU is set

[5]Although using running time cannot strictly reflect computational complexity, it can still well demonstrate the difference among all the compared algorithms.
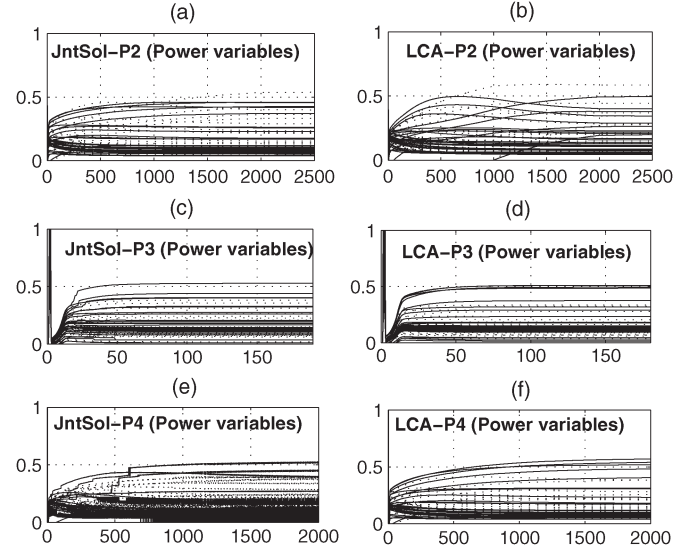

Fig. 3. Convergence behavior of primary variables (p_k, q_j) for optimization P2–P4. (a) JntSol: Optimization P2. (b) LCA: Optimization P2. (c) JntSol: Optimization P3. (d) LCA: Optimization P3. (e) JntSol: Optimization P4. (f) LCA: Optimization P4.
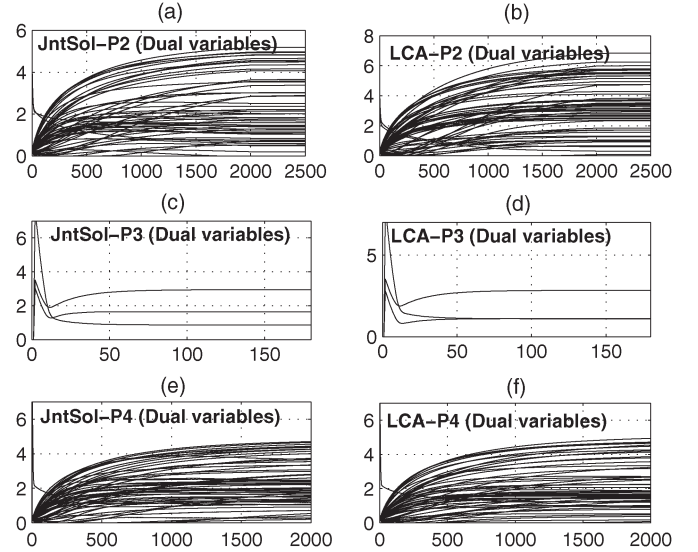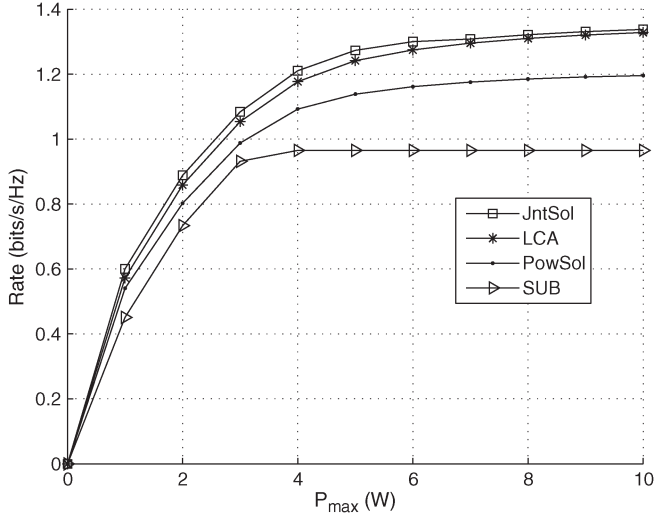

Fig. 4. Convergence behavior of dual variables for optimization P2–P4. (a) JntSol: Optimization P2. (b) LCA: Optimization P2. (c) JntSol: Optimization P3. (d) LCA: Optimization P3. (e) JntSol: Optimization P4. (f) LCA: Optimization P4.

as $I_{\text{th}}/K = 0.1$. We observe that as the dual variables converge, the primary variables also converge. Furthermore, both the joint optimal solution and the proposed low-complexity algorithms converge after an almost equal number of iterations, which supports our previous statement that I can be set close to I'.

Figs. 3 and 4 show the convergence behavior of variables for problems P2, P3, and P4. The different variables plotted in the subplots are summarized in Table II. In Fig. 3, all power variables at the SSN $(p_k, \forall k)$ are represented with solid blue lines, and all $q_j, \forall j$ are shown with dotted red lines. We notice that the problem under the sum power constraint and the sum interference constraint, i.e., P3, requires the least number of

TABLE II
VARIABLES PLOTTED IN FIGS. 2–4

Subplot	a	b	c	d	e	f
Fig. 2	JntSol-P1:$p_k, q_j, \forall k, j$	LCA-P1:$p_k, q_j, \forall k, j$	JntSol-P1:$\lambda, \mu, \tilde{\lambda}, \tilde{\mu}$	LCA-P1:$\lambda, \mu, \tilde{\lambda}, \tilde{\mu}$	-	-
Fig. 3	JntSol-P2:$p_k, q_j, \forall k, j$	LCA-P2:$p_k, q_j, \forall k, j$	JntSol-P3:$p_k, q_j, \forall k, j$	LCA-P3:$p_k, q_j, \forall k, j$	JntSol-P4:$p_k, q_j, \forall k, j$	LCA-P2:$p_k, q_j, \forall k, j$
Fig. 4	JntSol-P2:$\lambda, \mu, \tilde{\lambda}_k, \forall k, \tilde{\mu}_j, \forall j$	LCA-P2:$\lambda, \mu, \tilde{\lambda}_k, \forall k, \tilde{\mu}_j, \forall j$	JntSol-P3:λ, μ, ξ	LCA-P3:λ, μ, ξ	JntSol-P4:$\xi, \tilde{\lambda}_k, \forall k, \tilde{\mu}_j, \forall j$	LCA-P2:$\xi, \tilde{\lambda}_k, \forall k, \tilde{\mu}_j, \forall j$

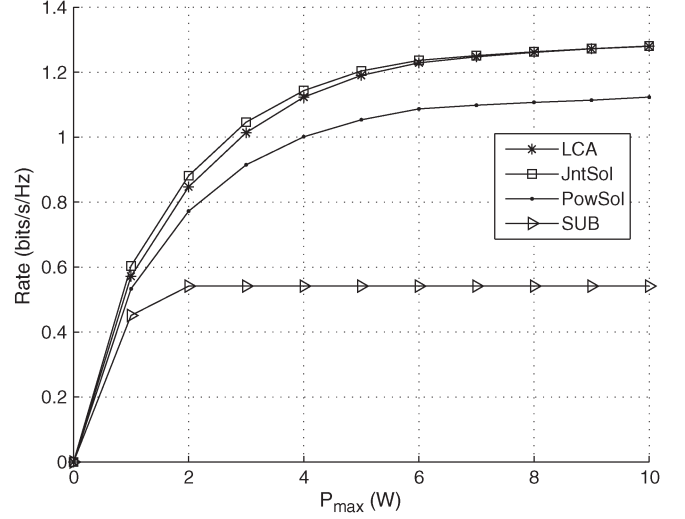
Fig. 5. Rate versus P_{max}. Optimization under the sum interference constraint and the per-node power constraint.

Fig. 6. Rate versus P_{max}. Optimization under the per-subcarrier interference constraint and the per-node power constraint.

iterations to converge, whereas P2 requires the utmost number of iterations. Further, we see that in all problems, JntSol and LCA require similar number of iterations. On the other hand, the running time of LCA is much less than that of JntSol in all of the four problems. This is due to the high time complexity of the Hungarian algorithm required in each iteration of JntSol. However, the difference in running times of LCA and PowSol comes from the fast convergence of the LCA, i.e., PowSol requires more iterations to converge than that of LCA.

B. Performance

Now, we check the performance of the proposed algorithms. In the first example, the performance comparison of four different algorithms under separate power and sum interference constraints is shown in Fig. 5. We display the secondary system's throughput versus P_{max} at $I_{th}/K = 0.1$. We observe that JntSol exhibits the best performance at all P_{max} values among all different schemes. Moreover, the proposed suboptimal schemes also provide much better performance over the trivial method (the SUB algorithm). The proposed suboptimal power allocation and subcarrier pairing method, i.e., LCA, achieves large performance gains over the PowSol method and gives very close performance to that of JntSol with much less computational complexity.

Next, we examine the performance of the end-to-end rate under separate power and per-subcarrier interference constraints, i.e., we consider the optimization in (10). The corresponding curves at $I_{th}^k = I_{th}^j = 0.1$ are shown in Fig. 6, and to compare with the results in Fig. 5, we set $KI_{th}^k = KI_{th}^j = I_{th}$. Similar

to the previous example, JntSol yields the best performance among the different methods, and the proposed suboptimal method (LCA) outperforms the solutions without subcarrier pairing (PowSol, SUB). Comparing the results with Fig. 5, all of the proposed schemes undergo a small performance loss at all P_{max} values, which is due to the strict protection criteria imposed by the PU. On the other hand, for the SUB method, we notice significant performance degradation in the high-P_{max} regime. Thus, the proposed optimization schemes are more resistive to the per-subcarrier interference threshold.

It is then of interest to check the impact of the interference threshold on the performance of the proposed methods. We consider optimization problem P1 and display the throughput curves versus I_{th} at $P_{max} = 2$ in Fig. 7. The key observation is that the gap between JntSol and SUB first increases with an increasing interference threshold and finally becomes constant for high values of I_{th}. This is due to the fact that in the high-I_{th} region, the system performance is restricted by power constraint (7) and becomes insensitive to I_{th}. Other observations are similar to those in Fig. 5.

To obtain more insight on the impact of subcarrier pairing, in this example, we compare JntSol with PowSol and SUB for the following four cases.

Case 1: This is the same as in Fig. 5.
Case 2: $|h_1|^2 > |h_2|^2 \ldots > |h_K|^2$ and $|g_1|^2 < |g_2|^2 \ldots < |g_K|^2$.
Case 3: $|h_1|^2 > |h_2|^2 \ldots > |h_K|^2$, $|\tilde{h}_1|^2 > |\tilde{h}_2|^2 \ldots > |\tilde{h}_K|^2$, $|g_1|^2 > |g_2|^2 \ldots > |g_K|^2$, and $|\tilde{g}_1|^2 < |\tilde{g}_2|^2 \ldots < |\tilde{g}_K|^2$.
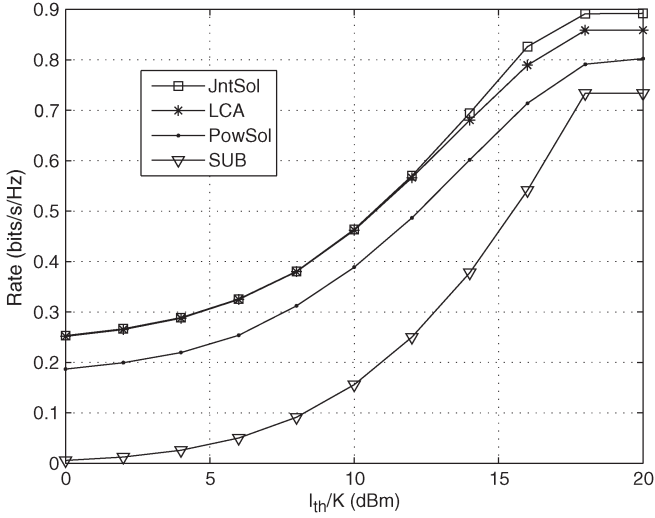Case 4: $|h_1|^2 > |h_2|^2 \ldots > |h_K|^2$, $|\tilde{h}_1|^2 < |\tilde{h}_2|^2 \ldots < |\tilde{h}_K|^2$, $|g_1|^2 < |g_2|^2 \ldots < |g_K|^2$, and $|\tilde{g}_1|^2 > |\tilde{g}_2|^2 \ldots > |\tilde{g}_K|^2$.

Fig. 7.   Rate versus I_{th}. Optimization under the sum interference constraint and the per-node power constraint.
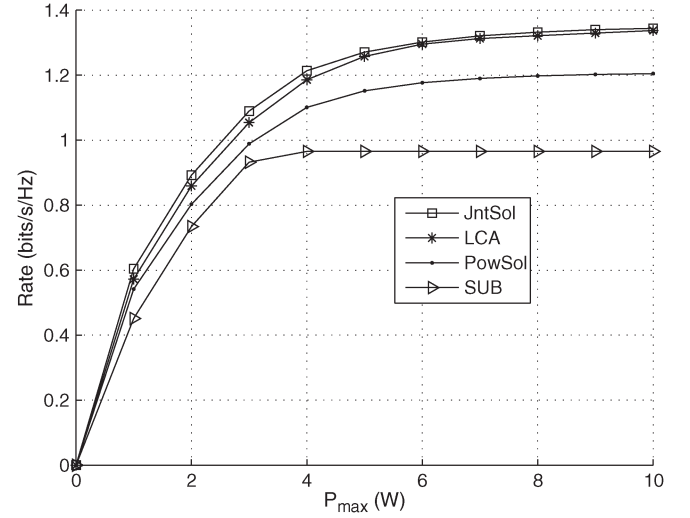

Fig. 9.   Rate versus $P_{\max}$. Optimization under the sum interference constraint and the aggregate power constraint.
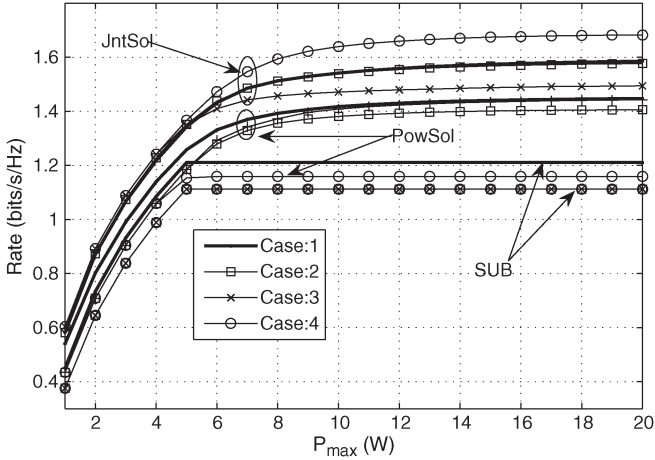

Fig. 8.   Rate versus $P_{\max}$. Optimization under the sum interference constraint and the per-node power constraint.

Fig. 10.   Rate versus $P_{\max}$. Optimization under the per-subcarrier interference constraint and the aggregate power constraint.

The throughput versus $P_{\max}$ for the four cases at $I_{\text{th}}/K = 0.15$ is shown in Fig. 8. It can be seen that in Case 2, JntSol exhibits almost the same performance as in Case 1, where PowSol and SUB experience performance loss. This is because in PowSol and SUB algorithms, the stronger carrier at the first hop will always be matched to the weaker carrier over the second hop. For Case 3, all of the schemes (JntSol, PowSol, and SUB) undergo rate loss as compared with Case 1. The PowSol and SUB algorithms observe loss for a similar reason as in Case 2, whereas JntSol experiences performance degradation due to the fact that a subcarrier with the highest interference gain is always paired with a subcarrier of the highest interference. Nevertheless, JntSol still exhibits the best performance over the others. In Case 4, we first observe that the solution with equal power allocation is the same as in Case 2 and Case 3. However, PowSol and JntSol exhibit the most interesting behavior, i.e., PowSol experiences significant performance degradation, and JntSol yields much better performance compared with all the other cases. The gain in JntSol is due to the fact that the strongest subcarrier (with the smallest interference) is paired with the
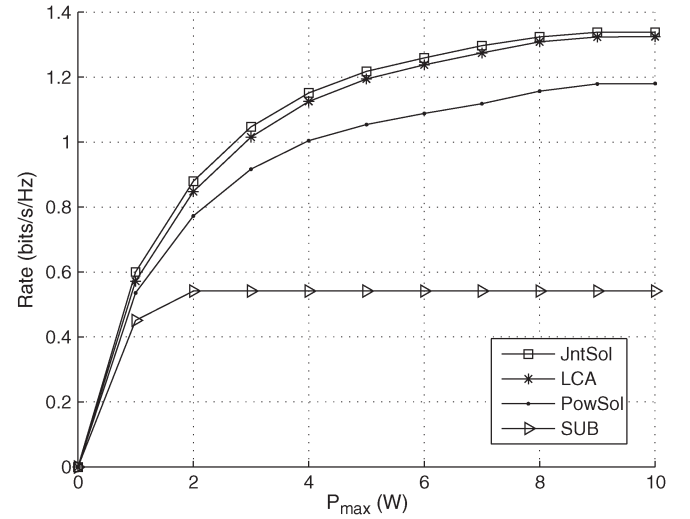
strongest subcarrier (with the weakest interference), whereas PowSol suffers from the loss because a stronger subcarrier (with low interference) is paired with a weaker subcarrier (with higher interference).

Finally, we check the performance of the different schemes under the sum power constraint (8). The results with sum interference and per-subcarrier interference constraints are shown in Figs. 9 and 10, respectively. For comparison, we assume $P_T = P_{\text{SSN}} + P_{\text{SRN}} = 2P_{\max}$ and set $I_{\text{th}}^k = I_{\text{th}}^j = 0.1$, $\forall k, j$. Similar to the previous examples, the proposed schemes yield significant gain over the trivial ones. Further, better performance is observed as compared with the results in Figs. 5 and 6. This is because the sum power constraint provides more flexibility in power allocation over the two hops. In Figs. 5, 6, 9, and 10, we observe that the throughput increases with $P_{\max}$ and becomes flat in the high-$P_{\max}$ region due to the interference constraints of the PU. It is further seen that in the SUB algorithm, the interference constraint becomes active

at low $P_{\max}$ values as compared with the optimized power-allocation schemes and results in a large performance gap.

In Figs. 2–4, we can see that the number of iterations required for the convergence of the joint resource-allocation algorithm and the suboptimal algorithm is almost equal. However, the number of computations under each iteration is much greater in the joint optimization solution, as shown in Algorithm 1 and Algorithm 2. Furthermore, the suboptimal algorithm requires much less running time as compared with JntSol, as shown in Table I. On the other hand, in Figs. 5, 6, 9, and 10, we notice that the proposed suboptimal algorithm sacrifices very little on performance in comparison to the joint solution and yields significant gain over the trivial solutions. Further, similar behavior can be observed in Fig. 7, which shows the robustness of the suboptimal algorithm toward interference temperature. Thus, we conclude that the proposed suboptimal algorithm enjoys significant less computational complexity for a very small performance loss over the joint optimization solution.

VI. CONCLUSION

In this paper, we have investigated the resource allocation problem in relay-aided OFDM CR networks. The SU's throughput is maximized via joint subcarrier pairing and power allocation. Meanwhile, the interference from both the secondary source and the SRN to the primary receiver is kept within an acceptable limit. A suboptimal resource-allocation algorithm was also designed to reduce computational complexity. The numerical examples demonstrated enhanced performance compared with trivial resource-allocation algorithms.

APPENDIX

The Lagrangian J associated with (17) is

$$J = c_{(k,j)} - p_k \left(\lambda + \tilde{\lambda}|\tilde{h}_k|^2\right) - q_j \left(\mu + \tilde{\mu}|\tilde{g}_j|^2\right)$$
$$+ \sum_{k=1}^{K} \eta_k p_k + \sum_{j=1}^{K} \gamma_j q_j \quad (32)$$

where η_k and γ_j are the Lagrange multipliers.

From KKT condition $\partial J / \partial p_k = 0$, we obtain

$$\eta_k = \left(\lambda + \tilde{\lambda}|\tilde{h}_k|^2\right) - \frac{a_k q_j^2 b_j^2}{\left(t_{(k,j)}\right)\left(t_{(k,j)} + a_k p_k b_j q_j\right)} \quad (33)$$

where $t_{k,j} \triangleq a_k p_k + b_j q_j$. From KKT condition $\eta_k \geq 0$, we obtain

$$\left(\lambda + \tilde{\lambda}|\tilde{h}_k|^2\right) \geq \frac{a_k q_j^2 b_j^2}{\left(t_{(k,j)}\right)\left(t_{(k,j)} + a_k p_k b_j q_j\right)}. \quad (34)$$

With the slackness condition $\alpha_{m,k} p_{m,k} = 0$, we further arrive at

$$p_k \left(\left(\lambda + \tilde{\lambda}|\tilde{h}_k|^2\right) - \frac{a_k q_j^2 b_j^2}{\left(t_{(k,j)}\right)\left(t_{(k,j)} + a_k p_k b_j q_j\right)}\right) = 0. \quad (35)$$

The following two cases hold.

a) If $(\lambda + \tilde{\lambda}|\tilde{h}_k|^2) < a_k$, then (34) can only be fulfilled when $p_k > 0$. In this case, (35) holds only if

$$\left(\lambda + \tilde{\lambda}|\tilde{h}_k|^2\right) = \frac{a_k q_j^2 b_j^2}{\left(t_{(k,j)}\right)\left(t_{(k,j)} + a_k p_k b_j q_j\right)}. \quad (36)$$

b) If $(\lambda + \tilde{\lambda}|\tilde{h}_k|^2) \geq a_k$, with $p_k > 0$, it is impossible to meet (35). This implies that $p_k = 0$.

Due to symmetry, for $q_j > 0$, we obtain

$$\left(\mu + \tilde{\mu}|\tilde{g}_j|^2\right) = \frac{b_j p_k^2 a_k^2}{\left(t_{(k,j)}\right)\left(t_{(k,j)} + a_k p_k b_j q_j\right)}. \quad (37)$$

Otherwise, $q_j = 0$.

From (36) and (37), we conclude that for a particular subcarrier pair (k, j), power values p_k and q_j are either both positive or both zero, i.e., if power allocated to a particular subcarrier over either hop is zero, no power is allocated to its corresponding subcarrier over the other hop, which meets the intuition very well. For $p_{m,k} > 0$ and $q_j > 0$, we can solve (36) and (37), which results in (18) and (19).

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