

An Improved Square-Root Algorithm for V-BLAST Based on Efficient Inverse Cholesky Factorization

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Abstract—A fast algorithm for inverse Cholesky factorization is proposed, to compute a triangular square-root of the estimation error covariance matrix for Vertical Bell Laboratories Layered Space-Time architecture (V-BLAST). It is then applied to propose an improved square-root algorithm for V-BLAST, which speedups several steps in the previous one, and can offer further computational savings in MIMO Orthogonal Frequency Division Multiplexing (OFDM) systems. Compared to the conventional inverse Cholesky factorization, the proposed one avoids the back substitution (of the Cholesky factor), and then requires only half divisions. The proposed V-BLAST algorithm is faster than the existing efficient V-BLAST algorithms. The expected speedups of the proposed square-root V-BLAST algorithm over the previous one and the fastest known recursive V-BLAST algorithm are $3.9 \sim 5.2$ and $1.05 \sim 1.4$, respectively.

Index Terms—MIMO, V-BLAST, square-root, fast algorithm, inverse Cholesky factorization.

I. INTRODUCTION

MULTIPLE-input multiple-output (MIMO) wireless communication systems can achieve huge channel capacities [1] in rich multi-path environments through exploiting the extra spatial dimension. Bell Labs Layered Space-Time architecture (BLAST) [2], including the relative simple vertical BLAST (V-BLAST) [3], is such a system that maximizes the data rate by transmitting independent data streams simultaneously from multiple antennas. V-BLAST often adopts the ordered successive interference cancellation (OSIC) detector [3], which detects the data streams iteratively with the optimal ordering. In each iteration, the data stream with the highest signal-to-noise ratio (SNR) among all undetected data streams is detected through a zero-forcing (ZF) or minimum mean-square error (MMSE) filter. Then the effect of the detected data stream is subtracted from the received signal vector.

Some fast algorithms have been proposed [4]–[11] to reduce the computational complexity of the OSIC V-BLAST detector [3]. An efficient square-root algorithm was proposed in [4] and then improved in [5], which also partially inspired

the modified decorrelating decision-feedback algorithm [6]. In addition, a fast recursive algorithm was proposed in [7] and then improved in [8]–[11]. The improved recursive algorithm in [8] requires less multiplications and more additions than the original recursive algorithm [7]. In [9], the author gave the “fastest known algorithm” by incorporating improvements proposed in [10], [11] for different parts of the original recursive algorithm [7], and then proposed a further improvement for the “fastest known algorithm”.

On the other hand, most future cellular wireless standards are based on MIMO Orthogonal Frequency Division Multiplexing (OFDM) systems, where the OSIC V-BLAST detectors [3]–[11] require an excessive complexity to update the detection ordering and the nulling vectors for each subcarrier. Then simplified V-BLAST detectors with some performance degradation are proposed in [12], [13], which update the detection [12] or the detection ordering [13] per group of subcarriers to reduce the required complexity.

In this letter, a fast algorithm for inverse Cholesky factorization [14] is deduced to compute a triangular square-root of the estimation error covariance matrix for V-BLAST. Then it is employed to propose an improved square-root V-BLAST algorithm, which speedups several steps in the previous square-root V-BLAST algorithm [5], and can offer further computational savings in MIMO OFDM systems.

This letter is organized as follows. Section II describes the V-BLAST system model. Section III introduces the previous square-root algorithm [5] for V-BLAST. In Section IV, we deduce a fast algorithm for inverse Cholesky factorization. Then in Section V, we employ it to propose an improved square-root algorithm for V-BLAST. Section VI evaluates the complexities of the presented V-BLAST algorithms. Finally, we make conclusion in Section VII.

In the following sections, $(\bullet)^T$, $(\bullet)^*$ and $(\bullet)^H$ denote matrix transposition, matrix conjugate, and matrix conjugate transposition, respectively. $\mathbf{0}_M$ is the $M \times 1$ zero column vector, while \mathbf{I}_M is the identity matrix of size M .

II. SYSTEM MODEL

The considered V-BLAST system consists of M transmit antennas and $N (\geq M)$ receive antennas in a rich-scattering and flat-fading wireless channel. The signal vector transmitted from M antennas is $\mathbf{a} = [a_1, a_2, \dots, a_M]^T$ with the covariance $E(\mathbf{a}\mathbf{a}^H) = \sigma_a^2 \mathbf{I}_M$. Then the received signal vector

$$\mathbf{x} = \mathbf{H} \cdot \mathbf{a} + \mathbf{w}, \quad (1)$$

where \mathbf{w} is the $N \times 1$ complex Gaussian noise vector with the zero mean and the covariance $\sigma_w^2 \mathbf{I}_N$, and

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_M] = [\underline{\mathbf{h}}_1, \underline{\mathbf{h}}_2, \dots, \underline{\mathbf{h}}_M]^H$$

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is the $N \times M$ complex channel matrix. \mathbf{h}_m and \mathbf{h}_n^H are the m^{th} column and the n^{th} row of \mathbf{H} , respectively.

Define $\alpha = \sigma_w^2 / \sigma_a^2$. The linear MMSE estimate of \mathbf{a} is

$$\hat{\mathbf{a}} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1} \mathbf{H}^H \mathbf{x}. \quad (2)$$

As in [4], [5], [7]–[11], we focus on the MMSE OSIC detector, which outperforms the ZF OSIC detector [7]. Let

$$\mathbf{R} = \mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M. \quad (3)$$

Then the estimation error covariance matrix [4]

$$\mathbf{P} = \mathbf{R}^{-1} = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I}_M)^{-1}. \quad (4)$$

The OSIC detection detects M entries of the transmit vector \mathbf{a} iteratively with the optimal ordering. In each iteration, the entry with the highest SNR among all the undetected entries is detected by a linear filter, and then its interference is cancelled from the received signal vector [3]. Suppose that the entries of \mathbf{a} are permuted such that the detected entry is a_M , the M^{th} entry. Then its interference is cancelled by

$$\mathbf{x}^{(M-1)} = \mathbf{x}^{(M)} - \mathbf{h}_M a_M, \quad (5)$$

where a_M is treated as the correctly detected entry, and the initial $\mathbf{x}^{(M)} = \mathbf{x}$. Then the reduced-order problem is

$$\mathbf{x}^{(M-1)} = \mathbf{H}_{M-1} \mathbf{a}_{M-1} + \mathbf{w}, \quad (6)$$

where the deflated channel matrix $\mathbf{H}_{M-1} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_{M-1}]$, and the reduced transmit vector $\mathbf{a}_{M-1} = [a_1, a_2, \dots, a_{M-1}]^T$. Correspondingly we can deduce the linear MMSE estimate of \mathbf{a}_{M-1} from (6). The detection will proceed iteratively until all entries are detected.

III. THE SQUARE-ROOT V-BLAST ALGORITHMS

The square-root V-BLAST algorithms [4], [5] calculate the MMSE nulling vectors from the matrix \mathbf{F} that satisfies

$$\mathbf{F} \mathbf{F}^H = \mathbf{P}. \quad (7)$$

Correspondingly \mathbf{F} is a square-root matrix of \mathbf{P} . Let

$$\mathbf{H}_m = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_m] \quad (8)$$

denote the first m columns of \mathbf{H} . From \mathbf{H}_m , we define the corresponding \mathbf{R}_m , \mathbf{P}_m and \mathbf{F}_m by (3), (4) and (7), respectively. Then the previous square-root V-BLAST algorithm in [5] can be summarized as follows.

The Previous Square-Root V-BLAST Algorithm

Initialization:

P1) Let $m = M$. Compute an initial $\mathbf{F} = \mathbf{F}_M$: Set $\mathbf{P}_0^{1/2} = (1/\sqrt{\alpha})\mathbf{I}_M$. Compute $\mathbf{\Pi}_i = \begin{bmatrix} 1 & \mathbf{h}_i^H \mathbf{P}_{i-1}^{1/2} \\ \mathbf{0}_M & \mathbf{P}_{i-1}^{1/2} \end{bmatrix}$ and

$\mathbf{\Pi}_i \mathbf{\Theta}_i = \begin{bmatrix} \times & \mathbf{0}_M^T \\ \times & \mathbf{P}_i^{1/2} \end{bmatrix}$ iteratively for $i = 1, 2, \dots, N$, where “ \times ” denotes irrelevant entries at this time, and $\mathbf{\Theta}_i$ is any unitary transformation that block lower-triangularizes the pre-array $\mathbf{\Pi}_i$. Finally $\mathbf{F} = \mathbf{P}_N^{1/2}$.

Iterative Detection:

- P2) Find the minimum length row of \mathbf{F}_m and permute it to the last row. Permute \mathbf{a}_m and \mathbf{H}_m accordingly.
P3) Block upper-triangularize \mathbf{F}_m by

$$\mathbf{F}_m \mathbf{\Sigma} = \begin{bmatrix} \mathbf{F}_{m-1} & \mathbf{u}_{m-1} \\ \mathbf{0}_{m-1}^T & \lambda_m \end{bmatrix}, \quad (9)$$

where $\mathbf{\Sigma}$ is a unitary transformation, \mathbf{u}_{m-1} is an $(m-1) \times 1$ column vector, and λ_m is a scalar.

- P4) Form the linear MMSE estimate of a_m , i.e.,

$$\hat{a}_m = \lambda_m [\mathbf{u}_{m-1}^H \quad (\lambda_m)^*] \mathbf{H}_m^H \mathbf{x}^{(m)}. \quad (10)$$

- P5) Obtain a_m from \hat{a}_m via slicing.
P6) Cancel the interference of a_m in $\mathbf{x}^{(m)}$ by (5), to obtain the reduced-order problem (6) with the corresponding $\mathbf{x}^{(m-1)}$, \mathbf{a}_{m-1} , \mathbf{H}_{m-1} and \mathbf{F}_{m-1} .
P7) If $m > 1$, let $m = m - 1$ and go back to step P2.

IV. A FAST ALGORITHM FOR INVERSE CHOLESKY FACTORIZATION

The previous square-root algorithm [5] requires extremely high computational load to compute the initial \mathbf{F} in step P1. So we propose a fast algorithm to compute an initial \mathbf{F} that is upper triangular.

If \mathbf{F}_m satisfies (7), any $\mathbf{F}_m \mathbf{\Sigma}$ also satisfies (7). Then there must be a square-root of \mathbf{P}_m in the form of

$$\mathbf{F}_m = \begin{bmatrix} \mathbf{F}_{m-1} & \mathbf{u}_{m-1} \\ \mathbf{0}_{m-1}^T & \lambda_m \end{bmatrix}, \quad (11)$$

as can be seen from (9). We apply (11) to compute \mathbf{F}_m from \mathbf{F}_{m-1} , while the similar equation (9) is only employed to compute \mathbf{F}_{m-1} from \mathbf{F}_m in [4] and [5].

From (11), we obtain

$$\mathbf{F}_m^{-1} = \begin{bmatrix} \mathbf{F}_{m-1}^{-1} & -\mathbf{F}_{m-1}^{-1} \mathbf{u}_{m-1} / \lambda_m \\ \mathbf{0}_{m-1}^T & 1 / \lambda_m \end{bmatrix}. \quad (12)$$

On the other hand, it can be seen that \mathbf{R}_m defined from \mathbf{H}_m by (3) is the $m \times m$ leading principal submatrix of \mathbf{R} [7]. Then we have

$$\mathbf{R}_m = \begin{bmatrix} \mathbf{R}_{m-1} & \mathbf{v}_{m-1} \\ \mathbf{v}_{m-1}^H & \beta_m \end{bmatrix}. \quad (13)$$

Now let us substitute (13) and (12) into

$$\mathbf{F}_m^{-H} \mathbf{F}_m^{-1} = \mathbf{R}_m, \quad (14)$$

which is deduced from (7) and (4). Then we obtain

$$\begin{bmatrix} \times & -\frac{\mathbf{F}_{m-1}^{-H} \mathbf{F}_{m-1}^{-1} \mathbf{u}_{m-1}}{\lambda_m} \\ \times & \frac{\mathbf{u}_{m-1}^H \mathbf{F}_{m-1}^{-H} \mathbf{F}_{m-1}^{-1} \mathbf{u}_{m-1} + 1}{\lambda_m \lambda_m^*} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{m-1} & \mathbf{v}_{m-1} \\ \mathbf{v}_{m-1}^H & \beta_m \end{bmatrix}, \quad (15)$$

where “ \times ” denotes irrelevant entries. From (15), we deduce

$$\begin{cases} -\mathbf{F}_{m-1}^{-H} \mathbf{F}_{m-1}^{-1} \mathbf{u}_{m-1} / \lambda_m = \mathbf{v}_{m-1}, \\ (\mathbf{u}_{m-1}^H \mathbf{F}_{m-1}^{-H} \mathbf{F}_{m-1}^{-1} \mathbf{u}_{m-1} + 1) / (\lambda_m \lambda_m^*) = \beta_m. \end{cases} \quad (16a)$$

From (16), finally we can derive

$$\begin{cases} \lambda_m = 1 / \sqrt{\beta_m - \mathbf{v}_{m-1}^H \mathbf{F}_{m-1} \mathbf{F}_{m-1}^H \mathbf{v}_{m-1}}, \\ \mathbf{u}_{m-1} = -\lambda_m \mathbf{F}_{m-1} \mathbf{F}_{m-1}^H \mathbf{v}_{m-1}. \end{cases} \quad (17a)$$

$$(17b)$$

We derive (17b) from (16a). Then (17b) is substituted into (16b) to derive

$$\lambda_m \lambda_m^* = (\beta_m - \mathbf{v}_{m-1}^H \mathbf{F}_{m-1} \mathbf{F}_{m-1}^H \mathbf{v}_{m-1})^{-1}, \quad (18)$$

while a λ_m satisfying (18) can be computed by (17a).

We can use (17) and (11) to compute \mathbf{F}_m from \mathbf{F}_{m-1} iteratively till we get \mathbf{F}_M . The iterations start from \mathbf{F}_1 satisfying (14), which can be computed by

$$\mathbf{F}_1 = \sqrt{\mathbf{R}_1^{-1}}. \quad (19)$$

Correspondingly instead of step P1, we can propose step N1 to compute an initial upper-triangular \mathbf{F} , which includes the following sub-steps.

The Sub-steps of Step N1

- N1-a) Assume the successive detection order to be t_M, t_{M-1}, \dots, t_1 . Correspondingly permute \mathbf{H} to be $\mathbf{H} = \mathbf{H}_M = [\mathbf{h}_{t_1}, \mathbf{h}_{t_2}, \dots, \mathbf{h}_{t_M}]$, and permute \mathbf{a} to be $\mathbf{a} = \mathbf{a}_M = [a_{t_1}, a_{t_2}, \dots, a_{t_M}]^T$.
- N1-b) Utilize the permuted \mathbf{H} to compute \mathbf{R}_M , where we can obtain all \mathbf{R}_{m-1} s, \mathbf{v}_{m-1} s and β_m s [7] (for $m = M, M-1, \dots, 2$), as shown in (13).
- N1-c) Compute \mathbf{F}_1 by (19). Then use (17) and (11) to compute \mathbf{F}_m from \mathbf{F}_{m-1} iteratively for $m = 2, 3, \dots, M$, to obtain the initial $\mathbf{F} = \mathbf{F}_M$.

The obtained upper triangular \mathbf{F}_M is equivalent to a Cholesky factor [14] of $\mathbf{P}_M = \mathbf{R}_M^{-1}$, since \mathbf{F}_M and \mathbf{P}_M can be permuted to the lower triangular \mathbf{F}_M and the corresponding \mathbf{P}_M , which still satisfy (7). Notice that the \mathbf{F}_M with columns exchanged still satisfies (7), while if two rows in \mathbf{F}_M are exchanged, the corresponding two rows and columns in \mathbf{P}_M need to be exchanged.

Now from (13), (7) and (4), it can be seen that (9) (proposed in [4]) and (11) actually reveal the relation between the m^{th} and the $(m-1)^{th}$ order inverse Cholesky factor of the matrix \mathbf{R} . This relation is also utilized to implement adaptive filters in [15], [16], where the m^{th} order inverse Cholesky factor is obtained from the m^{th} order Cholesky factor [15, equation (12)], [16, equation (16)]. Thus the algorithms in [15], [16] are still similar to the conventional matrix inversion algorithm [17] using Cholesky factorization, where the inverse Cholesky factor is computed from the Cholesky factor by the back-substitution (for triangular matrix inversion), an inherent serial process unsuitable for the parallel implementation [18]. Contrarily, the proposed algorithm computes the inverse Cholesky factor of \mathbf{R}_m from \mathbf{R}_m directly by (17) and (11). Then it avoids the conventional back substitution of the Cholesky factor.

In a word, although the relation between the m^{th} and the $(m-1)^{th}$ order inverse Cholesky factor (i.e. (9) and (11)) has been mentioned [4], [15], [16], our contributions in this letter include substituting this relation into (14) to find (18) and (17). Specifically, to compute the m^{th} order inverse Cholesky factor, the conventional matrix inversion algorithm using Cholesky factorization [17] usually requires $2m$ divisions (i.e. m divisions for Cholesky factorization and

the other m divisions for the back-substitution), while the proposed algorithm only requires m divisions to compute (19) and (17a).

V. THE PROPOSED SQUARE-ROOT V-BLAST ALGORITHM

Now \mathbf{R}_M has been computed in sub-step N1-b. Thus as the recursive V-BLAST algorithm in [11], we can also cancel the interference of the detected signal a_m in

$$\mathbf{z}_m = \mathbf{H}_m^H \mathbf{x}^{(m)} \quad (20)$$

by

$$\mathbf{z}_{m-1} = \mathbf{z}_m^{[-1]} - a_m \cdot \mathbf{v}_{m-1}, \quad (21)$$

where $\mathbf{z}_m^{[-1]}$ is the permuted \mathbf{z}_m with the last entry removed, and \mathbf{v}_{m-1} is in the permuted \mathbf{R}_m [9], [11], as shown in (13). Then to avoid computing $\mathbf{H}_m^H \mathbf{x}^{(m)}$ in (10), we form the estimate of a_m by

$$\hat{a}_m = \lambda_m \cdot \left[(\mathbf{u}_{m-1})^H \quad (\lambda_m)^* \right] \cdot \mathbf{z}_m. \quad (22)$$

It is required to compute the initial \mathbf{z}_M . So step N1 should include the following sub-step N1-d.

- N1-d) Compute $\mathbf{z}_M = \mathbf{H}_M^H \mathbf{x}^{(M)} = \mathbf{H}_M^H \mathbf{x}$.

The proposed square-root V-BLAST algorithm is summarized as follows.

The Proposed Square-root V-BLAST Algorithm

Initialization:

- N1) Set $m = M$. Compute \mathbf{R}_M , \mathbf{z}_M and the initial upper triangular $\mathbf{F} = \mathbf{F}_M$. This step includes the above-described sub-steps N1-a, N1-b, N1-c and N1-d.

Iterative Detection:

- N2) Find the minimum length row in \mathbf{F}_m and permute it to be the last m^{th} row. Correspondingly permute \mathbf{a}_m , \mathbf{z}_m , and rows and columns in \mathbf{R}_m [9].
- N3) Block upper-triangularize \mathbf{F}_m by (9).
- N4) Form the least-mean-square estimate \hat{a}_m by (22).
- N5) Obtain a_m from \hat{a}_m via slicing.
- N6) Cancel the effect of a_m in \mathbf{z}_m by (21), to obtain the reduced-order problem (6) with the corresponding \mathbf{z}_{m-1} , \mathbf{a}_{m-1} , \mathbf{R}_{m-1} and \mathbf{F}_{m-1} .
- N7) If $m > 1$, let $m = m - 1$ and go back to step N2.

Since \mathbf{F}_M obtained in step N1 is upper triangular, step N3 requires less computational load than the corresponding step P3 (described in Section III), which is analyzed as follows.

Suppose that the minimum length row of \mathbf{F}_M found in step N2 is the i^{th} row, which must be $[0 \ \dots \ 0 \ f_{ii} \ \dots \ f_{iM}]$ with the first $i-1$ entries to be zeros. Thus in step N3 the transformation Σ can be performed by only $(M-i)$ Givens rotations [14], i.e.,

$$\Sigma_M^g = \Omega_{i,i+1}^i \Omega_{i+1,i+2}^i \dots \Omega_{M-1,M}^i = \prod_{j=i}^{M-1} \Omega_{j,j+1}^i, \quad (23)$$

where the Givens rotation $\Omega_{k,n}^i$ rotates the k^{th} and n^{th} entries in each row of \mathbf{F}_M , and zeroes the k^{th} entry in the i^{th} row.

TABLE I
COMPARISON OF EXPECTED COMPLEXITIES BETWEEN THE PROPOSED
SQUARE-ROOT V-BLAST ALGORITHM AND THE PREVIOUS
SQUARE-ROOT V-BLAST ALGORITHM IN [5]

Step	The Algorithm in [5]	The Proposed Algorithm
1-b	$(3M^2N)$ [5] for step P1	$(\frac{M^2N}{2})$ [7]
1-c		$(\frac{M^3}{3})$
3	$(M^3, \frac{M^3}{3})$ or $(\frac{2}{3}M^3)$ [5]	From $(\frac{M^3}{3}, \frac{M^3}{9})$ to (0)
4	$(\frac{M^2N}{2})$ [5]	$(0(M^3))$
Sum	$(M^3 + \frac{7}{2}M^2N, \frac{M^3}{3} + \frac{7}{2}M^2N)$ or $(\frac{2}{3}M^3 + \frac{7}{2}M^2N)$	From $(\frac{2}{3}M^3 + \frac{M^2N}{2}, \frac{4}{9}M^3 + \frac{M^2N}{2})$ to $(\frac{M^3}{3} + \frac{M^2N}{2})$

In step N2, we can delete the i^{th} row in \mathbf{F}_M firstly to get $\bar{\mathbf{F}}_M$, and then add the deleted i^{th} row to $\bar{\mathbf{F}}_M$ as the last row to obtain the permuted \mathbf{F}_M . Now it is easy to verify that the \mathbf{F}_{M-1} obtained from $\mathbf{F}_M \Sigma_M^g$ by (9) is still upper triangular. For the subsequent $m = M-1, M-2, \dots, 2$, we also obtain \mathbf{F}_{m-1} from $\mathbf{F}_m \Sigma_m^g$ by (9), where Σ_m^g is defined by (23) with $M = m$. Correspondingly we can deduce that \mathbf{F}_{m-1} is also triangular. Thus \mathbf{F}_m is always triangular, for $m = M, M-1, \dots, 1$.

To sum up, our contributions in this letter include steps N1, N3, N4 and N6 that improve steps P1, P3, P4 and P6 (of the previous square-root V-BLAST algorithm [5]), respectively. Steps N4 and N6 come from the extension of the improvement in [11] (for the recursive V-BLAST algorithm) to the square-root V-BLAST algorithm. However, it is infeasible to extend the improvement in [11] to the existing square-root V-BLAST algorithms in [4], [5], since they do not provide \mathbf{R}_M that is required to get \mathbf{v}_{m-1} for (21).

VI. COMPLEXITY EVALUATION

In this section, (j, k) denotes the computational complexity of j complex multiplications and k complex additions, which is simplified to (j) if $j = k$. Similarly, $\langle \chi_1, \chi_2, \chi_3 \rangle$ denotes that the speedups in the number of multiplications, additions and floating-point operations (flops) are χ_1 , χ_2 and χ_3 , respectively, which is simplified to $\langle \chi_1 \rangle$ if $\chi_1 = \chi_2 = \chi_3$. Table I compares the expected complexity of the proposed square-root V-BLAST algorithm and that of the previous one in [5]. The detailed complexity derivation is as follows.

In sub-step N1-c, the dominant computations come from (17). It needs a complexity of $(\frac{(m-1)m}{2})$ to compute $\mathbf{y}_{m-1} = \mathbf{F}_{m-1}^H \mathbf{v}_{m-1}$ firstly, where \mathbf{F}_{m-1} is triangular. Then to obtain the m^{th} column of \mathbf{F} , we compute (17) by

$$\begin{cases} \lambda_m = \sqrt{1 / (\beta_m - \mathbf{y}_{m-1}^H \mathbf{y}_{m-1})}, \\ \mathbf{u}_{m-1} = -\lambda_m \mathbf{F}_{m-1} \mathbf{y}_{m-1}. \end{cases} \quad (24a)$$

$$(24b)$$

In (24), the complexity to compute $\mathbf{F}_{m-1} \mathbf{y}_{m-1}$ is $(\frac{(m-1)m}{2})$, and that to compute the other parts is $(O(m))$. So sub-step N1-c totally requires a complexity of $(\sum_{m=2}^M \frac{(m-1)m}{2} \times 2 + O(m)) = (\frac{M^3}{3} + O(M^2))$ to compute (17) for $M-1$ iterations, while sub-step N1-b requires a complexity of $(\frac{M^2N}{2})$ [7] to compute the Hermitian \mathbf{R}_M .

As a comparison, in each of the $N(> M-1)$ iterations, step P1 computes $\mathbf{h}_i^H \mathbf{P}_{i-1}^{1/2}$ to form the $(M+1) \times (M+1)$ pre-array $\mathbf{\Pi}_i$, and then block lower-triangularizes $\mathbf{\Pi}_i$ by the $(M+1) \times (M+1)$ Householder transformation [5]. Thus it can be seen that step P1 requires much more complexity than the proposed step N1.

In steps N3 and P3, we can apply the efficient complex Givens rotation [19] $\Phi = \frac{1}{q} \begin{bmatrix} c & s \\ -s^* & c \end{bmatrix}$ to rotate $\begin{bmatrix} d & e \end{bmatrix}$

into $\begin{bmatrix} 0 & (e/|e|)q \end{bmatrix}$, where $c = |e|$ and $q = \sqrt{|e|^2 + |d|^2}$ are real, and $s = (e/|e|)d^*$ is complex. The efficient Givens rotation equivalently requires [7] 3 complex multiplications and 1 complex additions to rotate a row. Correspondingly the complexity of step P3 is $(M^3, \frac{1}{3}M^3)$. Moreover, step P3 can also adopt a Householder reflection, and then requires a complexity of $(\frac{2}{3}M^3)$ [5]. On the other hand, the Givens rotation $\Omega_{j,j+1}^i$ in (23) only rotates non-zero entries in the first $j+1$ rows of the upper-triangular \mathbf{F}_M . Then (23) requires a complexity of $(\sum_{j=i}^{m-1} 3(j+1) \approx \frac{3(m^2-i^2)}{2}, \frac{(m^2-i^2)}{2})$.

When the detection order assumed in sub-step N1-a is statistically independent of the optimal detection order, the probabilities for $i = 1, 2, \dots, m$ are equal. Correspondingly the expected (or average) complexity of step N3 is $(\sum_{m=1}^M \frac{1}{m} \sum_{i=1}^m \frac{3(m^2-i^2)}{2} \approx \frac{M^3}{3}, \frac{M^3}{9})$. Moreover, when the probability for $i = 1$ is 100%, step N3 needs the worst-case complexity, which is $(\sum_{m=1}^M \frac{3(m^2-1^2)}{2} \approx \frac{M^3}{2}, \frac{M^3}{6})$. Correspondingly we can deduce that the worst-case complexity of the proposed V-BLAST algorithm is $(\frac{2}{3}M^3 + \frac{M^2N}{2}, \frac{M^3}{3} + \frac{7}{2}M^2N) - (\frac{M^3}{3}, \frac{M^3}{9}) + (\frac{M^2}{2}, \frac{M^3}{6}) = (\frac{5}{6}M^3 + \frac{1}{2}M^2N, \frac{1}{2}M^3 + \frac{1}{2}M^2N)$. It can be seen that the ratio between the worst-case and expected flops of the proposed square-root algorithm is only 1.125.

In MIMO OFDM systems, the complexity of step N3 can be further reduced, and can even be zero. In sub-step N1-a, we assume the detection order to be the optimal order of the adjacent subcarrier, which is quite similar or even identical to the actual optimal detection order [13]. Correspondingly the required Givens rotations are less or even zero. So the expected complexity of step N3 ranges from $(\frac{1}{3}M^3, \frac{1}{9}M^3)$ to zero, while the exact mean value depends on the statistical correlation between the assumed detection order and the actual optimal detection order.

The complexities of the ZF-OSIC V-BLAST algorithm in [6] and the MMSE-OSIC V-BLAST algorithms in [4], [7]–[9] are $(\frac{1}{2}M^3 + 2M^2N)$, $(\frac{2}{3}M^3 + 4M^2N + MN^2)$ [5], $(\frac{2}{3}M^3 + 3M^2N, \frac{1}{2}M^3 + \frac{5}{2}M^2N)$, $(\frac{2}{3}M^3 + \frac{5}{2}M^2N)$ and $(\frac{2}{3}M^3 + \frac{1}{2}M^2N)$, respectively. Let $M = N$. Also assume the transformation Σ in [5] to be a sequence of efficient Givens rotations [19] that are hardware-friendly [4]. When the expected complexities are compared, the speedups of the proposed square-root algorithm over the previous one [5] and the fastest known recursive algorithm [9] range from $\langle \frac{9}{2}/\frac{7}{6} = 3.86, \frac{23}{6}/\frac{17}{18} = 4.06, 3.9 \rangle$ to $\langle \frac{9}{2}/\frac{5}{6} = 5.4, \frac{23}{6}/\frac{5}{6} = 4.6, 5.2 \rangle$ and from $\langle \frac{7}{6}/\frac{7}{6} = 1, \frac{7}{6}/\frac{17}{18} = 1.24, 1.05 \rangle$ to $\langle \frac{7}{6}/\frac{5}{6} = 1.4 \rangle$, respectively.

Recently there is a trend to study the expected, rather than worst-case, complexities of various algorithms [20]. Even when the worst-case complexities are compared, the proposed square-root algorithm is still faster than the efficient V-BLAST algorithms in [4]–[8], [10], [11]. However, the worst-case flops of the proposed square-root algorithm are a little more than those of the fastest known recursive algorithm [9], while the ratio of the former to the latter is $[(5/6 + 1/2) \times 6 + (1/2 + 1/2) \times 2] / [(2/3 + 1/2) \times 8] = 1.07$.

For more fair comparison, we modify the fastest known recursive algorithm [9] to further reduce the expected complexity. We spend extra memories to store each intermediate \mathbf{P}_m ($m = 1, 2, \dots, M - 1$) computed in the *initialization* phase, which may be equal to the \mathbf{P}_m required in the *recursion* phase [9]. Assume the successive detection order and permute \mathbf{H} accordingly, as in sub-step N1-a. When the assumed order is identical to the actual optimal detection order, each \mathbf{P}_m required in the *recursion* phase is equal to the stored \mathbf{P}_m . Thus we can achieve the maximum complexity savings, i.e. the complexity of $(\frac{1}{6}M^3 + O(M^2))$ [9, equations (23) and (24)] to deflate \mathbf{P}_m s. On the other hand, when the assumed order is statistically independent of the actual optimal detection order, there is an equal probability for the m undetected antennas to be any of the C_m^M possible antenna combinations. Correspondingly $1/C_m^M = \frac{(M-m)!m!}{M!}$ is the probability for the stored \mathbf{P}_m to be equal to the \mathbf{P}_m required in the *recursion* phase. Thus we can obtain the minimum expected complexity savings, i.e. [9, equations (23) and (24)],

$$\left(\sum_{m=2}^M \frac{1}{C_m^M} \frac{(m-1)(m+2)}{2}, \sum_{m=2}^M \frac{1}{C_m^M} \frac{(m-1)m}{2} \right). \quad (25)$$

The ratio of the minimum expected complexity savings to the maximum complexity savings is 22% when $M = 4$, and is only 1.2% when $M = 16$. It can be seen that the minimum expected complexity savings are negligible when M is large. The minimum complexity of the recursive V-BLAST algorithm [9] with the above-described modification, which is $(\frac{2}{3}M^3 + \frac{1}{2}M^2N - \frac{1}{6}M^3) = (\frac{1}{2}M^3 + \frac{1}{2}M^2N)$, is still more than that of the proposed square-root V-BLAST algorithm. When $M = N$, the ratio of the former to the latter is $1/\frac{5}{6} = 1.2$.

Assume $M = N$. For different number of transmit/receive antennas, we carried out some numerical experiments to count the average flops of the OSIC V-BLAST algorithms in [4]–[9], the proposed square-root V-BLAST algorithm, and the recursive V-BLAST algorithm [9] with the above-described modification. The results are shown in Fig. 1. It can be seen that they are consistent with the theoretical flops calculation.

VII. CONCLUSION

We propose a fast algorithm for inverse Cholesky factorization, to compute a triangular square-root of the estimation error covariance matrix for V-BLAST. Then it is employed to propose an improved square-root algorithm for V-BLAST, which speedups several steps in the previous one [5], and can offer further computational savings in MIMO OFDM systems. Compared to the conventional inverse Cholesky factorization, the proposed one avoids the back substitution (of the Cholesky factor), an inherent serial process unsuitable for the parallel

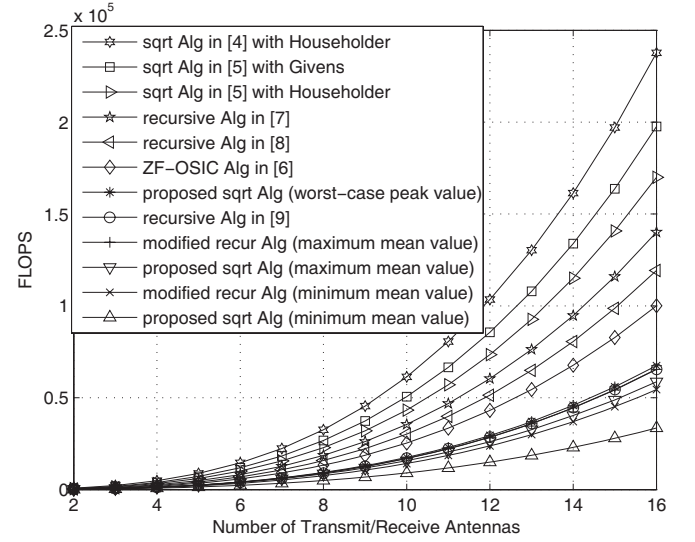


Fig. 1. Comparison of computational complexities among the MMSE-OSIC algorithms in [4], [5], [7]–[9] and this letter, and the ZF-OSIC algorithm in [6]. “sqrt” and “Alg” means square-root and algorithm, respectively. “... with Householder” and “... with Givens” adopt a Householder reflection and a sequence of Givens rotations, respectively. Moreover, “modified recur Alg” is the recursive algorithm [9] with the modification described in this letter.

implementation [18], and then requires only half divisions. Usually the expected, rather than worst-case, complexities of various algorithms are studied [20]. Then the proposed V-BLAST algorithm is faster than the existing efficient V-BLAST algorithms in [4]–[11]. Assume M transmitters and the equal number of receivers. In MIMO OFDM systems, the expected speedups (in the number of flops) of the proposed square-root V-BLAST algorithm over the previous one [5] and the fastest known recursive V-BLAST algorithm [9] are $3.9 \sim 5.2$ and $1.05 \sim 1.4$, respectively. The recursive algorithm [9] can be modified to further reduce the complexity at the price of extra memory consumption, while the minimum expected complexity savings are negligible when M is large. The speedups of the proposed square-root algorithm over the fastest known recursive algorithm [9] with the above-mentioned modification are 1.2, when both algorithms are assumed to achieve the maximum complexity savings. Furthermore, as shown in [21], the proposed square-root algorithm can also be applied in the extended V-BLAST with selective per-antenna rate control (S-PARC), to reduce the complexity even by a factor of M .

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