Joint Tx/Rx Energy-Efficient Scheduling in Multi-Radio Wireless Networks: A Divide-and-Conquer Approach

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***Abstract*—Most of the existing works on energy-efficient wireless communications only consider the transmitter (Tx) or the receiver (Rx) side power consumption, but not both. Moreover, the circuit power consumption is often assumed to be constant regardless of the transmission rate or the bandwidth. In this paper, we investigate the system-level energy-efficient transmission in multi-radio access networks by considering joint Tx and Rx power consumption and adopting link-dependent dynamic circuit power model. A combinatorial-type optimization problem for user scheduling, radio-link activation, and power control is formulated with the objective of maximizing joint Tx and Rx energy efficiency (EE). We tackle this problem using a divide-and-conquer approach. Specifically, the concepts of link EE and user EE are first introduced, which have structures similar to the system EE. Then, we explore their hierarchical relationships and propose an optimal algorithm whose complexity is linear in the product of the total number of users and radio links. Furthermore, we investigate the EE maximization problem with minimum user data rate constraints. The divide-and-conquer approach is also applied to find a sub-optimal but efficient solution. Finally, comprehensive numerical results are provided to validate the theoretical findings and demonstrate the effectiveness of the proposed algorithms.**

***Index Terms*—Energy efficiency, user scheduling, link management, power allocation, quality of service.**

I. INTRODUCTION

THE INCREASING number of wireless devices and new services lead to a significant increase in the demand for higher network capacity and higher user data rate. Meanwhile, they also result in higher energy consumption, which is a main concern in the development of future wireless communication systems. Recently, there has been an upsurge of interest in the energy efficiency (EE) optimization field, such as [1]–[11]. However, most of them only consider one side power consumption, i.e., either the transmitter (Tx) or the receiver (Rx) side. On the one hand, the expectation of limiting electric expenditure and reducing carbon emissions requires base stations or wireless access points to perform in an energy-efficient manner [1]. On the other hand, minimizing the user side energy consumption also deserves more efforts due to capacity limited batteries and various user experience requirements [12]–[14]. More importantly, according to [15]–[17], the techniques adopted to improve the EE of one end (Tx side or Rx side) may adversely affect the EE of the other end for practical communication systems. Therefore, it is necessary to study the joint Tx and Rx EE optimization [18], [19].

A key element in energy-efficient oriented research is the modeling of power consumption of communication systems [1], [15], [16]. Most of the existing works assume a constant circuit power model to simplify the system analysis and make the problem more tractable. Such constant circuit power model, however, may not be able to reflect the true behaviour of wireless devices. Moreover, it might provide very misleading conclusions as reported in [15], [20]. For instance, assume (as usually done in the revelent literature including [6]–[9]) that the overall power consumption is computed as the sum of the transmit power and the static constant power accounting for circuit processing. Given the fixed bandwidth of each radio link, increasing the number of active links N would lead to unbounded system EE as the average system data rate grows unboundedly when $N \to +\infty$. Obviously, it is impossible to achieve the infinite system EE [1]. This misleading conclusion is due to that the power model does not take into account the fact that the circuit power consumed by the baseband processing and analog circuits is linearly increasing with the number of active radio links (bandwidth) [16]. A comprehensive study aiming at quantifying the energy consumption of entire communication systems is provided in [16], which serves as a building block for analyzing and optimizing the EE from the whole system perspective. It suggests that energy-efficient designs based on the dynamic circuit power modeling are anticipated to provide trustable conclusions for practical communication systems.

From the optimization perspective, EE defined by the ratio of the total throughput to the total consumed power is generally formulated as a fractional form. The authors in [21] establish a general mathematical framework for EE maximization based on fractional programming theory. Specifically, with an additional parameter q, the fractional-form objective function is transformed into a subtractive form. In the inner layer, the subtractive form problem is solved for each q and in the outer layer,

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the parameter q is updated iteratively. This method has been adopted in most EE optimization works [3], [6]–[11]. However, such iterative nature prevents researchers to extract insights from the original problem, and thus only numerical results are available.

In this paper, we investigate the EE maximization problem for multi-radio networks where multiple users communicate with a common access point (AP) over mutually exclusive radio links. The problem is to schedule the active users and the active radio links through power control for maximizing the joint Tx and Rx EE. The distinct features and main contributions of this paper are summarized as follows:

- In our problem formulation, the considered power consumption consists of the transmission power and the dynamic circuit power, in which the latter includes the link dependent signal processing power and the static circuit power following the comprehensive study in [16]. The power models of existing works [2]–[8], [10], [21]–[23] are special cases of the one adopted in this paper.
- The EE maximization problem is formulated as a combinatorial-type problem due to the indicator function for user scheduling and radio link activation. We then explore the fractional structure of the system EE and propose a divide-and-conquer approach to solve the problem. Specifically, we first introduce the link-level EE and the user-level EE, and then utilize their hierarchical relationships to address the system EE. By doing so, an algorithm with linear complexity is proposed. It is worthwhile to mention that this algorithm can also be used to optimally solve the problem in [4] where only a quadratic complexity method is proposed.
- Through analysis, we find that the static receiving circuit power plays an important role in determining the optimal number of active users. In the extreme case when the static receiving power is negligible, time division multiplexing access (TDMA) is optimal for energy-efficient transmissions. In the other extreme case when the static receiving power is sufficiently large, all users will be scheduled.
- To guarantee the quality of service (QoS) of practical systems, we further consider the EE maximization problem with minimum individual user data rate constraints. The proposed divide-and-conquer approach can still be applied to find a sub-optimal solution based on the relationship of active links between maximizing the user EE and maximizing the system EE.

Previously, there are already some existing works considering the joint Tx and Rx EE optimization, including [4], [18], and [19]. The authors in [4] study the system EE for multi-user multiple input multiple output (MIMO) scenario, where the power allocation is optimized based on a fixed precoding matrix. However, the maximum transmit power constraint is not considered and the QoS is also not guaranteed, thus the solution is less practical for realistic communications. In addition, the proposed algorithm in [4] is claimed to have the quadratic complexity, but in our work, we propose a linear-complexity approach by exploiting the special structure of the system EE and the divide-and-conquer idea, which also helps to reveal interesting properties of the considered system. The authors in [18] focus on minimizing the system energy consumption in orthogonal frequency division multiple access (OFDMA) systems while we explicitly consider the system EE which is defined as the ratio of system data rate to the system power consumption. Therefore, the optimization problem in [18] is a convex one and can be efficiently solved by standard convex optimization tools. Compared with [19], this work differs in the following aspects. First, the number of scheduled users is optimized for the system EE while it is not in [19] due to the use of conventional circuit power model. Second, the minimum individual user data rate requirements are considered in this work while in [19] only the system sum-rate requirement is considered. Finally, the optimization method in [19] is generally based on the well-known Dinkelbach method [6]–[9], [24], but this work applies the divide-and-conquer approach which is based on exploiting the special structure of the EE.

TABLE I
SOME NOTATIONS USED IN THIS PAPER

Notation	Description
B	Bandwidth of each radio link
K	Number of users
$\mathcal{M}_k$	Set of radio links of user k
n_k	Number of radio links of user k, i.e., $\lvert\mathcal{M}_k\rvert$
n_k^o	Number of active radio links of user k
$P_{\text{sta},0}$	Static circuit power of the AP
$P_{\text{dyn},0}$	Per-link signal processing power of the AP
$P_{\text{sta},k}$	Static circuit power of user k
$P_{\text{dyn},k}$	Per-link signal processing power of user k
$ee_{k,i}$	EE of link i of user k
EE_k	EE of user k
EE	System EE
$g_{k,i}$	Channel gain of link i of user k
$p_{k,i}$	Transmission power of link i of user k
P_k	Overall power consumption of user k
P_0	Overall power consumption of the AP
Φ_k	Set of active radio links of user k
Φ	Set of active radio links of the system
Φ_k^*	Optimal solution of Φ_k
Φ^*	Optimal solution of Φ
$EE_{\Phi_k}^*$	Optimal user EE based on set Φ_k
EE_{Φ}^*	Optimal system EE based on set Φ

The remainder of this paper is organized as follows. In Section II, we introduce the system model and formulate the joint Tx and Rx EE maximization problem. In Section III, we study this problem by using the divide-and-conquer idea and reveal some insights of the solution. In Section IV, the EE optimization with QoS constraints is considered. Section V provides comprehensive simulation results and the paper is concluded in Section VI. Table I summarizes the major notations used throughout this paper.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this section, we introduce the system model and power consumption model, and then formulate the joint user scheduling, link activation, and power control problem for Tx/Rx EE maximization.

A. System Model

Consider a multi-radio network, where K users are communicating with one access point (AP) simultaneously and each user k, for $k = 1, \ldots, K$, occupies a pre-fixed and mutually exclusive subset of radio links, denoted as $\mathcal{M}_k$. The AP and users are all equipped with single antenna for the ease of implementation. The channel between the AP and each user is assumed to be quasi-static fading, which indicates that the channel coefficient remains constant during each block, but can vary from one block to another. It is assumed that global channel state information (CSI) of all users is perfectly known to the AP in order to explore the EE bound and extract possible insights of considered systems. In practice, the CSI can be estimated by each individual user and then fed back to the AP. Signaling overhead and incomplete CSI would result in performance loss, and the study on their impacts falls within the robust optimization field [25] and is thereby beyond the scope of this paper.

The channel gain of user k over its link $i \in \mathcal{M}_k$ and the corresponding power allocation on the link are denoted as $g_{k,i}$ and $p_{k,i}$, respectively. Without loss of generality, the receiver noise is modelled as a circularly symmetric complex Gaussian random variable with zero mean and variance σ^2 for all links. Then the data rate of user k over link $i \in \mathcal{M}_k$, denoted as $r_{k,i}$, can be expressed as

$$r_{k,i} = B \log_2 \left(1 + \frac{p_{k,i} g_{k,i}}{\Gamma \sigma^2}\right), \tag{1}$$

where B is the bandwidth of each radio link and Γ characterizes the gap between the actual achievable rate and the channel capacity due to a practical modulation and coding design. Consequently, the weighted overall system data rate can be expressed as

$$R_{\text{tot}} = \sum_{k=1}^{K} \omega_k R_k = \sum_{k=1}^{K} \omega_k \sum_{i \in \mathcal{M}_k} r_{k,i}, \tag{2}$$

where R_k denotes the data rate of user k and ω_k is provided by upper layers representing the priority of user k.

B. Power Consumption Model

An accurate modelling of the total power consumption is of primary importance for energy-efficient designs [1], [20]. In this work, we adopt the power consumption model established by the Energy Aware Radio and neTwork tecHnologies (EARTH) Project [16], which provides a comprehensive characterization of the power consumption for each component involved in the communication. Our considered overall system power consumption includes both the user side and the AP side.

At the user side, the power dissipation consists of two parts, i.e., the transmission power and the circuit power. Denote the transmission power of user k by P_{Tk} and it is given by

$$P_{Tk} = \frac{\sum_{i \in \mathcal{M}_k} p_{k,i}}{\xi}, \tag{3}$$

where $\xi \in (0, 1]$ is a constant which accounts for the efficiency of the power amplifier and it has been shown in [26] that this linear abstraction model for the amplifier is effective enough to characterize the reality. Denote P_{Ck} as the circuit power of user k and it contains a dynamic part for the signal processing which scales linearly with the number of active links and a static part for other circuit blocks. Let n_k^o be the number of active links of user k, i.e., the links on which the power allocation is non-zero. Then, the circuit power dissipation is modeled as

$$P_{Ck}(n_k^o) = n_k^o P_{\text{dyn},k} + \mathcal{I}(n_k^o) P_{\text{sta},k}. \tag{4}$$

Here, $\mathcal{I}(x)$ is the indicator function defined as

$$\mathcal{I}(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

and n_k^o can be thereby expressed as

$$n_k^o = \sum_{i \in \mathcal{M}_k} \mathcal{I}(p_{k,i}). \tag{6}$$

Note that if $p_{k,i} > 0$, then $\mathcal{I}(p_{k,i}) = 1$ and this means that link i of user k is active. Similarly, if $n_k^o > 0$, then $\mathcal{I}(n_k^o) = 1$ and this means that user k is scheduled. In (4), $P_{\text{dyn},k}$ is the per-link related dynamic component of the circuit power and $P_{\text{sta},k}$ is the static component of the circuit power for user k. Considering that different users may employ different types of terminals in practical systems [1], $P_{\text{dyn},k}$ and $P_{\text{sta},k}$ can be different for different user k. Now, the overall power consumption of user k, denoted as P_k, is

$$P_k = P_{Tk} + P_{Ck}(n_k^o). \tag{7}$$

At the AP side, the electronic circuit power is consumed to receive and decode signals. Similarly, the receiving power consumption also consists of two parts with one scaling linearly with the number of active links of all users, and the other independent of links. Denote $P_{\text{sta},0}$ and $P_{\text{dyn},0}$ as the static circuit power and the per-link receiving signal processing power, respectively. Then, the overall power consumption at the AP side is given by

$$P_0 = \sum_{k=1}^{K} n_k^o P_{\text{dyn},0} + P_{\text{sta},0}. \tag{8}$$

Finally, the weighted overall power consumption of the system can be expressed as

$$\begin{aligned} P_{\text{tot}} &= \Theta_{\text{t}} \sum_{k=1}^{K} P_k + \Theta_{\text{r}} P_0 \\ &= \sum_{k=1}^{K} \Bigg(\frac{\Theta_{\text{t}} \sum_{i \in \mathcal{M}_k} p_{k,i}}{\xi} + n_k^o \left(\Theta_{\text{t}} P_{\text{dyn},k} + \Theta_{\text{r}} P_{\text{dyn},0}\right) \\ &\qquad + \Theta_{\text{t}} \mathcal{I}(n_k^o) P_{\text{sta},k} \Bigg) + \Theta_{\text{r}} P_{\text{sta},0}, \end{aligned} \tag{9}$$

where the weights Θ_{t} and Θ_{r} characterize the priorities of Tx and Rx power consumption, respectively.

C. Problem Formulation

In our work, the system EE is defined by the ratio of the weighted overall system data rate R_{tot} to the overall system power consumption P_{tot}, i.e.,

$$EE = \frac{R_{\text{tot}}}{P_{\text{tot}}}, \tag{10}$$

where R_{tot} and P_{tot} are given in (2) and (9), respectively. Our goal is to maximize the EE of the considered system through power control $\boldsymbol{p} \triangleq \{p_{k,i}|k = 1, 2, \ldots K; i \in \mathcal{M}_k\}$. Mathematically, we can formulate the problem as

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \frac{\sum_{k=1}^{K} \omega_k \sum_{i\in\mathcal{M}_k} B \log_2\left(1 + \frac{p_{k,i} g_{k,i}}{\Gamma\sigma^2}\right)}{\Theta_{\text{t}} \sum_{k=1}^{K} P_k + \Theta_{\text{r}} P_0} \\ \text{s.t.} \quad & n_k^o = \sum_{i\in\mathcal{M}_k} \mathcal{I}(p_{k,i}), \quad 1 \le k \le K, i \in \mathcal{M}_k, \\ & p_{k,i} \le P_{\max}^{k,i}, \quad 1 \le k \le K, i \in \mathcal{M}_k, \\ & p_{k,i} \ge 0, \quad 1 \le k \le K, i \in \mathcal{M}_k, \end{aligned} \tag{11}$$

For practical consideration, we assume that each radio link i of user k has a peak power constraint $P_{\max}^{k,i}$ [1]. The authors in [4] consider a similar problem formulation as (11) but without peak power constraints. Thus, the problem in [4] is a special case of ours. Note that the power control problem formulated in (11) can be interpreted as a joint optimization of user scheduling, link activation, and power allocation. First, for each user k, if $p_{k,i} = 0$ for all $i \in \mathcal{M}_k$, or equivalently, the number of active links n_k^o defined in (6) is 0, then we say user k is not scheduled. Thus, $\mathcal{I}(n_k^o)$ in (11) can be regarded as the control variable for user scheduling. Second, for each user k and its link i, if $p_{k,i} = 0$, then we say link i of user k is inactive. Thus, $\mathcal{I}(p_{k,i})$ becomes the control variable for link activation. As such, for presentation clarity we refer problem (11) as the joint user scheduling, link activation, and power allocation problem. Such interpretation also facilitates the description of the proposed algorithm in Section III.

Observing problem (11), one can first find that, the fractional form of the objective function makes the problem non-convex. Second, the existence of the link activation indicator, i.e., $\mathcal{I}(p_{k,i})$ and the user scheduling indicator $\mathcal{I}(n_k^o) = \mathcal{I}\left(\sum_{i\in\mathcal{M}_k} \mathcal{I}(p_{k,i})\right)$ makes the objective function discontinuous and hence non-differentiable. The global optimal solution of (11) is generally difficult to obtain with efficient complexity. In the following section, we explore the particular structure of system EE and show that the global optimal solution can actually be obtained using a divide-and-conquer approach with low complexity.

III. Energy-Efficient User Scheduling, Link Activation, and Power Control

In this section, we solve the system EE maximization problem based on the divide-and-conquer idea, and propose an optimal joint user scheduling, link activation, and power control algorithm. We first introduce the definitions of link EE and user EE. Then, we show that we can first use the link EE to solve the user EE maximization problem and then use the user EE to solve the system EE maximization problem.

A. Link EE and User EE

Definition 1 (*Link EE*): The EE of link i of user k, for $i \in \mathcal{M}_k$, $k = 1, .., K$, is defined as the ratio of the achievable rate of the user on this link over the consumed power associated with this link, i.e.,

$$ee_{k,i} = \frac{\omega_k B \log_2\left(1 + \frac{p_{k,i} g_{k,i}}{\Gamma\sigma^2}\right)}{\frac{\Theta_t p_{k,i}}{\xi} + \Theta_{\text{t}} P_{\text{dyn},k} + \Theta_{\text{r}} P_{\text{dyn},0}}, \tag{12}$$

where the rate weight ω_k is defined in (2), the power weights Θ_{t} and Θ_{r} are defined in (9), and the link-level power consumption counts the transmission power of the user over the link, per-link dynamic circuit power of the user, and the per-link dynamic circuit power of the AP.

The link EE $ee_{k,i}$ is a strictly quasiconcave function of $p_{k,i}$ [25], and it is easy to prove that this fractional type function has the stationary point which is also the optimal point. Due to this property, we can find the maximum link EE by setting the partial derivative of $ee_{k,i}$ with respect to $p_{k,i}$ to zero, i.e.,

$$\frac{\partial ee_{k,i}}{\partial p_{k,i}} = \frac{\frac{B\omega_k g_{k,i}}{(\Gamma\sigma^2 + p_{k,i} g_{k,i})\ln 2}(P_{k,i} + \Theta_{\text{r}} P_{\text{dyn},0}) - \frac{\Theta_t \omega_k}{\xi} r_{k,i}}{\left(\frac{\Theta_t p_{k,i}}{\xi} + \Theta_{\text{t}} P_{\text{dyn},k} + \Theta_{\text{r}} P_{\text{dyn},0}\right)^2} = 0. \tag{13}$$

After some manipulations and given the peak power constraint, we obtain that the optimal power value $p_{k,i}$ and the maximum link EE $ee_{k,i}$ satisfy

$$p_{k,i}^* = \left[\frac{B\xi\omega_k}{\Theta_{\text{t}} ee_{k,i}^* \ln 2} - \frac{\Gamma\sigma^2}{g_{k,i}}\right]_0^{P_{\max}^{k,i}}, 1 \le k \le K, i \in \mathcal{M}_k, \tag{14}$$

where $[x]_b^a \triangleq \min\{\max\{x, b\}, a\}$. Based on (12) and (14), the numerical values of $ee_{k,i}^*$ and $p_{k,i}^*$, $\forall k$ and $i \in \mathcal{M}_k$, can be easily obtained by the bisection method [25].

Definition 2 (*User EE*): The EE of user k, for $k = 1, \ldots, K$, is defined as the ratio of the total achievable rate of the user on all its preassigned radio links over the total power consumption associated with this user, i.e.,

$$EE_k = \frac{\omega_k \sum_{i\in\mathcal{M}_k} B \log_2\left(1 + \frac{p_{k,i} g_{k,i}}{\Gamma\sigma^2}\right)}{\frac{\Theta_{\text{t}} \sum_{i\in\mathcal{M}_k} p_{k,i}}{\xi} + n_k^o(\Theta_{\text{t}} P_{\text{dyn},k} + \Theta_{\text{r}} P_{\text{dyn},0}) + \Theta_{\text{t}} P_{\text{sta},k}}, \tag{15}$$

where the user-level power consumption counts the total transmission power of the user, the overall circuit power of the user, and the dynamic circuit power of the AP related to this user.

Now, we elaborate on how the maximum user EE for each user can be computed from the maximum link EE on all its links. We first formulate the user EE maximization problem as:

$$\begin{aligned}
\max_{\{p_{k,i}\}} \quad & \frac{\omega_k \sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\frac{\Theta_\mathrm{t}\sum_{i\in\mathcal{M}_k} p_{k,i}}{\xi} + n_k^o(\Theta_\mathrm{t}P_{\mathrm{dyn},k}+\Theta_\mathrm{r}P_{\mathrm{dyn},0})+\Theta_\mathrm{t}P_{\mathrm{sta},k}} \\
\text{s.t.} \quad & n_k^o = \sum_{i\in\mathcal{M}_k} \mathfrak{I}(p_{k,i}), \quad 1\le k\le K, i\in\mathcal{M}_k, \\
& p_{k,i} \le P_{\max}^{k,i}, \quad 1\le k\le K, i\in\mathcal{M}_k, \\
& p_{k,i} \ge 0, \quad 1\le k\le K, i\in\mathcal{M}_k.
\end{aligned} \tag{16}$$

Define Φ_k as the set of active links for user k with $p_{k,i} > 0$. Then, we have $n_k^o = |\Phi_k|$. Given any Φ_k, it is easy to prove that EE_k is strictly quasiconcave in $p_{k,i}$. Thus, similar to the link EE, we obtain that the optimal power value $p_{k,i}$ and the maximum user EE EE_{Φ_k} under set Φ_k satisfy

$$p_{k,i} = \left[\frac{B\xi\omega_k}{\Theta_\mathrm{t}EE_k^*\ln 2} - \frac{\Gamma\sigma^2}{g_{k,i}}\right]_0^{P_{\max}^{k,i}}, 1\le k\le K, i\in\Phi_k. \tag{17}$$

Let $EE_{\Phi_k}^*$ denote the maximum intermediate user EE of user k when its current set of active radio links is Φ_k. The numerical values of $p_{k,i}^*$ and $EE_{\Phi_k}^*$, $\forall k$ and $i\in\mathcal{M}_k$, can be easily obtained by the bisection method [25]. The following theorem provides a sufficient and necessary condition for determining whether an arbitrary link should be scheduled.

Theorem 1 (Link Activation Condition): Given any active link set $\Phi_k \subseteq \mathcal{M}_k$ and any not-yet-activated link $i \in \mathcal{M}_k\backslash\Phi_k$ of user k, the link i can be activated and added to Φ_k if and only if $EE_{\Phi_k}^* \le ee_{k,i}^*$.

Proof: Please see Appendix A. ■

The interpretation of Theorem 1 is that if the new link i wants to join the active link set, it should have a better utilization of the power than existing activated links of the user it is associated with. Given Theorem 1, we introduce an algorithm to gradually obtain the optimal active link set and the maximum user EE based on the link EE. The details of this procedure are summarized in Algorithm 1 and described below.

Sort all radio links of user k according to their link EE $ee_{k,i}^*$ in descending order, i.e., $ee_{k,1}^* \ge ee_{k,2}^* \ge \ldots \ge ee_{k,n_k}^*$, where $n_k = |\mathcal{M}_k|$, and set the initial $\Phi_k = $ ffl. Then, we successively take one link from the order and decide whether it can be added to Φ_k. The maximum user EE is reached when one link is found unable to be added to Φ_k or all links are added to Φ_k.

Remark 1: Algorithm 1 is optimal in the sense that it can reach the maximum user EE. This is ensured by the ordering of the link EE as well as Theorem 1. Algorithm 1 opens up a new way to address the fractional-form EE maximization problems. Note that the conventional way to treat this kind of problems is to transform it into a subtractive-form problem and then solve it by a sequence of parameterized problems [3], [7].

B. User Scheduling and Power Control

In this subsection, we show how to solve the original system EE maximization problem (11) based on the link EE and user EE defined earlier. For the explanation convenience, we first introduce two auxiliary sets. Denote Φ as the set of active

Algorithm 1. Link Activation for User EE maximization for User k

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Compute the maximum link EE ee*_{k,i} for all i ∈ M_k, by (12)
  and (14);
Sort all links of user k in the descending order of ee*_{k,i}, i.e.,
  ee*_{k,1} ≥ ee*_{k,2} ≥ ... ≥ ee*_{k,n_k}
Set EE*_{Φ*_k} = 0 and Φ*_k = ffl
for i = 1 : n_k
    if   EE*_{Φ*_k} ≤ ee*_{k,i}   do
        Φ*_k = Φ*_k ⋃{i};
        Compute p*_{k,i} and EE*_{Φ*_k} by (17);
    else return
end
```

links of all users, i.e., $\Phi = \{(k,i)\,|\,p_{k,i} > 0, \forall i, k\}$, with its optimum denoted as Φ^*. Denote U as the set of scheduled users which have at least one active link belonging to set Φ, i.e., $U = \{k\,|\,(k,i)\in\Phi, \forall k, i\}$. Apparently, U can be sufficiently determined by Φ.

Given the set of overall active links Φ, and accordingly the set of scheduled users U, then n_k^o can be readily calculated and problem (11) is simplified into the following problem

$$\begin{aligned}
\max_{\boldsymbol{p}} \quad & \frac{\sum_{k\in U}\omega_k\sum_{i\in\Phi} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\Theta_\mathrm{t}\sum_{k\in U} P_k + \Theta_\mathrm{r}P_0} \\
\text{s.t.} \quad & 0 < p_{k,i} \le P_{\max}^{k,i}, \quad 1\le k\le K, i\in\Phi.
\end{aligned} \tag{18}$$

It can be verified that problem (18) is a standard quasiconcave optimization problem and therefore can be readily solved as (16). Then our task is transformed to finding the scheduled users and their corresponding active links. Similar to Theorem 1, we shall present a theorem to determine whether a user should be scheduled for system EE maximization in what follows.

Note that the links that are not activated for user EE maximization may become activated in the system EE maximization problem. This is because the optimal power allocation for user EE maximization may not be optimal for system EE maximization. Therefore, to schedule users according to their individual maximum user EE alone cannot guarantee the global optimality. Therefore, before presenting Theorem 2 formally, we introduce an important concept of "virtual user" to deal with those inactive links in the stage of user EE maximization. We first let (k, i') denote the inactive link i' of user k in the optimal solution of maximum user EE. Then, we define each inactive link (k, i') as a virtual user and index it using ℓ. These virtual users are treated just like the real users in the system. Each virtual user only contains one link. For notation consistence, we define the link set of virtual user ℓ as $\{(k, i')\} = \Phi_\ell$. Therefore, the EE of this virtual user ℓ is equivalent to the EE of link i' of user k, i.e., $EE_{\Phi_\ell}^* = ee_{k,i'}^*$. In the rest of this subsection, unless specified otherwise, term "user" refers to both real users and virtual users. The difference between real users and virtual users is that each real user may contain multiple links (links with strictly positive power for user EE maximization) and its circuit power includes the static circuit power $P_{\mathrm{sta},k}$ as well as

Algorithm 2. User Scheduling for System EE Maximization

Sort all users (include both real users and virtual users) in the descending order of $EE^*_{\Phi^*_k}$, i.e., $EE^*_{\Phi^*_1} \geq EE^*_{\Phi^*_2} \geq, \ldots, \geq EE^*_{\Phi^*_L}$;
Set $EE^*_{\Phi^*} = 0$, $\Phi^* = ffl$, and $U = ffl$;
for $k = 1 : L$
  if $EE^*_{\Phi^*} \leq EE^*_{\Phi^*_k}$ **do**
    $\Phi^* = \Phi^* \bigcup \Phi^*_k$ and $U = U \bigcup \{k\}$;
    Obtain $p^*_{k,i}$ and $EE^*_{\Phi^*}$ by solving problem (18);
  else return
end

the link-dependent power $\Theta_{\mathrm{t}} P_{\mathrm{dyn},k} + \Theta_{\mathrm{r}} P_{\mathrm{dyn},0}$, while each virtual user only contains one link and its circuit power is given by the link-dependent power $\Theta_{\mathrm{t}} P_{\mathrm{dyn},k} + \Theta_{\mathrm{r}} P_{\mathrm{dyn},0}$. Note that the number of virtual users derived from a real user k can vary from 0 to $n_k - 1$ since at least one link is activated in achieving the maximum user EE for each user.

By treating those inactive links in the stage of maximizing user EE as virtual users, the original problem is simplified to finding users that maximizes the system EE just like the problem of finding links maximizing the user EE. Now, we readily present Theorem 2 which provides a sufficient and necessary condition for determining whether an arbitrary user (including both real user and virtual user) should be activated.

Theorem 2 (User Scheduling Condition): Given any scheduled user set U with its corresponding active link set Φ, and any not-yet-scheduled user $k \notin U$ with its corresponding active link set $\Phi^*_k \nsubseteq \Phi$, the user k should be scheduled and added to U if and only if $EE^*_{\Phi} \leq EE^*_{\Phi^*_k}$. Moreover, if any user k is scheduled, then all the active links in Φ^*_k in terms of maximum user EE will be activated and added to Φ in achieving the maximum system EE.

Proof: Please see Appendix B. ■

Based on Theorem 2, we can now present an algorithm to solve the system EE maximization problem. Its outline is summarized in Algorithm 2 and described in what follows. We first sort all users in descending order according to the maximum user EE $EE^*_{\Phi^*_k}$, i.e., $EE^*_{\Phi^*_1} \geq EE^*_{\Phi^*_2} \geq, \ldots, \geq EE^*_{\Phi^*_L}$, where L is the total number of users including real users and virtual users. Then, we have the following proposition to characterize a property of the order.

Proposition 1: Assume that the virtual user ℓ is derived from the link i' of the real user k. Following the descending order of the user EE, this virtual user ℓ must line up behind its corresponding real user k, i.e. the index of virtual user ℓ must be larger than that of real user k.

Proof: According to Algorithm 1, we have $EE^*_{\Phi^*_k} > ee^*_{k,i'}$, i.e, $EE^*_{\Phi^*_k} > EE^*_{\Phi_\ell}$. Therefore, when they line up together according to the descending order, the virtual user ℓ (inactive link) must be ranked after its corresponding real user k.

This proposition guarantees that virtual users (inactive links) are less likely to be active than their corresponding real users in maximizing the system EE, otherwise it may lead to the situation that some link is activated but its associated real user is not scheduled in the optimal solution, which does not make sense. However, the virtual users derived from a real user k can be more favorable than other real users or virtual users derived from other real users due to different channel conditions and power consumptions. Based on Theorem 2 and Proposition 1, the optimality of Algorithm 2 can be easily shown by an extension of the proof of Corollary 1 in [19].

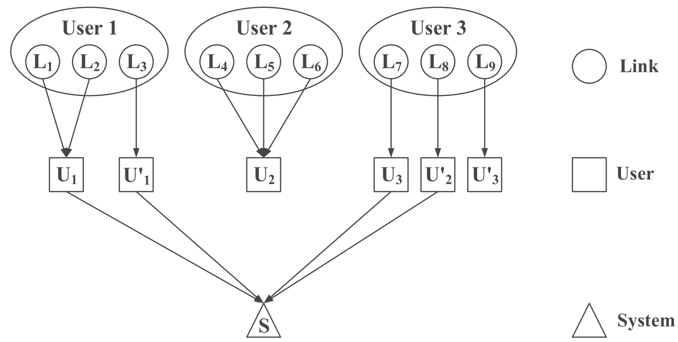

Fig. 1. An illustration of the process for obtaining the scheduled users and the active links. For example, link L_1, L_2, and L_3 belongs to user 1. In the second level, $\mathbf{U}'_1$ (L_3) can be regarded as a virtual user of user 1. $\mathbf{U_1}$ is composed of active links in maximizing EE_1. Similar interpretation applies to other users.

Remark 2: In summary, the overall procedure to solve the original non-convex system EE maximization problem (11) involves three steps. In step 1, solve the link EE maximization problem (12) by using the bisection method for all links of all users. In step 2, solve the user EE maximization problem (16) by using Algorithm 1 and the results of step 1 for all users. In step 3, solve the system EE maximization problem (11) by using Algorithm 2 and the results of step 2. It is seen that the original master problem of system EE maximization is broken into small subproblems of the user EE maximization, and each small subproblem is further broken into smaller sub-subproblems of link EE maximization. Thus, we name the proposed approach as a divide-and-conquer approach. Fig. 1 illustrates an example of the divide-and-conquer approach for for three users and each having three links. As we can see that in maximizing the user EE of user 1, i.e., EE_1, link 1 and link 2 are activated while link 3 is not. However, link 3 (virtual user $\mathbf{U}'_1$) is then reactivated in maximizing the system EE. For user 2, although all its links (link 4, 5, and 6) are active in maximizing EE_2, they are not active in maximizing the system EE. For user 3, only its link 7 is active in maximizing EE_3, but both link 7 and link 8 are activated in maximizing the system EE.

Corollary 1: Following the user EE order, the system EE is either always increasing or first increasing and then decreasing with the order index.

Proof: Please see Appendix C. ■

This corollary suggests that the bisection method can be employed to search the optimal index, which can further reduce the computational complexity of Algorithm 2.

C. Impact of Static Receiving Power and User Scheduling Analysis

The next theorem states the important role of the static receiving power $P_{\mathrm{sta},0}$ in the energy-efficient transmission.

Theorem 3: The optimal number of scheduled users for maximizing the system EE is nondecreasing with the static receiving power $P_{\text{sta},0}$. When $P_{\text{sta},0}$ is negligible or zero, only one user should be scheduled, namely, TDMA is the optimal scheduling strategy. When $P_{\text{sta},0}$ is sufficiently large, all users will be scheduled for the energy-efficient transmission.

Proof: Please see Appendix D. ■

The intuition is that when $P_{\text{sta},0}$ is larger, the power consumption of other components is less dominant, and hence it is more effective in obtaining higher EE by increasing the system throughput. In the extreme case, when $P_{\text{sta},0}$ is sufficient large, the additional power consumption brought by increasing throughput only has trivial impact on the total power consumption, and thus the EE maximization problem (11) is approximate to the throughput maximization problem where all users will be scheduled. On the other hand, if $P_{\text{sta},0} = 0$, the optimal strategy is only to schedule the 'best' user where the best is in terms of the user EE. It has the similar interpretation as that of the throughput maximization problem in TDMA systems: only the user with the best channel gain will be scheduled. From Theorem 3, it is also interesting to note that although the weights have been imposed on users to enforce a certain notion of fairness, the number of scheduled users still can not be guaranteed, especially for the case with a low static receiving power. This explicitly suggests that it is necessary to consider strict QoS constraints for practical energy-efficient applications.

D. Special Cases for Tx or Rx Side EE Maximization Problem

1) The Tx side EE maximization, i.e., $\Theta_{\text{t}} = 1$ and $\Theta_{\text{r}} = 0$,

$$\begin{aligned}
\max_{\boldsymbol{p}} \quad & EE = \frac{\sum_{k=1}^{K} \omega_k \sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\sum_{k=1}^{K}\left(\frac{\sum_{i\in\mathcal{M}_k} p_{k,i}}{\xi} + n_k^o P_{\text{dyn},k} + \mathcal{I}(n_k^o)P_{\text{sta},k}\right)} \\
\text{s.t.} \quad & n_k^o = \sum_{i\in\mathcal{M}_k} \mathcal{I}(p_{k,i}), \quad 1\le k\le K, i\in\mathcal{M}_k, \\
& p_{k,i} \le P_{\max}^{k,i}, \quad 1\le k\le K, i\in\mathcal{M}_k, \\
& p_{k,i} \ge 0, \quad 1\le k\le K, i\in\mathcal{M}_k.
\end{aligned} \tag{19}$$

Based on Theorem 3, it is found that the optimal solution to (19) is to schedule the user with the maximum user EE. Moreover, as we can see that the user static circuit power, $P_{\text{sta},k}$, also plays the similar role in the link activation as the role of the system static receiving power $P_{\text{sta},0}$ played in the user scheduling. More specifically, if $P_{\text{sta},k}$ is not considered in the user power consumption, then the optimal solution to the user EE maximization problem (16) is only to activate the link with the maximum link EE.

2) The Rx side EE maximization, i.e., $\Theta_{\text{t}} = 0$ and $\Theta_{\text{r}} = 1$,

$$\begin{aligned}
\max_{\boldsymbol{p}} \quad & EE = \frac{\sum_{k=1}^{K} \omega_k \sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\sum_{k=1}^{K} n_k^o P_{\text{dyn},0} + P_{\text{sta},0}}, \\
\text{s.t.} \quad & n_k^o = \sum_{i\in\mathcal{M}_k} \mathcal{I}(p_{k,i}), \quad 1\le k\le K, i\in\mathcal{M}_k, \\
& p_{k,i} \le P_{\max}^{k,i}, \quad 1\le k\le K, i\in\mathcal{M}_k, \\
& p_{k,i} \ge 0, \quad 1\le k\le K, i\in\mathcal{M}_k.
\end{aligned} \tag{20}$$

When only the receiving power is considered, increasing the transmission power will not bring additional power consumption in the Rx side, while only activating one more radio link would lead to that. Therefore, each link, if active, will transmit with its maximum power $P_{\max}^{k,i}$ and thereby only the link activation needs to be considered. Following the link EE order, the optimal solution can be easily obtained.

Remark 3: Note that the EE maximization problem formulated in [4] in multiuser MIMO systems does not consider the peak power constraints, and is thus a special case of our problem formulation of (11). The authors in [4] proposed a quadratic complexity method to obtain the optimal solution, while our proposed method is able to solve that problem with linear complexity. In addition, the solution of the Tx side case is also applicable to [27], where the EE of a single user and multi-link system is considered, but only a suboptimal approach is proposed.

E. Complexity Analysis

In this subsection, we provide the complexity analysis for the proposed method in comparison with existing methods. Without loss of generality, we assume that each user is configured with N_0 radio links. The optimal solution to problem (11) can be obtained within $2KN_0 + K$ iterations of power control by the proposed divide-and-conquer approach. First, the computation of maximum link EE requires KN_0 times of power control. Second, the computation of maximum user EE only requires $\sum_{k=1}^{K} n_k^o$ times of power control as can be seen from Algorithm 1. Third, the computation of maximum system EE at most requires $K + \sum_{k=1}^{K}(N_0 - n_k^o)$ times of power control since there are K real users and $\sum_{k=1}^{K}(N_0 - n_k^o)$ virtual users (inactive links in maximizing the user EE) in the system.Therefore, the overall complexity is at most $KN_0 + \sum_{k=1}^{K} n_k^o + K + \sum_{k=1}^{K}(N_0 - n_k^o) = 2KN_0 + K$ times of power control. Note that the power control of (14) or (17) is generally based on the bisection search of $ee_{k,i}^*$ or $EE_{\Phi_k}^*$ which is a single variable search between an interval and thus almost has the same computational complexity. Since the power control by using bisection method [25], the link activation by using Algorithm 1, and the user scheduling by using Algorithm 2 are all guaranteed to converge, the proposed method is guaranteed to converge.

The complexity of the exhaustive search method for all the possibilities is about 2^{KN_0} times the power control. Based on Dinkelbach method [24], if each user further exploits the channel quality among the multiple radio links, each user has $N_0 + 1$ possibilities of link activation. Thus, the total computational complexity is about $I_{\max}(N_0 + 1)^K$, where $I_{\max}$ is the iterations for the convergence of the Dinkelbach method in the outer layer [7]. The complexity reduction of the proposed method stems from exploiting the special structure of the introduced link EE, user EE, and the system EE.

IV. Energy Efficient Scheduling With QoS Constraints

In this section, we address the energy-efficient user scheduling, link activation, and power allocation problem with QoS constraints. Similar to [28], we consider two typical classes of user traffics. One is the delay-constrained (DC) class, in which each user has an individual minimum data rate requirement and the other is the best-effort (BE) class, in which each user does not have any specific rate requirement and can be scheduled for system EE maximization. Then, the joint Tx and Rx EE maximization problem with QoS constraints can be formulated as follows

$$\begin{aligned}\max_{\boldsymbol{p}} \quad & \frac{\sum_{k=1}^{K}\sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\Theta_{\mathrm{t}}\sum_{k=1}^{K}P_k+\Theta_{\mathrm{r}}P_0}\\ \text{s.t.} \quad & n_k^o=\sum_{i\in\mathcal{M}_k}\mathfrak{I}(p_{k,i}), \quad 1\le k\le K, i\in\mathcal{M}_k,\\ & p_{k,i}\le P_{\max}^{k,i}, \quad 1\le k\le K, i\in\mathcal{M}_k,\\ & \sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)\ge R_{\min}^k, 1\le k\le K_1,\\ & p_{k,i}\ge 0, \quad 1\le k\le K, i\in\mathcal{M}_k,\end{aligned} \tag{21}$$

where K_1 is the number of DC users out of the total K users, $R_{\min}^k$ is the required minimum data rate of DC user k, and $P_{\max}^{k,i}$ has the same meaning as that in problem (11). With the QoS constraints, problem (21) becomes much more challenging. Inspired by the study in Section III, we propose a low complexity algorithm at the cost of a slight performance loss, which will be demonstrated by numerical results in Section V.

From Theorem 2, we have revealed that if one user is scheduled in the system, then all the active links in maximizing the user EE will surely be active in maximizing the system EE. Therefore, since DC users are guaranteed to be scheduled, we can find their active radio links as well as power control to satisfy their minimum data rate requirements first. Specifically, for each DC user, we solve the user EE maximization problem subject to the data rate constraint and obtain the corresponding active links, i.e.,

$$\begin{aligned}\max_{\{p_{k,i}\}} \quad & \frac{\omega_k\sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\frac{\Theta_{\mathrm{t}}\sum_{i\in\mathcal{M}_k}p_{k,i}}{\xi}+n_k^o(\Theta_{\mathrm{t}}P_{\mathrm{dyn},k}+\Theta_{\mathrm{r}}P_{\mathrm{dyn},0})+\Theta_{\mathrm{t}}P_{\mathrm{sta},k}}\\ \text{s.t.} \quad & n_k^o=\sum_{i\in\mathcal{M}_k}\mathfrak{I}(p_{k,i}), \quad 1\le k\le K, i\in\mathcal{M}_k,\\ & \sum_{i\in\mathcal{M}_k} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)\ge R_{\min}^k, \quad 1\le k\le K_1,\\ & p_{k,i}\le P_{\max}^{k,i}, \quad 1\le k\le K, i\in\mathcal{M}_k,\\ & p_{k,i}\ge 0, \quad 1\le k\le K, i\in\mathcal{M}_k.\end{aligned} \tag{22}$$

Note that problem (22) is in fact problem (16) with minimum date rate constraints. Since each link of the same user k has the same $\Theta_{\mathrm{t}}P_{\mathrm{dyn},k}+\Theta_{\mathrm{r}}P_{\mathrm{dyn},0}$, it is easy to show that Theorem 1 is still valid for problem (22) and hence a similar optimal algorithm like Algorithm 1 can be developed. We refer interested readers to [19] (Section IV-C) for the details. The following theorem reveals the relationship between those links and the optimal solution.

Theorem 4: For users whose EE is limited by user data rates, i.e., the achieved user data rate in maximizing EE_k is exactly $R_{\min}^k$, the active links in maximizing user EE are also guaranteed to be active in the optimal solution to problem (21).

Proof: Please see Appendix E. ■

Algorithm 3. Energy-Efficient Scheduling with QoS Constraints

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1: Obtain $\Phi_k^*, \forall k\in\{1,\ldots,K_1\}$, by solving problem (22) with a modification of Algorithm 1;
2: Obtain EE_{DC}^* by solving problem (23) with given Φ_k^*;
3: Obtain Φ_k^*, $\forall k\in\{K_1+1,\ldots,K\}$ by using Algorithm 1 and define virtual users for $i\in\mathcal{M}_k\backslash\Phi_k^*, \forall k$;
4: Obtain the solution by using Algorithm 2.

Theorem 4 indicates that the active links in the user EE maximization of those kind of DC users are actually a subset of links in maximizing system EE. Therefore, we can solve the following problem with those links of DC users as a basis, i.e.,

$$\begin{aligned}\max_{\boldsymbol{p}} \quad & EE_{DC}=\frac{\sum_{k=1}^{K_1}\sum_{i\in\Phi_k^*} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)}{\sum_{k=1}^{K_1}\Theta_{\mathrm{t}}\sum_{k\in U}P_k+\Theta_{\mathrm{r}}P_0}\\ \text{s.t.} \quad & n_k^o=\sum_{i\in\mathcal{M}_k}\mathfrak{I}(p_{k,i}), \quad 1\le k\le K_1, i\in\mathcal{M}_k,\\ & p_{k,i}\le P_{\max}^{k,i}, \quad 1\le k\le K_1, i\in\Phi_k^*,\\ & \sum_{i\in\Phi_k^*} B\log_2\left(1+\frac{p_{k,i}g_{k,i}}{\Gamma\sigma^2}\right)\ge R_{\min}^k, \quad 1\le k\le K_1,\\ & p_{k,i}\ge 0, \quad 1\le k\le K_1, i\in\Phi_k^*.\end{aligned} \tag{23}$$

As Φ_k^* and n_k^o are fixed, the combinatorial characteristic is then eliminated and only the transmit power needs to be optimized. Thus, problem (23) is easily verified to be a standard quasiconcave optimization problem and can be solved by the well-known bisection based waterfilling method [10]. Taking EE_{DC}^* as an initialization of the system EE, the remaining links and BE users can be further exploited to improve the system EE. Specifically, we sort those remaining links according to their link EE and then successively add them to the system following the conclusion in Theorem 1 and Theorem 2. The algorithm finishes until adding one link or BE user will decrease the system EE. The details of the procedure is summarized in Algorithm 3.

V. Numerical Results

In this section, we provide comprehensive simulation results to validate our theoretical findings and demonstrate the effectiveness of proposed methods. We consider a network with hexagonal coverage with a radius of 1000 meters. The users are randomly and uniformly distributed in the coverage except the concentric circle with a radius of 100 meters. Without loss of generality, $P_{\max}^{k,i}$ and $R_{\max}^k$ are assumed the same for all equally weighted users. The total number of radio links of each user is also assumed to be the same, given by N_0. The main system parameters are listed in Table II according to [23], [29] unless specified otherwise. All simulation results are averaged over 5000 channel realizations. We first demonstrate the impact of the dynamic circuit power model

TABLE II
SIMULATION PARAMETERS

Parameter	Description
Bandwidth of each radio link, B	15 kHz
Maximum allowed transmit power, $P_{\max}^{k,i}$	25 dBm
Static circuit power of the AP, $P_{\text{sta},0}$	5000 mW
Link dependent power of the AP, $P_{\text{dyn},0}$	45 mW
Static circuit power of the user k, $P_{\text{sta},k}$	100 mW
Link dependent power of the user k, $P_{\text{dyn},k}$	$5-30$ mW
Power spectral density of thermal noise	-174 dBm/Hz
Power amplifier efficiency, ξ	0.38
Path loss model	Okumura-Hata
Lognormal Shadowing	8 dB
Penetration loss	20 dB
Fading	Rayleigh flat fading
$\{\Theta_{\text{t}}, \Theta_{\text{r}}\}$	$\{1, 1\}$
Γ	1

for energy-efficient communication. We then compare the proposed divide-and-conquer approach for joint Tx and Rx EE maximization (denoted as "EE optimal" in simulation figures) with several benchmarks at different system parameters. The benchmark scheduling methods include: 1) Dinkelbach method with exhaustive search[6]–[9]; 2) EE Transmitter: based on the Tx side optimization; 3) EE Receiver: based on the Rx side optimization; 4) Throughput Optimal: based on the conventional throughput maximization [28].

A. *Impact of Dynamic Circuit Power Model*

In order to show the impact of adopting dynamic circuit power model, we compare the system EE of the following three cases at $K = 20$ users: 1) "Dynamic": the proposed algorithm with the dynamic circuit power model in (9). That is, both the user scheduling and the link activation are considered. 2) "Semi-dynamic": the proposed algorithm with the modified power model in (9) where $\mathcal{I}(n_k^o)$ is replaced by 1. That is, only the link activation is considered while assuming that all users are scheduled as in [23]. 3) "Static": the proposed algorithm with the modified power control in (9) where both $\mathcal{I}(p_{k,i})$ and $\mathcal{I}(n_k^o)$ are replaced by 1. That is, only the power control is performed while assuming that all users are scheduled and all links are activated as in [6]–[9]. We obtain the power control solutions to the system EE maximization problem based on "Semi-dynamic" or "Static" scheme and then substitute them into problem (11) to obtain their actual system EE for comparison. As observed from Fig. 2, the "Static" scheme suffers from some performance loss, and with the increasing of radio links N_0, the performance loss becomes larger. This is because the energy-efficient design based on the non-dynamic power model does not capture the energy consumption saving in the dimension of the number of radio-links or users. This further demonstrates the necessity of adopting accurate power consumption model in the energy-efficient design as pointed out in [15], [16], [20].

B. *System EE and System Throughput Versus Maximum Transmit Power*

Fig. 3 and Fig. 4 evaluate the performance comparison of different scheduling designs from the EE perspective and

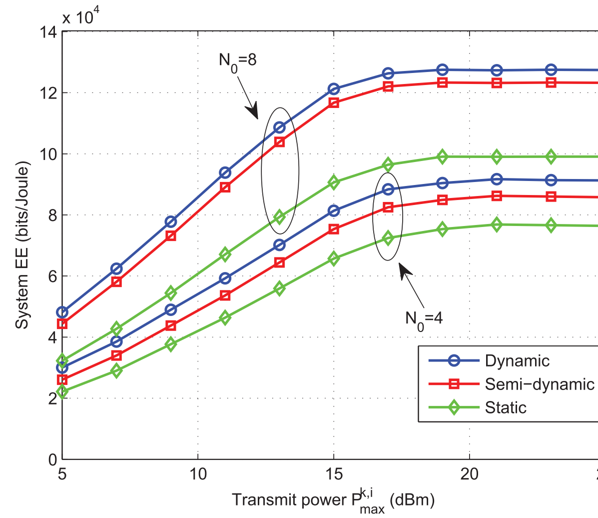

Fig. 2. The impact of dynamic circuit model on energy-efficient transmissions ($K = 20$ users).

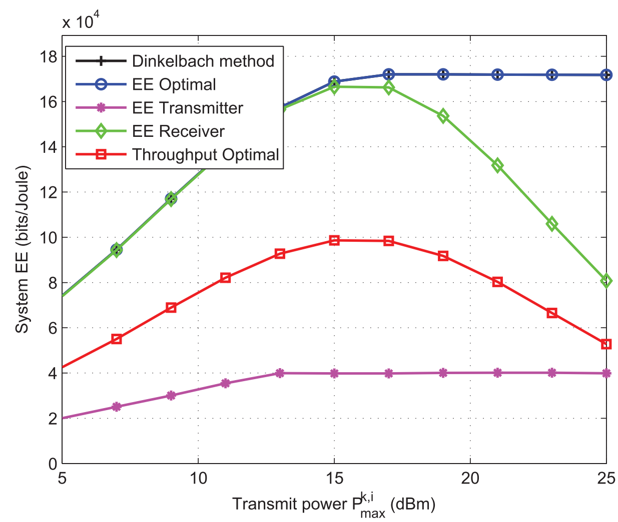

Fig. 3. The system EE versus the transmit power of each radio link ($K = 8$ users, $N_0 = 20$ radio links).

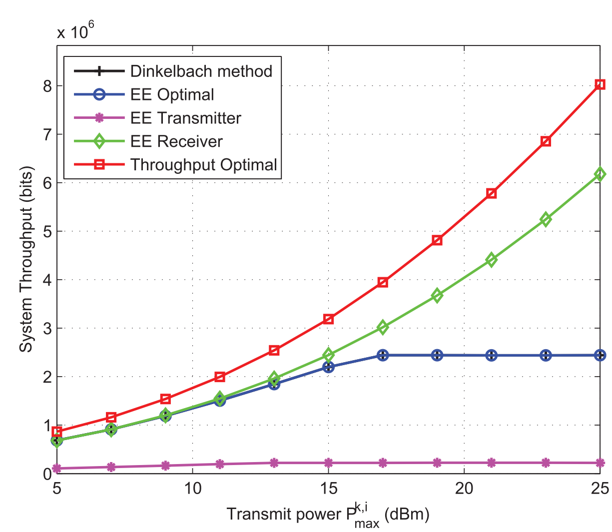

Fig. 4. The system throughput versus the transmit power of each radio link ($K = 8$ users, $N_0 = 20$ radio links).

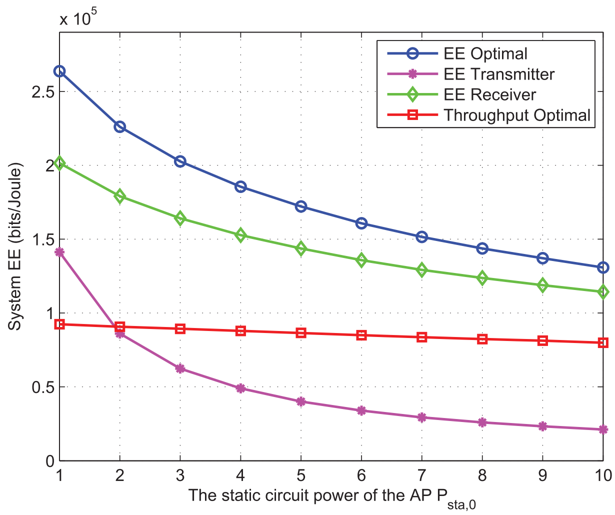

Fig. 5. The system EE versus the static circuit power of the AP ($K = 8$ users, $N_0 = 20$ radio links).

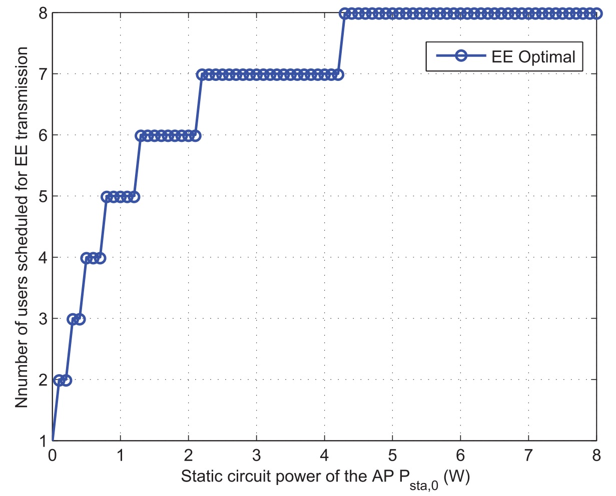

Fig. 6. The number of users scheduled versus the static circuit power of the AP ($K = 8$ users , $N_0 = 20$ radio links).

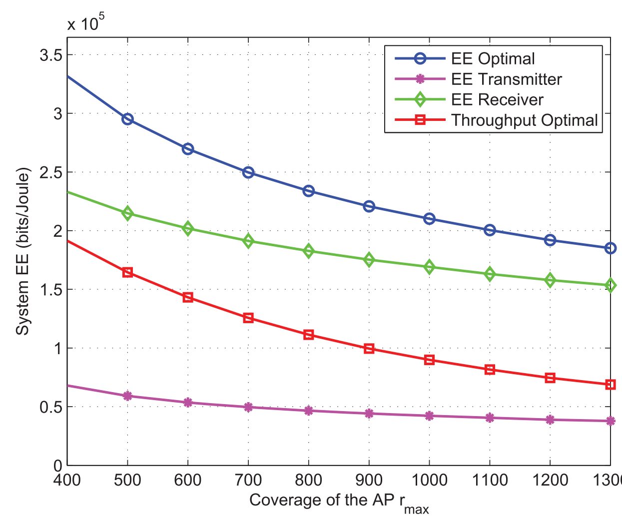

Fig. 7. The system EE versus the coverage of the AP ($K = 8$ users, $N_0 = 20$ radio links).

the spectral efficiency perspective, respectively. Specifically in Fig. 3, we can first observe that our proposed method performs the same as the Dinkelbach method, which demonstrates its optimality. Moreover, as the transmit power increases, the performance of the EE Optimal scheme first increases and then approaches a constant. This is because the system EE is defined as the ratio of the overall weighted system data rate to the system power consumption, exceedingly large transmit power would result in low EE. This means that when the maximum EE is achieved, the energy efficient design is not willing to consume more power even if the maximum allowed transmit power is sufficiently large, and thus the actual transmit power and the EE both keep constant. However, those of the Throughput Optimal scheme and the EE Receiver scheme first increase and then decrease due to their greedy use of the transmit power.

It is also interesting to note that the EE Receiver scheme approaches the EE Optimal scheme in the low transmit power regime while it is closer to the Throughput Optimal scheme in the high transmit power regime. This is because that in the high transmit power region, the nature of the EE Receiver scheme is to transmit with the peak power, which makes it close to the Throughput Optimal scheme, while in the low transmit power regime, the transmit power plays a less dominant role in the total power consumption and this thereby makes the EE Optimal scheme and the EE Receiver scheme perform almost the same. A similar phenomenon can also be found in Fig. 4 in terms of the system throughput. Moreover, the EE Transmitter scheme is poor in both EE and spectral efficiency due to the fact that *only one user is scheduled*, as found in Section III-C.

C. Comparison at Different Static Receiving Power $P_{\text{sta},0}$

Fig. 5 shows the system EE versus the static receiving power $P_{\text{sta},0}$. We can clearly observe that the EE of all designs decrease with the increasing of static circuit power of the AP. Moreover, we notice that the performance gaps between the EE Optimal scheme, the EE Receiver scheme, and the Throughput Optimal scheme reduce as $P_{\text{sta},0}$ increases. Since when $P_{\text{sta},0}$ becomes larger, the transmit power as well as the user scheduling power becomes less denominated, which weakens the effectiveness of energy-efficient power control and user scheduling, and thereby makes performance of the EE Optimal scheme and the EE Receiver scheme approach that of the Throughput Optimal scheme.

Fig. 6 validates our theoretical findings in Theorem 3 which characterizes the monotonicity of the number of scheduled users with $P_{\text{sta},0}$. As we can see that when $P_{\text{sta},0}$ is small enough or zero, the optimal energy-efficient strategy is to schedule only one user. As $P_{\text{sta},0}$ increases, more users are scheduled to improve the system EE through boosting the system throughput.

D. Comparison at Different Cell Size $r_{\max}$ and Minimum Data Rate Requirement $R_{\max}^k$

As expected in Fig. 7, the system EE of all schemes decreases with the increasing of cellsize due to the path loss. In addition,

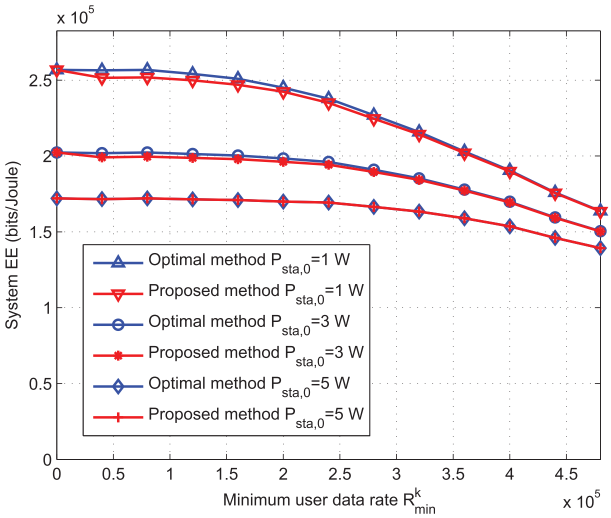

Fig. 8. The system EE versus the minimum user data rate requirement ($K = 8$ users, $K_1 = 4$ users, $N_0 = 20$ radio links).

we also find that as $r_{\max}$ increases, the performance of the EE Receiver scheme approaches from the Throughput Optimal scheme to the EE Optimal scheme. The interpretation is that more links will transmit with its maximum power for the EE Receiver scheme when the channel quality is good, which results in the Throughput Optimal scheme. When the channel quality becomes worse, links will be selectively activated for the EE receiver scheme, and the active links of the EE Optimal scheme will also transmit with their maximum power in order to compensate for the channel degradation, which makes the performance of the EE Receiver scheme close to that of the EE Optimal scheme.

In Fig. 8, we evaluate the performance behaviour of the proposed divide-and-conquer method (Algorithm 3) and the optimal method using Dinkelbach method [7], [24]) with data rate requirements. Compared with the optimal method, the proposed method has slight performance loss in the low data rate regime and they almost perform the same in the high data rate regime. Moreover, when $P_{\text{sta},0}$ becomes larger, the performance gap between the proposed method and the optimal method becomes less. It is interesting to note that when the user data rate requirement $R^k_{\min}$ is lower, the performance gap between the proposed method and the optimal method becomes larger. This is due to the non-strict equivalence of the user EE maximization and the system EE maximization. As for lower $R^k_{\min}$, the maximization of user EE is very likely to schedule more links than required, thus leading to higher user data rate R_k than the minimum requirement $R^k_{\min}$. However, these links' channel conditions may be inferior to other users' links from a system perspective. Therefore, some of these scheduled links may be unfavorable in achieving a higher system EE and thus results in a performance gap between the proposed scheme and the optimal scheme in the low $R^k_{\min}$ region. Thus, the smaller the minimum user data rate requirement $R^k_{\min}$ ($R^k_{\min} > 0$) is, the above phenomenon is more likely to happen and therefore leads a larger performance gap.

VI. Conclusions

In this paper, we investigated the joint Tx and Rx EE maximization problem in multi-radio networks via joint user scheduling, link activation, and power control. A dynamic power consumption model for the considered system was established by including transmission power, link dependent signal processing power, and static circuit power. We then introduced two sub-level's EE and explored their hierarchical relationships. The system EE maximization problem was directly addressed from the fractional perspective, which results in a divide-and-conquer approach of linear complexity. Moreover, we revealed that the static receiving power has an implicit interpretation for the optimal number of scheduled users. In the extreme case when the static receiving power is negligible, TDMA is the optimal scheduling strategy. In order to meet the QoS in practice, we then extended the proposed approach to solve the problem with user minimal data rate constraints, which exhibits good performance. It was shown that joint Tx and Rx optimization scheme outperforms the one side optimization schemes from the EE perspective. In addition, the Tx side optimization scheme results in both low EE and low SE, which further demonstrates the importance of the joint Tx and Rx design.

Appendix A
Proof of Theorem 1

Denote $p^*_{k,i}$ as the optimal power allocation corresponding to the link EE $ee^*_{k,i}$ by (12) and (14). Also, denote $\hat{p}_{k,i}$ and $\check{p}_{k,i}$ as the optimal power allocations corresponding to $EE^*_{\Phi_k \bigcup \{(k,i)\}}$ and $EE^*_{\Phi_k}$ by (18), respectively. Let $S_k \triangleq \{p_{k,i} | 0 \leq p_{k,i} \leq P^{k,i}_{\max}, \forall i \in \mathcal{M}_k, k = 1, \ldots, K\}$, and $P_{k,i}(p_{k,i}) = \frac{\Theta_t p_{k,i}}{\xi} + \Theta_t P_{\text{dyn},k}$. Then, we have the following

$$
\begin{aligned}
EE^*_{\Phi_k \bigcup \{(k,i)\}} &= \max_{\mathbf{p}_k \in S_k} \frac{\sum_{\ell=1}^{i} \omega_k r_{k,\ell}(p_{k,\ell})}{\sum_{\ell=1}^{i} P_{k,i}(p_{k,\ell}) + \Theta_t P_{\text{sta},k}} \\
&= \frac{\sum_{\ell=1}^{i-1} \omega_k r_{k,\ell}(\hat{p}_{k,\ell}) + \omega_k r_{k,i}(\hat{p}_{k,i})}{\sum_{\ell=1}^{i-1} P_{k,\ell}(\hat{p}_{k,\ell}) + \Theta_t P_{\text{sta},k} + P_{k,i}(\hat{p}_{k,i})} \\
&\geq \frac{\sum_{\ell=1}^{i-1} \omega_k r_{k,\ell}(\check{p}_{k,\ell}) + \omega_k r_{k,i}(p^*_{k,i})}{\sum_{\ell=1}^{i-1} P_{k,\ell}(\check{p}_{k,\ell}) + \Theta_t P_{\text{sta},k} + P_{k,i}(p^*_{k,i})} \\
&\geq \min\left\{ \frac{\sum_{\ell=1}^{i-1} \omega_k r_{k,\ell}(\check{p}_{k,\ell})}{\sum_{\ell=1}^{i-1} P_{k,\ell}(\check{p}_{k,\ell}) + \Theta_t P_{\text{sta},k}}, \frac{\omega_k r_{k,i}(p^*_{k,i})}{P_{k,i}(p^*_{k,i})} \right\} \\
&= \min\left\{ EE^*_{\Phi_k}, ee^*_{k,i} \right\}. \qquad (24)
\end{aligned}
$$

On the other hand,

$$
\begin{aligned}
EE^*_{\Phi_k \bigcup \{(k,i)\}} &= \frac{\sum_{\ell=1}^{i-1} \omega_k r_{k,\ell}(\hat{p}_{k,\ell}) + \omega_k r_{k,i}(\hat{p}_{k,i})}{\sum_{\ell=1}^{i-1} P_{k,\ell}(\hat{p}_{k,\ell}) + \Theta_t P_{\text{sta},k} + P_{k,i}(\hat{p}_{k,i})} \\
&\leq \max\left\{ \frac{\sum_{\ell=1}^{i-1} \omega_k r_{k,\ell}(\hat{p}_{k,\ell})}{\sum_{\ell=1}^{i-1} P_{k,\ell}(\hat{p}_{k,\ell}) + \Theta_t P_{\text{sta},k}}, \frac{\omega_k r_{k,i}(\hat{p}_{k,i})}{P_{k,i}(\hat{p}_{k,i})} \right\} \\
&\leq \max\left\{ \frac{\sum_{\ell=1}^{i-1} \omega_k r_{k,\ell}(\check{p}_{k,\ell})}{\sum_{\ell=1}^{i-1} P_{k,\ell}(\check{p}_{k,\ell}) + \Theta_t P_{\text{sta},k}}, \frac{\omega_k r_{k,i}(p^*_{k,i})}{P_{k,i}(p^*_{k,i})} \right\} \\
&= \max\left\{ EE^*_{\Phi_k}, ee^*_{k,i} \right\}. \qquad (25)
\end{aligned}
$$

Based on (24) and (25), we have

$$\min\left\{EE^*_{\Phi_k}, ee^*_{k,i}\right\} \leq EE^*_{\Phi_k \bigcup\{(k,i)\}} \leq \max\left\{EE^*_{\Phi_k}, ee^*_{k,i}\right\}. \tag{26}$$

Therefore, if $EE^*_{\Phi_k} \leq ee^*_{k,i}$, from (26) it follows that

$$EE^*_{\Phi_k} \leq EE^*_{\Phi_k \bigcup\{(k,i)\}} \leq ee^*_{k,i}. \tag{27}$$

In this case, activating link i of user k can achieve no less user EE and hence link i can be included in the active link set Φ_k.

Remark 4: The reason of putting the sign "=" in "$EE^*_{\Phi_k} \leq ee^*_{k,i}$" in Theorem 1 is to let the users schedule more links and thereby achieve higher throughput while not decreasing the EE. It can also be put in the other case.

On the other hand, if $EE^*_{\Phi_k} > ee^*_{k,i}$, it follows that

$$EE^*_{\Phi_k} > EE^*_{\Phi_k \bigcup\{(k,i)\}} > ee^*_{k,i}. \tag{28}$$

In this case, activating link i would decrease the user EE $EE^*_{\Phi_k}$, and thus link i should not be activated. This completes the proof.

APPENDIX B
PROOF OF THEOREM 2

The first part in Theorem 2 can be similarly proved by an extension of Theorem 1, thus we omit it for brevity. We now prove the second part, i.e., if user k is scheduled, then all the active links in Φ^*_k in terms of the maximum user EE will also be activated and added to in achieving the maximum system EE. Assume that user k is scheduled, but the link i, for $i \in \Phi^*_k$, is not activated in maximizing system EE, i.e., $(k, i) \notin \Phi$. In Theorem 1, we have shown that the sufficient and necessary condition of any $i \in \Phi^*_k$ is that $EE^*_{\Phi_k} \leq ee^*_{k,i}$. Thus, it follows that

$$EE^*_{\Phi^*_k} \leq ee^*_{k,n^o_k} \leq ee^*_{k,i}, \quad \forall i \in \Phi^*_k, \tag{29}$$

where n^o_k also denotes the last link activated according to the link EE order since user k overall has n^o_k links activated. On the other hand, since user k is scheduled, we must have $EE^*_\Phi \leq EE_{\Phi^*_k}$ by the first part of Theorem 2. Combining with (29), it follows that

$$EE^*_\Phi \leq EE^*_{\Phi^*_k} \leq ee^*_{k,n^o_k} \leq ee^*_{k,i} = EE^*_{\Phi_\ell}, \quad \forall i \in \Phi^*_k, \tag{30}$$

where the virtual user expression is adopted, i.e.,$\{(k, i)\} = \Phi^*_\ell$. According to 1) in Theorem 2, there must be

$$EE^*_\Phi \leq EE^*_{\Phi^* \bigcup \Phi^*_\ell} = EE^*_{\Phi \bigcup\{(k,i)\}}. \tag{31}$$

Therefore, from (31), we can conclude that scheduling link i of user k can help to maximize the system EE, which contradicts the assumption that $(k, i) \notin \Phi$. Theorem 2 is thus proved.

APPENDIX C
PROOF OF COROLLARY 1

According to the descending order of user EE, assume that the optimal termination index which denotes the index of the last activated user, is m. Here, note that the termination index is the re-order index in terms of the user EE where real users and virtual users are ranked together. It is no longer the original user index that only indicates real users. Let $EE^*_\Phi(m) = EE_{\Phi^*_1 \bigcup \cdots \bigcup \Phi^*_m}$ represents the system EE of mth round. From Theorem 2, we have that for any user order index m satisfying $1 \leq m < m^*$, there must be $EE^*_\Phi(m) \leq EE^*_\Phi(m+1)$ since it is exactly the way to improve system EE successively. On the other hand, since m^* is the termination index, it follows that $EE^*_\Phi(m^*) > EE^*_{\Phi_{m^*+1}}$. Thus, from Theorem 2, we have

$$EE^*_\Phi(m^*) > EE^*_\Phi(m^*+1) > EE^*_{\Phi_{m^*+1}} \geq EE^*_{\Phi^*_{m^*+2}}. \tag{32}$$

With $EE^*_\Phi(m^*+1) > EE^*_{\Phi^*_{m^*+2}}$, we can further have

$$EE^*_\Phi(m^*+1) > EE^*_\Phi(m^*+2) > EE^*_{\Phi_{m^*+2}} \geq EE^*_{\Phi^*_{m^*+3}}. \tag{33}$$

Following this procedure, we can successively prove that for any order index m satisfying $m^* \leq m < L$, it always follows that $EE^*_\Phi(m) > EE^*_\Phi(m+1)$.

Therefore, if $m^* = L$, then we can conclude that the system EE is always increasing with the order index m; else if $1 \leq m^* < L$, we conclude that the system EE is first increasing and then decreasing with the order index m. Corollary 1 is thus proved.

APPENDIX D
PROOF OF THEOREM 3

For the first statement, we only have to prove that if $P^1_{\text{sta},0} < P^2_{\text{sta},0}$, then $m^*_1 \leq m^*_2$, where m^*_1 and m^*_2 are the termination indices of the proposed method for $P^1_{\text{sta},0}$ and $P^2_{\text{sta},0}$, respectively. For notational simplicity, let $EE^*_\Phi(P_{\text{sta},0}, m)$ as the maximum system EE in the mth round when the static circuit power is $P_{\text{sta},0}$. Note that $P_{\text{sta},0}$ does not affect the descending order of the user EE. From (18), it is easy to show that the system EE is strictly decreasing with $P_{\text{sta},0}$. Thus, for the same order index m^*_1, we have

$$EE^*_\Phi\left(P^1_{\text{sta},0}, m^*_1\right) > EE^*_\Phi\left(P^2_{\text{sta},0}, m^*_1\right). \tag{34}$$

Assume $(m^*_1 + \ell)$ is the next *real* user order index behind m^*_1, for $\ell = 1, \ldots, L - m^*_1$. Since m_1 is the optimal termination order index, from Theorem 2, we know that the $(m^*_1 + \ell)$th user is not scheduled when $P_{\text{sta},0} = P^1_{\text{sta},0}$. Then, it must follows that

$$EE^*_\Phi\left(P^1_{\text{sta},0}, m^*_1\right) > EE^*_{\Phi^*_{m^*_1+\ell}}. \tag{35}$$

Therefore, if $EE^*_\Phi(P^1_{\text{sta},0}, m^*_1) > EE^*_{\Phi^*_{m^*_1+\ell}} \geq EE^*_\Phi(P^2_{\text{sta},0}, m^*_1)$, from 1) in Theorem 2, we can conclude that the $(m^*_1 + \ell)$th user will be scheduled when $P_{\text{sta},0} = P^2_{\text{sta},0}$.Thus, the system with $P^2_{\text{sta},0}$ would at least schedule one more user compared with that of $P^1_{\text{sta},0}$, i.e., $m^*_2 \geq m^*_1 + 1$. On the other hand, if $EE^*_\Phi(P^1_{\text{sta},0}, m^*_1) > EE^*_\Phi(P^2_{\text{sta},0}, m^*_1) > EE^*_{\Phi^*_{m^*_1+\ell}}$, since $(m^*_1 + \ell)$ is assumed to be the index of the first *real* user behind m_1^*. Thus, the $(m_1^*+\ell)$th user and all users behind this *real* user also can not be scheduled when $P_{\text{sta},0}=P_{\text{sta},0}^2$. Thus, we have $m_1^*=m_2^*$. Based on the above analysis, we conclude that $m_1^*\le m_2^*$ and the first statement is thus proved.

We next prove the second statement in Theorem 3. If $P_{\text{sta},0}=0$, we have that $EE_\Phi^*(0,1)=EE_{\Phi_1^*}^*$. Since all the users and links are sorted according to the descending order, it follows that $EE_\Phi^*(0,1)=EE_{\Phi_1^*}^*>EE_{\Phi_k^*}^*$, for $k=2,\dots,L$. Therefore, by Theorem 2, the algorithm would finishes and only the first user is scheduled. Then, we can separate the power allocation and the user scheduling as follows

$$\begin{aligned}
\max_k \quad \max_{\boldsymbol{p}_k} \quad & EE_k \\
\text{s.t.} \quad & n_k^o=\sum_{i\in\mathcal{M}_k}\mathfrak{I}(p_{k,i}), \quad i\in\mathcal{M}_k,\\
& n_k^o=\sum_{i\in\mathcal{M}_k}\mathfrak{I}(p_{k,i}), \quad i\in\mathcal{M}_k,\\
& p_{k,i}\le P_{\max}^{k,i}, \quad i\in\mathcal{M}_k,\\
& p_{k,i}\ge 0, \quad i\in\mathcal{M}_k.
\end{aligned} \tag{36}$$

Therefore, only the user with the highest user EE will be scheduled.

We now prove the third by contradiction. Assume that for any $P_{\text{sta},0}$, there is one user k not scheduled in maximizing the system EE. Then by Theorem 2, it must have $EE_\Phi^*(P_{\text{sta},0},m^*)>EE_{\Phi_k}^*$. Recall that $EE_\Phi^*(P_{\text{sta},0},m^*)$ is strictly decreasing with $P_{\text{sta},0}$, with $EE_\Phi^*(P_{\text{sta},0},m^*)\to 0$ as $P_{\text{sta},0}\to+\infty$. Thus, we can always find a sufficiently large $P'_{\text{sta},0}$, satisfying $EE_\Phi^*(P'_{\text{sta},0},m^*)<EE_{\Phi_k}^*$, then the user k should be scheduled to improve the system EE, which contradicts the assumption. Theorem 3 is thus proved.

APPENDIX E
PROOF OF THEOREM 4

We prove this theorem by contradiction. Assume for user k, the achieved data rate R_k^u in maximizing the EE of user k satisfies $R_k^u=R_{\min}^k$ and link $m\in\Phi_k^*$ is not active in the optimal solution to problem (21). The maximum system EE is denoted as EE^*, and the corresponding data rate and power of user k are R_k^* and P_k^*, respectively. The total data rate and total power of all the rest users are R_{rest}^* and P_{rest}^*, respectively. Let $P_{k,i}=\frac{\Theta_t p_{k,i}}{\xi}+\Theta_{\text{t}}P_{\text{dyn},k}$. Then, the maximum system EE can be expressed as

$$EE^*=\frac{R_{\text{rest}}^*+R_k^*}{P_{\text{rest}}^*+P_k^*}=\frac{R_{\text{rest}}^*+\sum_{i\in\Phi_k^*\backslash m}r_{k,i}}{P_{\text{rest}}^*+\sum_{i\in\Phi_k^*\backslash m}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}. \tag{37}$$

Therefore, we have the following two cases:

1) if $\frac{R_{\text{rest}}^*}{P_{\text{rest}}^*}\ge\frac{R_k^*}{P_k^*}$, since link m is active in maximizing the user EE, it follows that

$$\frac{R_k^*}{P_k^*}=\frac{\sum_{i\in\Phi_k^*\backslash m}r_{k,i}}{\sum_{i\in\Phi_k^*\backslash m}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}\le \frac{\sum_{i\in\Phi_k^*}r_{k,i}}{\sum_{i\in\Phi_k^*}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}=\frac{R_k^u}{P_k^u}. \tag{38}$$

where R_k^u and P_k^u are the corresponding data rate and power consumption of user k, respectively. Moreover, recall that R_k^* should also satisfy the minimum date rate of user k, i.e., $R_k^*\ge R_{\min}^k=R_k^u$. Then with (38) and using the fractional property, we can easily prove that

$$\frac{R_{\text{rest}}^*+R_k^*}{P_{\text{rest}}^*+P_k^*}\le\frac{R_{\text{rest}}^*+R_k^u}{P_{\text{rest}}^*+P_k^u}. \tag{39}$$

From (39), we can see that scheduling link m can increase or at least maintain the maximum system EE. 2) else if $\frac{R_{\text{rest}}^*}{P_{\text{rest}}^*}<\frac{R_k^*}{P_k^*}$, as we know that, the maximum user EE meeting the same data rate requirement R_k^u base on set $\Phi_k^*\backslash m$ must be less than the that of based on Φ_k^* due to the optimality of the latter, i.e.,

$$\frac{R_k^u}{\sum_{i\in\Phi_k^*\backslash m}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}\le\frac{R_k^u}{\sum_{i\in\Phi_k^*}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}. \tag{40}$$

Since $R_k^*\ge R_{\min}^k=R_k^u$, it is easy to show $\sum_{i\in\Phi_k^*\backslash m}P_{k,i}\ge\sum_{i\in\Phi_k^*}P_{k,i}$. Therefore, under $\frac{R_{\text{rest}}^*}{P_{\text{rest}}^*}<\frac{R_k^*}{P_k^*}$, we can similarly prove that

$$\frac{R_{\text{rest}}^*+R_k^*}{P_{\text{rest}}^*+\sum_{i\in\Phi_k^*\backslash m}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}\le\frac{R_{\text{rest}}^*+R_k^*}{P_{\text{rest}}^*+\sum_{i\in\Phi_k^*}P_{k,i}+\Theta_{\text{t}}P_{\text{sta},k}}. \tag{41}$$

From (41), we know that scheduling link m can increase or at least maintain the maximum system EE.

Based on 1) and 2), we conclude that link m should keep active in achieving the maximum system EE which contradicts the assumption. Theorem 4 is thus proved.

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