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PAPER

Joint Source and Relay Beamformer Design for General MIMO Relaying Broadcast Channel with Imperfect Channel State Information^{*,**}

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SUMMARY Effective communication strategies with a properly designed source precoding matrix (PM) and a properly designed relay beamforming matrix (BM) can significantly improve the spectral efficiency of multiple-input multiple-output (MIMO) relaying broadcast channels (RBCs). In the present paper, we first propose a general communication scheme with non-regenerative relay that can overcome the half-duplex relay constraint of the general MIMO-RBC. Based on the proposed scheme, the robust source PM and relay BM are designed for imperfect channel state information at the transmitter (CSIT). In contrast to the conventional non-regenerative relaying communication scheme for the MIMO-RBC, in the proposed scheme, the source can send information continuously to the relay and users during two phases. Furthermore, in conjunction with the advanced precoding strategy, the proposed scheme can achieve a full-degree-of-freedom (DoF) MIMO-RBC with that each entry in the related channel matrix is considered to an i.i.d. complex Gaussian variable. The robust source PM and relay BM designs were investigated based on both throughput and fairness criteria with imperfect CSIT. However, solving the problems associated with throughput and fairness criteria for the robust source PM and relay BM designs is computationally intractable because these criteria are non-linear and non-convex. In order to address these difficulties, we first set up equivalent optimization problems based on a tight lower bound of the achievable rate. We then decompose the equivalent throughput problem into several decoupled subproblems with tractable solutions. Finally, we obtain the suboptimal solution for the throughput problem by an alternating optimization approach. We solve the fairness problem by introducing an adjusted algorithm according to the throughput problem. Finally, we demonstrate that, in both cases of throughput and fairness criteria, the proposed relaying communication scheme with precoding algorithms outperforms existing methods.

key words: MIMO, source precoding, relay beamforming matrices, degree

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of freedom, relay half-duplex constraint, imperfect channel state information

1. Introduction

Wireless cooperative communication can improve network capacity and service coverage and reduce energy consumption by using relays [1]. However, the half-duplex constraint (HDC), i.e., the inability of relay nodes to transmit and receive signals simultaneously at the same frequency, is a major potential weakness because system bandwidth resources are used inefficiently due to the necessity of extra dedicated bandwidth for relay retransmissions [2]. In order to overcome the HDC, recent studies [3]–[5] have focused on the full-duplex relaying approach, which assumes that the relay node can transmit and receive signals simultaneously at the same frequency. However, for a relay to receive and transmit on the same frequency band at the same time is generally recognized as impossible because doing so will result in strong self-interference from the transmitter to the receiver of the relay node [5]. Moreover, most studies [3]–[5] considered only the three-terminal model (i.e., one source, one relay, and one destination). In the present study, we focus on a transparent half-duplex non-regenerative relaying scheme for multiple-input multiple-output (MIMO) relaying broadcast channels (RBCs), where two communication phases are required for one transmission due to the relay's HDC. In the first phase, the source transmits the signal to both the relay and the destinations, and in the second phase, the relay forwards the obtained signal to the destinations, while the source also simultaneously transmits the signal to the destinations. Note that the protocol is inefficient if the source remains silent during the second phase.

Existing studies have investigated the multiple-input multiple-output (MIMO) non-regenerative relaying broadcast channel to design effective source precoding and relay processing matrices. In [6], the authors examined the scenario of MIMO fixed relays by applying linear processing to improve link capacity in a cellular system. In addition, an implementable architecture integrating Tomlinson-Harashima precoding and adaptive modulation, which can adjust transmit streams adaptively, was proposed. In [7], considering the quality-of-service constraint, the authors jointly investigated linear beamforming and power saving for wireless cellular networks with multi-antenna relays. In

[8], the authors studied the joint optimization of precoding design for source and relay to improve system capacity under the constraint of transmit power, and the formulated non-convex problem is solved by quadratic programming approaches. Moreover, in [9], the capacity optimization for an amplify-and-forward relay network was considered, and they demonstrated that the duality relationships hold when a node is equipped with multiple antennas. However, the above-mentioned studies ignored the receiver's direct links (DLs) and each receiver was assumed to be a single antenna. In [10], a precoding scheme was proposed to optimize power allocation under the constraint of quality-of-service and transmit power budgets by applying a joint zero forcing (ZF) strategy and the DLs' contribution of receivers. However, this study considered only the ZF scheme with single-antenna receivers. In [11] and [12], the authors proposed a two-phase relaying model for a MIMO relaying broadcast network that considers the DLs' contribution of receivers when the source transmits a signal to both the relay and destinations. However, their studies only considered a source to be inactive during the retransmission phase of the relay, which causes the loss of half the degrees of freedom (DoFs) in the two transmission phases. In [13], the authors considered the source to be active during the retransmission phase of the relay, but, considered only the ZF strategy with a regenerative relaying scheme and ignored the receiver's direct links. In [14]–[17], in the presence of imperfect CSI, robust precoding algorithms were developed in order to deal with CSI quantization or estimation errors, but only an inefficient protocol in which the source remained silent during the second phase was considered. In practice, DLs can provide valuable spatial diversity to the MIMO relay system and should not be ignored. In addition, the source can be active during relay retransmissions in order to overcome the relay's HDC and achieve full DoF.

In the present study, in order to overcome the relay's HDC by allowing the source to be active during the second phase, we consider a general MIMO-RBC with two types of users, i.e., direct users and relay users. The direct user is defined as a user having better DL (*source-destination*) gain than relay link (RL) (*source-relay-destination*) gain, whereas the relay user is defined as a user having better RL gain than DL gain. In the present study, we focus on a general design strategy to deal with the precoding matrix (PM) for the source and the beamforming matrix (BM) for the relay to serve direct users and relay users simultaneously by allowing the source to be active during relay retransmissions within the proposed scheme, which can overcome the relay's HDC and achieve full DoF. Hence, we need not distinguish direct users from relay users in the remainder of the paper. To the best of our knowledge, the present study is the first analytical study on source and relay matrices designed for general MIMO-RBC with coordinated users (direct users and relay users) to overcome the relay's HDC and achieve full DoF during two phases. We first introduce a theorem to prove that the proposed scheme can achieve the maximum DoF of a MIMO-RBC with full rank, and then

consider two-system performance criteria for designing the source PM and relay BM: the summed throughput maximization and the minimum user's rate maximization subjected to transmitted power constraints at both the source and relay with imperfect channel state information at the transmitter (CSIT). However, this problem is computationally intractable due to the fact that the throughput and fairness criteria for the robust source PM and relay BM designs are non-linear and non-convex [18]. In order to address these difficulties, we first set up the equivalent optimization problems based on a tight lower bound of the achievable rate. We then decompose the equivalent throughput problem into several decoupled subproblems with tractable solutions. Finally, we can achieve a suboptimal solution for the throughput problem using an alternating optimization approach. Accordingly, the fairness problem can be resolved using an adjusted algorithm according to the throughput problem.

The remainder of the present paper is organized as follows. Section 2 specifies the system model. The full DoF of the MIMO-RBC with the proposed scheme and a general description of the optimization problems are given in Sect. 3. The equivalent optimization problems are presented in Sect. 4. In Sect. 5, we describe the design of the base station and relay matrices using the WMMSE method for the throughput optimization problem. Section 6 provides the design method for the base station and relay matrices for the fairness optimization problem based on the obtained method by dealing with the throughput optimization problem. Finally, numerical results and conclusions are presented in Sects. 7 and 8, respectively.

Notation: The following notational conventions are followed throughout the paper. Vectors and matrices are presented in boldface lowercase and uppercase letters, respectively. The trace, expectation, inverse, transpose, conjugate transpose, determinant, rank, and pseudoinverse of a matrix are denoted by $\text{Tr}(\cdot)$, $\mathbf{E}(\cdot)$, $(\cdot)^{-1}$, $(\cdot)^T$, $(\cdot)^\dagger$, $|\cdot|$, $\text{Rank}(\cdot)$, and $\text{Pinv}(\cdot)$, respectively. $\text{diag}(a_1, \dots, a_N)$ is a diagonal matrix, for which a_i is the i th diagonal entry. $\text{diag}[a]_N$ is an $N \times N$ diagonal matrix with diagonal entry a . A set of $M \times N$ matrices over a complex field is expressed as $\mathbf{C}^{M \times N}$, and \mathbf{I} is the identity matrix with appropriate dimensions. $\mathcal{CN}(x, y)$ is used to indicate a circularly symmetric complex Gaussian distribution with mean x and covariance y . Moreover, i.i.d. indicates independent and identically distributed. The set of mobile-users is expressed as $\mathcal{U} = \{1, 2, \dots, K\}$, where K is the number of mobile-users.

2. System Model

The system model considered in the present study, which is a general MIMO-RBC consisting of one source (i.e., base station (BS)), one relay station (RS), and K multi-antenna mobile users (MUs), is shown in Fig. 1. The numbers of antennas for the BS and RS are N_b and N_r , respectively, and the number of receiver antennas for the k th MU is N_k . There are two kinds of MUs in the MIMO-RBC considered herein:

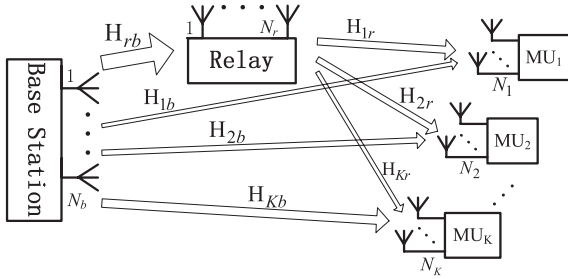


Fig. 1 System model of the MIMO-RBC consisting of three parts: the base station (which acts as source), the relay station, and mobile users. The number of mobile users (direct users and relay users) is K .

direct users and relay users. For the direct users, the DLs gains are better than the RLs gains, and the opposite is true for the relay users. The sets \mathcal{B} and \mathcal{R} contain the indices of the direct users and relay users, respectively. The total number of MUs is $|\mathcal{B} \cup \mathcal{R}| = K$. However, it is not necessary to distinguish direct users from relay users in the remainder of the present paper. The RS is used to aid the data communication between the BS and the relay users. Hence, the number of antennas for the RS must satisfy $N_r \geq \sum_{k \in \mathcal{R}} N_k$. Note that this system model is used for the relay architectures of the third-generation partnership project long-term evolution advanced (3GPP LTE-A) [19]. We consider the BS and RS to be in a flat fading channel. The BS (or RS) and MUs are also in flat fading channels in the case of a non-line-of-sight (NLOS) scenario[†]. Note that, in the proposed scheme, it is only necessary to satisfy $\sum_{k=1}^K N_k \leq 2N_b$ in order to simultaneously support N_k independent substreams for the k th user, which is remarkably different from the conventional schemes for MIMO-RBC [6]–[12] that require $\sum_{k=1}^K N_k \leq N_b$. The considered relaying scheme can be categorized as a non-regenerative, half-duplex scheme [20], and the data transmission scheme has two phases.

2.1 First Phase

Suppose that \mathbf{s}_k is the symbol intended for the k th MU, and the signal vector of one MU is independent from those of the other MUs. In the first phase, by applying a linear PM $\mathbf{P} = [\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K]$ to the data vector \mathbf{s} , where $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_K^T]^T \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, and $\mathbf{P}_k \in \mathbb{C}^{N_b \times N_k}$ is a PM acting on signal vector \mathbf{s}_k for the k th MU, the BS broadcasts the precoded data streams to the RS and MUs. Accordingly, the obtained signal vector for the k th MU and RS can be formulated as follows:

$$\mathbf{y}_{1k} = \mathbf{H}_{kb} \mathbf{P}_k \mathbf{s}_k + \sum_{i=1, i \neq k}^K \mathbf{H}_{kb} \mathbf{P}_i \mathbf{s}_i + \mathbf{n}_k, \quad (1)$$

$$\mathbf{y}_r = \mathbf{H}_{rb} \mathbf{P} \mathbf{s} + \mathbf{n}_r. \quad (2)$$

In the above expressions, the channel matrix between the BS

[†]In fact, the line-of-sight (LOS) scenario can be seen as a special case of the NLOS, and our results can be extended directly to the LOS scenario.

and the k th MU (RS) is denoted by \mathbf{H}_{kb} (\mathbf{H}_{rb}), and each entry in \mathbf{H}_{kb} (\mathbf{H}_{rb}) is an i.i.d. complex Gaussian variable^{††} mean zero and variance σ_{kb}^2 (σ_{rb}^2). Moreover, $\mathbf{n}_x \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ ($x = k$ and r) represents the Gaussian noise vectors for the k th MU and RS, respectively.

2.2 Second Phase

By applying a linear BM \mathbf{G} to the received signal vector, the received signals are forwarded to MUs by the RS during the second phase. At the same time, the BS applies a new linear PM $\mathbf{F} = [\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_K]$ to the data vector $\mathbf{s}^{\dagger\dagger\dagger}$, and then broadcasts the precoded data streams to MUs, where $\mathbf{F}_k \in \mathbb{C}^{N_b \times N_k}$ is a PM acting on signal vector \mathbf{s}_k for the k th MU. This is another remarkable difference from conventional MIMO-RBC schemes [6]–[12], in which the BS remains silent during the second phase. Note that the proposed scheme will definitely increase the diversity gain by allowing the BS to transmit continuously during the second phase. If we assume that the BS-receiver channels experience slow fading and remain unchanged during the second phase [23], the received signal vector at the k th MU during the second phase can be written as follows:

$$\mathbf{y}_{2k} = \mathbf{H}_{kb} \mathbf{F}_k \mathbf{s}_k + \mathbf{H}_{kr} \mathbf{G} \mathbf{H}_{rb} \mathbf{P}_k \mathbf{s}_k + \sum_{i=1, i \neq k}^K (\mathbf{H}_{kb} \mathbf{F}_i + \mathbf{H}_{kr} \mathbf{G} \mathbf{H}_{rb} \mathbf{P}_i) \mathbf{s}_i + \mathbf{H}_{kr} \mathbf{G} \mathbf{n}_r + \mathbf{z}_k, \quad (3)$$

where \mathbf{H}_{kr} is the channel matrix from the RS to the k th MU, in which the entry is assumed to be an i.i.d. complex Gaussian variable with mean zero and variance σ_{kr}^2 , and $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is the Gaussian noise vector observed at the k th MU.

Suppose that P_b and P_r are power constraints for the BS and the RS, respectively, in the two phases. Then, we have

$$\text{Tr}(\mathbf{P}^\dagger \mathbf{P}) \leq P_b, \text{ and } \text{Tr}(\mathbf{F}^\dagger \mathbf{F}) \leq P_b, \quad (4a)$$

$$\text{Tr}(\mathbf{G} \mathbf{H}_{rb} \mathbf{P} \mathbf{P}^\dagger \mathbf{H}_{rb}^\dagger \mathbf{G}^\dagger + \mathbf{G} \mathbf{G}^\dagger) \leq P_r. \quad (4b)$$

For simplicity, (1) and (3) can be rewritten in matrix form as

$$\begin{bmatrix} \mathbf{y}_{1k} \\ \mathbf{y}_{2k} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{kb} & \mathbf{0} \\ \mathbf{H}_{krb} & \mathbf{H}_{kb} \end{bmatrix}}_{\mathbf{H}_k} \underbrace{\begin{bmatrix} \mathbf{P}_k \\ \mathbf{F}_k \end{bmatrix}}_{\bar{\mathbf{P}}_k} \mathbf{s}_k + \underbrace{\begin{bmatrix} \mathbf{n}_k \\ \mathbf{H}_{kr} \mathbf{G} \mathbf{n}_r + \mathbf{z}_k \end{bmatrix}}_{\mathbf{N}_k} + \sum_{i=1, i \neq k}^K \underbrace{\begin{bmatrix} \mathbf{H}_{kb} & \mathbf{0} \\ \mathbf{H}_{krb} & \mathbf{H}_{kb} \end{bmatrix}}_{\bar{\mathbf{P}}_i} \underbrace{\begin{bmatrix} \mathbf{P}_i \\ \mathbf{F}_i \end{bmatrix}}_{\bar{\mathbf{P}}_i} \mathbf{s}_i, \quad (5)$$

where $\mathbf{H}_{krb} \triangleq \mathbf{H}_{kr} \mathbf{G} \mathbf{H}_{rb}$.

^{†††}The same assumption can be found in [21], [22], etc.

^{††††}The reason for the BS transmitting the same data vector \mathbf{s} to the MUs during the second phase is that the RS needs the BS to cooperatively transmit the \mathbf{s} to MUs. In fact, the data vector \mathbf{s} is transmitted primarily to the RS by the BS during the first phase and is transmitted primarily to MUs by the BS during the second phase.

Due to the errors introduced by channel estimation, quantization, reciprocity mismatch, and delay, for example, the CSIT is an estimate of the true channel response \mathbf{H} [21], [24]. Then, the channel can be formulated as

$$\mathbf{H}_{rb} = \widehat{\mathbf{H}}_{rb} + e_{rb}\mathbf{\Omega}_{rb}, \quad (6a)$$

$$\mathbf{H}_{kb} = \widehat{\mathbf{H}}_{kb} + e_{kb}\mathbf{\Omega}_{kb}, \quad (6b)$$

$$\mathbf{H}_{kr} = \widehat{\mathbf{H}}_{kr} + e_{kr}\mathbf{\Omega}_{kr}, \quad (6c)$$

where $\widehat{\mathbf{H}}_x$ ($x \triangleq rb, kb$ and kr) is the estimated CSIT, e_x is the estimation error, $\mathbf{\Omega}_x$ is independent of \mathbf{H}_x , and each entry in $\mathbf{\Omega}_x$ is an i.i.d. complex Gaussian variable with mean zero and variance σ_x^2 . The entries of $\widehat{\mathbf{H}}_x$ are also i.i.d. complex Gaussian variables with mean zero and variance $(1 - e_x^2)\sigma_x^2$ [21]. Assume that $e_{kb} = e_{kr}$ for $k \in \mathcal{U}$, and $e_x\sigma_x^2$ is known to the BS [21]. Accordingly, it is necessary to design the BS PM and RS BM using imperfect CSIT.

Considering Gaussian signaling at the source, we can obtain the achievable rate for the k th MU during the two phases as follows:

$$\mathcal{R}_k = \log \det \left[\mathbf{I} + \widetilde{\mathbf{P}}_k^\dagger \mathbf{H}_k^\dagger \mathbf{R}_k^{-1} \mathbf{H}_k \widetilde{\mathbf{P}}_k \right], \quad (7)$$

where $\mathbf{R}_k = \mathbf{N}_k \mathbf{N}_k^\dagger + \sum_{i=1, i \neq k}^K \mathbf{H}_k \widetilde{\mathbf{P}}_i \widetilde{\mathbf{P}}_i^\dagger \mathbf{H}_k^\dagger$, and \mathbf{N}_k and \mathbf{H}_k are given in Eq. (5). A concise proof of (7) is given in the Appendix.

3. Problem Formulation

Before formulating the objective problems, we first introduce a theorem that demonstrates a significantly different feature of the proposed scheme as compared with the conventional MIMO-RBC [6]–[12]. This theorem indicates that the proposed scheme can support a maximum of $\sum_{k=1}^K N_k$ (if $\sum_{k=1}^K N_k \leq 2N_b$) substreams, while existing schemes can only support a maximum of N_b substreams.

Theorem 1: Let $\widetilde{\mathbf{H}}_{kb} \triangleq [\mathbf{H}_{krb} \ \mathbf{H}_{kb}]$, and $\widetilde{\mathbf{H}}_{\Sigma j} \triangleq [\widetilde{\mathbf{H}}_{1b}^T, \dots, \widetilde{\mathbf{H}}_{jb}^T]^T$, ($k = 1, \dots, K$; $1 \leq j \leq K$). It is feasible for the considered scheme to support a maximum of $\sum_{k=1}^K N_k$ substreams if $\text{Rank}(\widetilde{\mathbf{H}}_{\Sigma K}) = \sum_{k=1}^K N_k \leq 2N_b$, i.e., the rank of the correlated channel is full.

Proof: The multiple-antenna user can be seen as a simple combination of multiple single antenna users in full rank channels, and it will not affect the maximum of substreams which can be supported for the considered scheme, since the post-coding of multiple-antenna user only affects the rate of gain and Theorem 1 focuses on the maximum of supported substreams. Therefore without loss of generality [7], we can only consider the case that each user is only equipped with single antenna to simplify the proof. Due to the fact that each user is with single antenna, we can get $\text{Rank}(\widetilde{\mathbf{H}}_{kb}) = 1$, ($k = 1, \dots, K$), and $\text{Rank}(\widetilde{\mathbf{H}}_{\Sigma K}) = K \leq 2N_b$. Let $\mathbf{G} = \alpha_r \mathbf{I}$ (α_r is a power control factor for satisfying the RS power control), and based on the zero-forcing precoding theory [10], we can obtain $\widetilde{\mathbf{P}} \triangleq \begin{bmatrix} \mathbf{P} \\ \mathbf{F} \end{bmatrix} =$

$\text{Pinv}(\widetilde{\mathbf{H}}_{\Sigma K})(\text{diag}(\gamma_1, \dots, \gamma_K))^{\frac{1}{2}}$, where γ_i is a power control factor for satisfying the BS power control. Then, substituting $\mathbf{G} = \alpha_r \mathbf{I}$, and $\widetilde{\mathbf{P}}$ into (7), it can be directly obtained that it is feasible to support $\sum_{k=1}^K N_k$ substreams with appropriate power control factors α_r and diagonal matrix $\text{diag}(\gamma_1, \dots, \gamma_K)$, since the \mathcal{R}_k in (7) can be represented as $\log \det \left(1 + \frac{\gamma_k}{\zeta_k} \right)$, where ζ_k can be seen as an invariant factor which is mainly related to the channel matrix $\widetilde{\mathbf{H}}_{\Sigma K}$ and noise [10], [22]. \square

Based on Theorem 1, we can obtain the following corollary.

Corollary 1: The maximum DoF for the relaying scheme considered herein is $\min\{2N_b, \sum_{k=1}^K N_k\}$. In other words, the maximal spatial multiplexing gain of the MIMO-RBC is $\min\{2N_b, \sum_{k=1}^K N_k\}$.

Note that the maximum DoF in the conventional relaying scheme is $\min\{N_b, \sum_{k=1}^K N_k\}$ [6]–[12]. In other words, the maximal spatial multiplexing gain of the existing MIMO-RBC scheme is $\min\{N_b, \sum_{k=1}^K N_k\}$. Hence, the proposed scheme significantly increases the DoF of the MIMO-RBC.

Next, we formulate two optimization problems according to the throughput and fairness criteria, respectively. The throughput-based problem is referred as *Problem Tp*, and the fairness-based problem is referred as *Problem Fp*.

3.1 Problem Tp

Under the constraint of the BS transmission power budget P_b for the two phases and the constraint of the RS transmission power budget P_r , jointly design BS PM \mathbf{P} and \mathbf{F} and RS BM \mathbf{G} so that the throughput of the system can be maximized according to the imperfect CSIT, i.e.,

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}] = \arg \max_{\{\mathbf{P}, \mathbf{F}, \mathbf{G}\}} \mathcal{R}_{sum}, \quad (8a)$$

$$\text{s.t. : } (4), \quad (8b)$$

where $\mathcal{R}_{sum} = \sum_{k=1}^K \left(\widehat{\mathcal{R}}_k = \mathbb{E} \left[\mathcal{R}_k \left| \widehat{\mathbf{H}}_x, e_x \right. \right] \right)$, $x \triangleq rb, kb$, and kr .

3.2 Problem Fp

Under the constraint of the BS transmission power budget P_b for the two phases and the constraint of the RS transmission power budget P_r , jointly design BS PM \mathbf{P} and \mathbf{F} and RS BM \mathbf{G} so that the minimum achievable rate among all MUs can be maximized according to the imperfect CSIT, i.e.,

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}] = \arg \max_{\{\mathbf{P}, \mathbf{F}, \mathbf{G}\}} \mathcal{R}_{min}, \quad (9a)$$

$$\text{s.t. : } (4), \quad (9b)$$

where $\mathcal{R}_{min} = \min_{k \in \mathcal{U}} \widehat{\mathcal{R}}_k$, and $\widehat{\mathcal{R}}_k = \mathbb{E} \left[\mathcal{R}_k \left| \widehat{\mathbf{H}}_x, e_x \right. \right]$, $x \triangleq rb, kb$ and kr .

It can be verified that both optimization problems are

non-linear and non-convex, and the optimal closed-form solutions are difficult to obtain directly [25].[†]

4. Equivalent Optimization Problems

Since the above-mentioned optimization problems are non-linear and non-convex, we are finding suboptimal solutions rather than optimal solutions for the two optimization problems. Hence, we first expand the expressions of the achievable data rate for MUs according to the imperfect CSIT. We then construct two equivalent optimization problems based on a tight lower bound of the achievable rate to replace the original optimization problems. Finally, we propose a general linear iterative design algorithm by alternating optimization in order to solve the equivalent optimization problems. Let

$$\begin{aligned}\xi_x &\triangleq e_x \sigma_x \quad (x \triangleq rb, kb \text{ and } kr), \\ \widehat{\mathbf{H}}_{krb} &\triangleq \widehat{\mathbf{H}}_{kr} \mathbf{G} \widehat{\mathbf{H}}_{rb}, \\ \mathbb{A}_k &\triangleq \widehat{\mathbf{H}}_{kb} \mathbf{P}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{kb}^\dagger + \text{diag} \left[\xi_{kb}^2 \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) \right]_{N_k}, \\ \mathbb{B}_k &\triangleq \widehat{\mathbf{H}}_{kb} \mathbf{P}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{krb}^\dagger + \widehat{\mathbf{H}}_{kb} \mathbf{P}_k \mathbf{F}_k^\dagger \widehat{\mathbf{H}}_{kb}^\dagger \\ &\quad + \text{diag} \left[\xi_{kb}^2 \text{Tr}(\mathbf{P}_k \mathbf{F}_k^\dagger) \right]_{N_k}, \\ \mathbb{C}_k &\triangleq \mathbb{B}_k^\dagger, \\ \mathbb{E}_k &\triangleq \widehat{\mathbf{H}}_{kr} \mathbf{G} \mathbf{G}^\dagger \widehat{\mathbf{H}}_{kr}^\dagger + \text{diag} \left[\xi_{kr}^2 \text{Tr}(\mathbf{G} \mathbf{G}^\dagger) \right]_{N_r} + \mathbf{I}_{N_k}, \\ \mathbb{D}_k &\triangleq \widehat{\mathbf{H}}_{kr} \mathbf{G} \text{diag} \left[\xi_{rb}^2 \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) \right]_{N_r} \mathbf{G}^\dagger \widehat{\mathbf{H}}_{kr}^\dagger \\ &\quad + \widehat{\mathbf{H}}_{krb} \mathbf{P}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{krb}^\dagger \\ &\quad + \text{diag} \left[\xi_{kr}^2 \text{Tr}(\mathbf{G} \widehat{\mathbf{H}}_{rb} \mathbf{P}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \mathbf{G}^\dagger) \right]_{N_k} \\ &\quad + \widehat{\mathbf{H}}_{kb} \mathbf{F}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{krb}^\dagger + \widehat{\mathbf{H}}_{krb} \mathbf{P}_k \mathbf{F}_k^\dagger \widehat{\mathbf{H}}_{kb}^\dagger \\ &\quad + \widehat{\mathbf{H}}_{kb} \mathbf{F}_k \mathbf{F}_k^\dagger \widehat{\mathbf{H}}_{kb}^\dagger + \text{diag} \left[\xi_{kb}^2 \text{Tr}(\mathbf{F}_k \mathbf{F}_k^\dagger) \right]_{N_k}.\end{aligned}$$

We obtain

$$\begin{aligned}\widehat{\mathcal{R}}_k &= \mathbb{E} \left[\mathcal{R}_k \left| \widehat{\mathbf{H}}_x, e_x \right. \right], \quad (x \triangleq rb, kb \text{ and } kr) \\ &= \log \det \left[\sum_{k=1}^K \begin{bmatrix} \mathbb{A}_k & \mathbb{B}_k \\ \mathbb{C}_k & \mathbb{D}_k \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{N_k} & \mathbf{0} \\ \mathbf{0} & \mathbb{E}_k \end{bmatrix} \right] - \\ &\quad \log \det \left[\sum_{i=1, i \neq k}^K \begin{bmatrix} \mathbb{A}_i & \mathbb{B}_i \\ \mathbb{C}_i & \mathbb{D}_i \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{N_k} & \mathbf{0} \\ \mathbf{0} & \mathbb{E}_k \end{bmatrix} \right], \quad (10)\end{aligned}$$

where the following property has been used:

$$\mathbb{E} \left[\boldsymbol{\Omega}_x \mathbf{X} \boldsymbol{\Omega}_x^\dagger \right] = \text{diag} \left[\sigma_x^2 \text{Tr}(\mathbf{X}) \right]_{N_k}, \quad (x = rb, kb, \text{ and } kr).$$

According to Eq. (10), problem (8) is a very difficult optimization problem. In order to obtain an efficient solution for the original problem, we first focus on finding the tight lower bound for $\widehat{\mathcal{R}}_k$. We then set up another optimization problem in terms of this lower bound, which enables our solution to proceed. Let

$$\begin{aligned}\overline{\mathbb{A}}_k &\triangleq \text{diag} \left[\xi_{kb}^2 \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) \right]_{N_k}, \\ \overline{\mathbb{B}}_k &\triangleq \text{diag} \left[\xi_{kb}^2 \text{Tr}(\mathbf{P}_k \mathbf{F}_k^\dagger) \right]_{N_k}, \\ \overline{\mathbb{C}}_k &= \overline{\mathbb{B}}_k^\dagger, \\ \overline{\mathbb{D}}_k &\triangleq \widehat{\mathbf{H}}_{kr} \mathbf{G} \text{diag} \left[\xi_{rb}^2 \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) \right]_{N_r} \mathbf{G}^\dagger \widehat{\mathbf{H}}_{kr}^\dagger \\ &\quad + \text{diag} \left[\xi_{kr}^2 \text{Tr}(\mathbf{G} \widehat{\mathbf{H}}_{rb} \mathbf{P}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \mathbf{G}^\dagger) \right]_{N_k} \\ &\quad + \text{diag} \left[\xi_{kb}^2 \text{Tr}(\mathbf{F}_k \mathbf{F}_k^\dagger) \right]_{N_k}, \\ \widehat{\mathbf{H}}_k &\triangleq \begin{bmatrix} \widehat{\mathbf{H}}_{kb} & \mathbf{0} \\ \widehat{\mathbf{H}}_{krb} & \widehat{\mathbf{H}}_{kb} \end{bmatrix}, \\ \widehat{\mathbf{R}}_{\mathbf{I}_k} &\triangleq \sum_{i=1, i \neq k}^K \begin{bmatrix} \mathbb{A}_i & \mathbb{B}_i \\ \mathbb{C}_i & \mathbb{D}_i \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{N_k} & \mathbf{0} \\ \mathbf{0} & \mathbb{E}_k \end{bmatrix}, \\ \mathbf{Z}_k &\triangleq \begin{bmatrix} \overline{\mathbb{A}}_k & \overline{\mathbb{B}}_k \\ \overline{\mathbb{C}}_k & \overline{\mathbb{D}}_k \end{bmatrix}, \\ \widehat{\mathbf{R}}_{\Sigma_k} &\triangleq \widehat{\mathbf{R}}_{\mathbf{I}_k} + \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger + \mathbf{Z}_k.\end{aligned}$$

Then, we have

$$\begin{aligned}\widehat{\mathcal{R}}_k &= \log \det \left| \mathbf{I}_{N_k} + (\widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger + \overline{\mathbb{A}}_k) \widehat{\mathbf{R}}_{\mathbf{I}_k}^{-1} \right| \\ &\stackrel{a}{\geq} \log \det \left| \mathbf{I}_{N_k} + \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \widehat{\mathbf{R}}_{\mathbf{I}_k}^{-1} \right| \\ &\stackrel{b}{\geq} \log \det \left| \mathbf{I}_{N_k} + \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger (\widehat{\mathbf{R}}_{\mathbf{I}_k} + \mathbf{Z}_k)^{-1} \right| \\ &\stackrel{c}{=} \log \det \left| \mathbf{I}_{N_k} + \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger (\widehat{\mathbf{R}}_{\mathbf{I}_k} + \mathbf{Z}_k)^{-1} \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \right| \\ &\stackrel{d}{=} -\log \det \left| \mathbf{I}_{N_k} - \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger (\widehat{\mathbf{R}}_{\mathbf{I}_k} + \mathbf{Z}_k \right. \\ &\quad \left. + \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger)^{-1} \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \right| \\ &= -\log \det \left| \mathbf{I}_{N_k} - \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \widehat{\mathbf{R}}_{\Sigma_k}^{-1} \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k \right| \\ &\triangleq -\log \det |\mathbf{E}_k|, \quad (11)\end{aligned}$$

where (a) is obtained by ignoring $\overline{\mathbb{A}}_k$ caused by errors, (b) is obtained because $\mathbf{Z}_k \geq 0$, (c) is based on the property whereby $\det[\mathbf{I} + \mathbf{A}\mathbf{B}] = \det[\mathbf{I} + \mathbf{B}\mathbf{A}]$, and (d) originates from the Woodbury matrix identity, i.e., $(\mathbf{B} + \mathbf{U}\mathbf{A}\mathbf{V})^{-1} = \mathbf{B}^{-1} - \mathbf{B}^{-1}\mathbf{U}(\mathbf{A}^{-1} + \mathbf{V}\mathbf{B}^{-1}\mathbf{U})^{-1}\mathbf{V}\mathbf{B}^{-1}$ [26]. We have used \mathbf{E}_k to indicate $\mathbf{I}_{N_k} - \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \widehat{\mathbf{R}}_{\Sigma_k}^{-1} \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k$. Based on the above equation, the lower bound is tight, because $\overline{\mathbb{A}}_k$ and \mathbf{Z}_k , which determine the differences produced in the expressions of (a) and (b), respectively, are both small for the case in which $e_x^2 \ll 1$ ($x = kb, rb$ and kr). On the other hand, the equation also reveals that the lower bound is tight, i.e., when $e_x = 0$, we can obtain $R_k = \widehat{\mathcal{R}}_k = -\log \det |\mathbf{E}_k|$.

4.1 Equivalent Problem Tp

Therefore, the optimization problem Tp based on the lower bound (11) can be shown as follows:

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}] = \arg \max_{\{\mathbf{P}, \mathbf{F}, \mathbf{G}\}} \sum_{k=1}^K -\log \det |\mathbf{E}_k|, \quad (12a)$$

[†]This can be proven by convex optimization based on [25].

$$\text{s.t. : (4).} \quad (12b)$$

In order to obtain the solution for the relaxed problem in (12), we need the following theorem.

Theorem 2: Let

$$\mathbf{M}_k \triangleq \mathbf{I} - \mathbf{A}_k \widehat{\mathbf{H}}_k \widetilde{\mathbf{P}}_k - \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \mathbf{A}_k^\dagger + \mathbf{A}_k \widehat{\mathbf{R}}_{\Sigma_k} \mathbf{A}_k^\dagger, \quad (13)$$

where \mathbf{A}_k is a matrix variable. Then, the optimal solution for (14), as formulated below, is also the solution for the relaxed problem given in (12):

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}, \mathbf{A}] = \arg \max_{(\mathbf{P}, \mathbf{F}, \mathbf{G}, \mathbf{A})} \sum_{k=1}^K -\log \det |\mathbf{M}_k|, \quad (14a)$$

$$\text{s.t. : (4),} \quad (14b)$$

where $\mathbf{A} \triangleq \{\mathbf{A}_k\}_{k=1}^K$.

Proof: For any \mathbf{P} , \mathbf{F} , and \mathbf{G} , we can readily find the optimal \mathbf{A}_k that is equal to $\widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \widehat{\mathbf{R}}_{\Sigma_k}^{-1}$. Then, substituting $\mathbf{A}_k = \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \widehat{\mathbf{R}}_{\Sigma_k}^{-1}$ into \mathbf{M}_k , we can obtain $\mathbf{E}_k = \mathbf{M}_k$. \square

The closed-form solution of problem Tp , formulated in (14), is still intractable, because it is also a non-linear problem. Actually, according to the proof for Theorem 2, when \mathbf{P} , \mathbf{F} , and \mathbf{G} are given, we can obtain the optimal \mathbf{A}_k as

$$\mathbf{A}_k = \widetilde{\mathbf{P}}_k^\dagger \widehat{\mathbf{H}}_k^\dagger \widehat{\mathbf{R}}_{\Sigma_k}^{-1}. \quad (15)$$

Second, in order to solve problem Tp formulated in (14), we first set up the following optimization problem [12]:

$$\begin{aligned} & [\mathbf{P}, \mathbf{F}, \mathbf{G}, \mathbf{W}, \mathbf{A}] \\ & = \arg \min_{(\mathbf{P}, \mathbf{F}, \mathbf{G}, \mathbf{W}, \mathbf{A})} \sum_{k=1}^K w_k (\text{Tr}(\mathbf{W}_k \mathbf{M}_k) - \log \det(\mathbf{W}_k)), \\ & \text{s.t. : (4),} \end{aligned} \quad (16)$$

where $\mathbf{A} \triangleq \{\mathbf{A}_k\}_{k=1}^K$, $\mathbf{W} \triangleq \{\mathbf{W}_k\}_{k=1}^K$, and $\mathbf{W}_k \geq \mathbf{0}$ is a weight matrix for the k th MU. Then, we have the following lemma [27].

Lemma 1: Let the weighting factors $w_1 = \dots = w_K = 1$. Then, the optimal solution for problem (16) is also the solution for problem Tp formulated in (14).

Proof: Suppose that \mathbf{P}^{opt} , \mathbf{F}^{opt} , and \mathbf{G}^{opt} are the optimal matrices for the new optimization problem in (16) with $w_1 = \dots = w_K = 1$. Then, we need only confirm that they are also the optimal matrices for the optimization problem Tp in (14). For the given optimal matrices \mathbf{P}^{opt} , \mathbf{F}^{opt} , and \mathbf{G}^{opt} , the optimal \mathbf{A}_k for the k th MU to minimize the problem (16) can be verified to also be the optimal \mathbf{A}_k for the k th MU to minimize the problem (15). It can also be verified that the optimum weight matrix \mathbf{W}_k for the k th MU with the given \mathbf{P}^{opt} , \mathbf{G}^{opt} , \mathbf{F}^{opt} , and \mathbf{A} can be expressed as

$$\mathbf{W}_k = \mathbf{M}_k^{-1}. \quad (17)$$

Therefore, substituting \mathbf{W}_k into (16) with $w_1 = \dots = w_K = 1$, we can obtain the following equations:

$$\begin{aligned} [\mathbf{P}^{\text{opt}}, \mathbf{G}^{\text{opt}}, \mathbf{F}^{\text{opt}}] &= \arg \min - \sum_{k=1}^K \log \det(\mathbf{M}_k^{-1}) \\ &= \arg \max \sum_{k=1}^K -\log \det(\mathbf{M}_k). \end{aligned} \quad (18)$$

Combining (18) and (14), the proof of Lemma 1 is completed [27]. \square

4.2 Equivalent Problem Fp

Optimization problem Fp based on lower bound (11) can be rewritten as follows:

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}] = \arg \min_{(\mathbf{P}, \mathbf{F}, \mathbf{G})} \max_{k \in \mathcal{U}} \log \det |\mathbf{E}_k|, \quad (19a)$$

$$\text{s.t. : (4).} \quad (19b)$$

In order to obtain the solution for this min-max optimization problem, the following theorem is necessary.

Theorem 3: The optimal solution for (20), as formulated below, is also the solution for the min-max problem given in (19):

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}] = \arg \min_{(\mathbf{P}, \mathbf{F}, \mathbf{G})} \sum_{k=1}^K \log \det |\mathbf{E}_k|, \text{ and} \quad (20a)$$

$$\text{satisfy : } \det |\mathbf{E}_1| = \dots = \det |\mathbf{E}_K|, \quad (20b)$$

$$\text{s.t. : (4).} \quad (20c)$$

Proof: Since the power P_b can be distributed on-demand at the BS to find the optimal solution, if \mathbf{P}^{opt} , \mathbf{F}^{opt} , and \mathbf{G}^{opt} are the optimal matrices for problem Fp in (19), they must make $\det |\mathbf{E}_1| = \dots = \det |\mathbf{E}_K|$ the smallest achievable value, because the logarithm function is a monotonically increasing function. Therefore, \mathbf{P}^{opt} , \mathbf{F}^{opt} , and \mathbf{G}^{opt} are also the optimal matrices for problem (20), and vice versa. \square

Although both equivalent optimization problems in (16) and (20) are non-linear and non-convex, we can first solve the equivalent optimization problem in (16) by an alternating optimization method because we can decouple this equivalent problem into three subproblems and solve each of them by the alternating optimization approach [27].

5. Base Station Precoding Matrices Design by the Weighted MMSE Method

In this section, we consider the BS PM \mathbf{P} and \mathbf{F} design by an alternating optimization approach, which has been effectively used for solving optimization problems in signal processing and information theory, because this approach is iterative in nature and simple [27], [28]. Based on Theorem 3, the optimization problem Tp formulated in (16) is the basic optimization problem for problem Fp . Accordingly, we first solve optimization problem Tp formulated in (16). Since this basic optimization problem is also a non-linear and non-convex optimization problem, it is difficult to directly obtain the optimal solution, especially the closed-form solution. However, the problem can be decoupled into

three subproblems and solved using an alternating optimization approach. The solution obtained by this method is sub-optimal.

5.1 BS PM \mathbf{P} Design for the First Phase

For given \mathbf{F} , \mathbf{G} , \mathbf{W} , and \mathbf{A} , the problem in (16) w.r.t. \mathbf{P} can be reformulated as the following weighted minimum problem:

$$\min_{\mathbf{P}} \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)), \quad (21a)$$

$$\text{s.t. : } \text{Tr}(\mathbf{P}^\dagger \mathbf{P}) \leq P_b, \quad (21b)$$

$$\text{Tr}(\mathbf{G} \mathbf{H}_{rb} \mathbf{P} \mathbf{P}^\dagger \mathbf{H}_{rb}^\dagger \mathbf{G}^\dagger + \mathbf{G} \mathbf{G}^\dagger) \leq P_r. \quad (21c)$$

Since an iterative design is considered herein, the relay power constraint in (21c) can be ignored in order to simplify the BS PM \mathbf{P} design in the first phase, which does not affect the final result. Next, we present the following lemma to illustrate that subproblem (21) with respect to PM \mathbf{P} is convex without considering the relay power constraint.

Lemma 2: For given BS PM \mathbf{F} , relay BM \mathbf{G} , \mathbf{A} , and weight matrices \mathbf{W}_k , the subproblem of BS PM \mathbf{P} with fixed relay power to minimize the matrix-weighted sum-MSE in the considered MIMO-RBC formulated in (21) is convex.

Proof: We first prove the objective function is convex. Since

$$\mathcal{J}(\{\mathbf{P}_k\}_{k=1}^K) = \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)) = \sum_{k=1}^K \mathcal{J}_k(\mathbf{P}_k), \quad (22)$$

where

$$\begin{aligned} \mathcal{J}_k(\mathbf{P}_k) = & \text{Tr} \left(\mathbf{W}_k \left(\mathbf{I} - [\mathbf{A}_{k1} \widehat{\mathbf{H}}_{kb} \mathbf{P}_k + \mathbf{A}_{k2} \widehat{\mathbf{H}}_{krb} \mathbf{P}_k \right. \right. \\ & \left. \left. + \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kb} \mathbf{F}_k \right] \right. \\ & \left. - [\mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{kb}^\dagger \mathbf{A}_{k1}^\dagger + \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{krb}^\dagger \mathbf{A}_{k2}^\dagger + \mathbf{F}_k^\dagger \widehat{\mathbf{H}}_{kb}^\dagger \mathbf{A}_{k2}^\dagger] \right. \\ & \left. + \mathbf{A}_{k1} \mathbf{A}_{k1}^\dagger + \mathbf{A}_{k2} \mathbb{E}_k \mathbf{A}_{k2}^\dagger + \mathbf{A}_k \mathbf{Z}_k \mathbf{A}_k^\dagger \right) \\ & + \sum_{i=1}^K \text{Tr} \left(\mathbf{W}_i [\mathbf{A}_{i1} \widehat{\mathbf{H}}_{ib} \mathbf{P}_k + \mathbf{A}_{i2} \widehat{\mathbf{H}}_{irb} \mathbf{P}_k \right. \\ & \left. + \mathbf{A}_{i2} \widehat{\mathbf{H}}_{ib} \mathbf{F}_k] [\mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{ib}^\dagger \mathbf{A}_{i1}^\dagger + \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{irb}^\dagger \mathbf{A}_{i2}^\dagger + \mathbf{F}_k^\dagger \widehat{\mathbf{H}}_{ib}^\dagger \mathbf{A}_{i2}^\dagger] \right), \end{aligned}$$

in which the \mathbf{A}_k is needed to be split into two matrices, i.e., $\mathbf{A}_k = [\mathbf{A}_{k1}, \mathbf{A}_{k2}]$. Hence, we need only verify that $\mathcal{J}_k(\mathbf{P}_k)$ is convex because the sum of two convex functions is also a convex function. According to the differential rule for vectors and the definition of Hessian matrices [25], [29], we can obtain the following matrix:

$$\mathcal{H}(\mathcal{J}_k) = \begin{bmatrix} \mathcal{H}_{\mathbf{p}_k \mathbf{p}_k^*} & \mathcal{H}_{\mathbf{p}_k \mathbf{p}_k} \\ \mathcal{H}_{\mathbf{p}_k^* \mathbf{p}_k} & \mathcal{H}_{\mathbf{p}_k^* \mathbf{p}_k^*} \end{bmatrix} \geq 0, \quad (23)$$

where $\mathcal{H}(\mathcal{J}_k)$ is a Hessian matrix, $\mathbf{p}_k = \text{Vec}(\mathbf{P}_k)$, $\text{Vec}(\cdot)$ signifies the matrix vectorization operator, \mathbf{p}_k^* is the conjugate of \mathbf{p}_k , and $\mathcal{H}_{\mathbf{p}_k \mathbf{p}_k^*}$ is the partial derivative of $\mathbf{p}_k \mathbf{p}_k^*$.

Hence, the objective function in (21a) is convex due to

$\mathcal{H}(\mathcal{J}_k) \geq 0$. Similarly, it can be verified that the feasible region of the BS power constraint in (21b) is also convex. \square

Thus, the Lagrangian function of (21) for \mathbf{P} is given as

$$\mathcal{L}_{\mathbf{P}} = \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)) + \lambda_p (\text{Tr}(\mathbf{P} \mathbf{P}^\dagger) - P_b).$$

Accordingly, the first-order necessary condition of $\mathcal{L}_{\mathbf{P}}$ w.r.t. each \mathbf{P}_k yields the KKT conditions, as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}_{\mathbf{P}}}{\partial \mathbf{P}_k^*} = & -\widetilde{\mathbf{H}}_{kb}^\dagger \mathbf{A}_k^\dagger \mathbf{W}_k + \widetilde{\mathbf{H}}_{kb}^\dagger \mathbf{A}_k^\dagger \mathbf{W}_k \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kb} \mathbf{F}_k \\ & + \left(\sum_{i=1}^K \widetilde{\mathbf{H}}_{ib}^\dagger \mathbf{A}_i^\dagger \mathbf{W}_i \mathbf{A}_i \widetilde{\mathbf{H}}_{ib} + \lambda_p \mathbf{I} \right) \mathbf{P}_k = 0, \end{aligned} \quad (24)$$

$$\lambda_p \left(\sum_{k=1}^K \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) - P_b \right) = 0, \quad (25)$$

$$\sum_{k=1}^K \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) \leq P_b. \quad (26)$$

Based on the above KKT conditions, we can obtain each \mathbf{P}_k as:

$$\begin{aligned} \mathbf{P}_k = & \left(\sum_{i=1}^K (\widetilde{\mathbf{H}}_{ib}^\dagger \mathbf{A}_i^\dagger \mathbf{W}_i \mathbf{A}_i \widetilde{\mathbf{H}}_{ib}) \right. \\ & \left. + \lambda_p \mathbf{I} \right)^{-1} \widetilde{\mathbf{H}}_{kb}^\dagger \mathbf{A}_k^\dagger \mathbf{W}_k (\mathbf{I} - \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kb} \mathbf{F}_k), \end{aligned} \quad (27)$$

where $\widetilde{\mathbf{H}}_{ib} = [\widehat{\mathbf{H}}_{ib}^T, \widehat{\mathbf{H}}_{irb}^T]^T$, and λ_p is the Lagrangian multiplier. We can obtain the value of λ_p through a 1-D search mechanism because $\text{Tr}(\mathbf{P}(\lambda_p) \mathbf{P}(\lambda_p)^\dagger)^\dagger$ decreases monotonically with λ_p .

5.2 BS PM \mathbf{F} Design for the Second Phase

For given \mathbf{P} , \mathbf{G} , \mathbf{W} , and \mathbf{A} , the problem in (8) w.r.t. \mathbf{F} can be reformulated as

$$\min_{\mathbf{F}} \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)), \text{ s.t. : } \text{Tr}(\mathbf{F}^\dagger \mathbf{F}) \leq P_b. \quad (28)$$

Lemma 3: With given BS PM \mathbf{P} , relay BM \mathbf{G} , \mathbf{A} , and weight matrices \mathbf{W}_k , the subproblem of the BS PM \mathbf{F} design to minimize the matrix-weighted sum-MSE in the MIMO-RBC considered here and formulated in (28) is convex.

Proof: The proof is similar to that of Lemma 2 and is omitted here. \square

Thus, the Lagrangian function of (28) for \mathbf{F} is shown by

$$\mathcal{L}_{\mathbf{F}} = \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)) + \lambda_f (\text{Tr}(\mathbf{F} \mathbf{F}^\dagger) - P_b).$$

$^\dagger \mathbf{P}(\lambda_p)$ means \mathbf{P} is the function of λ_p .

Then, the first-order necessary condition of \mathcal{L} w.r.t. each \mathbf{F}_k yields the KKT conditions as follows:

$$\frac{\partial \mathcal{L}_F}{\partial \mathbf{F}_k^*} = -\widehat{\mathbf{H}}_{kb}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k + \widehat{\mathbf{H}}_{kb}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k (\mathbf{A}_{k1} \widehat{\mathbf{H}}_{kb} + \mathbf{A}_{k2} \widehat{\mathbf{H}}_{krb}) \mathbf{P}_k + \left(\sum_{i=1}^K \widehat{\mathbf{H}}_{ib}^\dagger \mathbf{A}_{i2}^\dagger \mathbf{W}_i \mathbf{A}_{i2} \widehat{\mathbf{H}}_{ib} + \lambda_f \mathbf{I} \right) \mathbf{F}_k^\dagger = 0, \quad (29)$$

$$\lambda_f \left(\sum_{k=1}^K \text{Tr}(\mathbf{F}_k \mathbf{F}_k^\dagger) - P_b \right) = 0, \quad (30)$$

$$\sum_{k=1}^K \text{Tr}(\mathbf{F}_k \mathbf{F}_k^\dagger) \leq P_b. \quad (31)$$

Based on the above KKT conditions, we can obtain each \mathbf{F}_k as:

$$\mathbf{F}_k = \left(\sum_{i=1}^K \widehat{\mathbf{H}}_{ib}^\dagger \mathbf{A}_{i2}^\dagger \mathbf{W}_i \mathbf{A}_{i2} \widehat{\mathbf{H}}_{ib} + \lambda_f \mathbf{I} \right)^{-1} \left(\widehat{\mathbf{H}}_{kb}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k (\mathbf{I} - (\mathbf{A}_{k1} \widehat{\mathbf{H}}_{kb} + \mathbf{A}_{k2} \widehat{\mathbf{H}}_{krb}) \mathbf{P}_k) \right), \quad (32)$$

where λ_f is the Lagrangian multiplier, which can be obtained through a 1-D search mechanism, because $\text{Tr}(\mathbf{F}(\lambda_f) \mathbf{F}(\lambda_f)^\dagger)$ monotonically decreases with λ_f .

5.3 RS BM G Design

For given \mathbf{P} , \mathbf{F} , \mathbf{W} , and \mathbf{A} , the problem in (8) w.r.t. \mathbf{G} can be reformulated as follows:

$$\min_{\mathbf{G}} \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)), \quad \text{s.t. : (4b)}. \quad (33)$$

Lemma 4: Given matrices \mathbf{P} , \mathbf{F} , and \mathbf{A} and weight matrices \mathbf{W}_k , the subproblem of the RS BM G design to minimize the matrix-weighted sum-MSE in the MIMO-RBC considered here and formulated in (33) is convex.

Proof: The proof is similar to that of the Lemma 2, so it is omitted here. \square

Therefore, the Lagrangian function of (33) for \mathbf{G} is shown by

$$\mathcal{L}_g = \sum_{k=1}^K (\text{Tr}(\mathbf{W}_k \mathbf{M}_k)) + \lambda_g \left(\text{Tr}(\mathbf{G} \widehat{\mathbf{H}}_{rb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \mathbf{G}^\dagger + e_{rb}^2 \sigma_{rb}^2 \mathbf{G} \text{diag}[P_b]_{N_r} \mathbf{G}^\dagger + \mathbf{G} \mathbf{G}^\dagger) - P_r \right).$$

Accordingly, based on \mathcal{L} 's first-order necessary condition with respect to \mathbf{G} , we can derive the KKT conditions as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}_g}{\partial \mathbf{G}^\dagger} = & \sum_{k=1}^K \left(-\widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{rb}^\dagger + \widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{A}_{k1} \widehat{\mathbf{H}}_{kb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \right) \\ & + \left(\sum_{k=1}^K \widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kr} + \lambda_g \mathbf{I} \right) \mathbf{G} (\mathbf{I} + \widehat{\mathbf{H}}_{rb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger) \\ & + \sum_{k=1}^K \sum_{i=1}^K \left(\widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kb} \mathbf{F}_i \mathbf{P}_i^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \right) = 0. \end{aligned} \quad (34)$$

$$\lambda_g \left(\text{Tr}(\mathbf{G} \widehat{\mathbf{H}}_{rb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \mathbf{G}^\dagger + e_{rb}^2 \sigma_{rb}^2 \mathbf{G} \text{diag}[P_b]_{N_r} \mathbf{G}^\dagger + \mathbf{G} \mathbf{G}^\dagger) - P_r \right) = 0. \quad (35)$$

$$\text{Tr}(\mathbf{G} \widehat{\mathbf{H}}_{rb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger \mathbf{G}^\dagger + e_{rb}^2 \sigma_{rb}^2 \mathbf{G} \text{diag}[P_b]_{N_r} \mathbf{G}^\dagger + \mathbf{G} \mathbf{G}^\dagger) \leq P_r. \quad (36)$$

Based on the above KKT conditions, we can obtain relay matrix \mathbf{G} as:

$$\mathbf{G} = \left(\sum_{k=1}^K \mathcal{D}_k + \lambda_g \mathbf{I} \right)^{-1} \left(\sum_{k=1}^K (\mathcal{O}_k - \mathcal{Q}_k - \sum_{i=1}^K \mathcal{T}_{ki}) \right) (\mathbf{I} + \widehat{\mathbf{H}}_{rb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger)^{-1}, \quad (37)$$

where

$$\begin{aligned} \mathcal{D}_k & \triangleq \widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kr}, \\ \mathcal{O}_k & \triangleq \widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{P}_k^\dagger \widehat{\mathbf{H}}_{rb}^\dagger, \\ \mathcal{Q}_k & \triangleq \widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{A}_{k1} \widehat{\mathbf{H}}_{kb} \mathbf{P} \mathbf{P}^\dagger \widehat{\mathbf{H}}_{rb}^\dagger, \\ \mathcal{T}_{ki} & \triangleq \widehat{\mathbf{H}}_{kr}^\dagger \mathbf{A}_{k2}^\dagger \mathbf{W}_k \mathbf{A}_{k2} \widehat{\mathbf{H}}_{kb} \mathbf{F}_i \mathbf{P}_i^\dagger \widehat{\mathbf{H}}_{rb}^\dagger, \end{aligned}$$

and the Lagrangian multiplier of λ_g is obtained through a 1-D search mechanism.

5.4 General Iterative Joint Design Algorithm for Problem Tp

In the aforementioned discussion, by fixing three of the four matrices (\mathbf{P} , \mathbf{F} , \mathbf{G} and \mathbf{A}/\mathbf{W}), the remaining matrix can be optimized. Therefore, we propose a joint design algorithm to jointly optimize \mathbf{P} , \mathbf{F} , \mathbf{G} , and \mathbf{A}/\mathbf{W} based on alternating optimization. The joint design algorithm is outlined in Algorithm 1 (In our algorithm, feedback of the communication state from the receiver to the base station is necessary. This feedback is only performed one time and is not necessary for each loop of the algorithm. We assume that this information can be transmitted from the receiver to the base station through a dedicated control channel allocated by the base station.).

This algorithm is always convergent to a stationary point. The convergence analysis of this algorithm is also

Algorithm 1 : Joint Design Algorithm for Problem Tp

-
- 1: **Initialize:** $\mathbf{P} = \mathbf{F} = \sqrt{\frac{P_b}{N_b}} \mathbf{I}$, $\mathbf{G} = \rho \mathbf{I}$, and ρ is set under the relay's power constraints, $\mathbf{A}_k/\mathbf{W}_k \leftarrow (15)/(17)$ with $\mathbf{P} = \mathbf{F} = \sqrt{\frac{P_b}{N_b}} \mathbf{I}$ and $\mathbf{G} = \rho \mathbf{I}$, $\forall k \in \mathcal{U}$.
 - 2: **For:**
 - 3: $\mathbf{P}_k \leftarrow (27)$ for fixed \mathbf{F} , \mathbf{G} , \mathbf{W} , and \mathbf{A} , $\forall k \in \mathcal{U}$;
 - 4: $\mathbf{F}_k \leftarrow (32)$ for fixed \mathbf{P} , \mathbf{G} , \mathbf{W} , and \mathbf{A} , $\forall k \in \mathcal{U}$;
 - 5: $\mathbf{G} \leftarrow (37)$ for fixed \mathbf{P} , \mathbf{F} , \mathbf{W} , and \mathbf{A} ;
 - 6: $\mathbf{A}_k \leftarrow (15)$ for fixed \mathbf{P} , \mathbf{F} , and \mathbf{G} , $\forall k \in \mathcal{U}$;
 - 7: $\mathbf{W}_k \leftarrow (17)$ for fixed \mathbf{P} , \mathbf{F} , and \mathbf{G} , $\forall k \in \mathcal{U}$;
 - 8: **End:** The convergence criterion is satisfied.
-

referred to as the block coordinate method [18]. The numerical results of the present study will demonstrate the convergence. In addition, the computational complexity of this algorithm is $O(N_b^3)$ times the number of iterations [18], where N_b is the number of antennas at the BS.

6. Matrices Design for Problem Fp

The constrained forms of the newly formulated problem of (20) will be relaxed to obtain the optimal matrices design for the above problem *Fp* at first. Next, we can use the results based on the problem *Tp* to solve the relaxed problem. Finally, a jointly designed general algorithm (Algorithm 2), which is similar to that based on problem *Tp*, can also be obtained for the source PM and the relay BM based on problem *Fp*.

First, we can rewrite the constrained forms of the (20) in a relaxed manner as

$$[\mathbf{P}, \mathbf{F}, \mathbf{G}] = \arg \min_{(\mathbf{P}, \mathbf{F}, \mathbf{G})} \sum_{k=1}^K (\zeta_k = \log \det |\mathbf{E}_k|), \quad (38a)$$

$$\text{s.t. : } \quad \text{Tr}(\mathbf{P}_k \mathbf{P}_k^\dagger) = p_k, \quad \text{and } \text{Tr}(\mathbf{F}_k \mathbf{F}_k^\dagger) = c_k, \\ k \in \{1, 2, \dots, K\}, \quad (38b)$$

$$\text{Tr}(\mathbf{G} \mathbf{H}_{rb} \mathbf{P} \mathbf{P}^\dagger \mathbf{H}_{rb}^\dagger \mathbf{G}^\dagger + \mathbf{G} \mathbf{G}^\dagger) \leq P_r, \quad (38c)$$

where $\mathbf{p} \triangleq \{p_k\}_{k=1}^K$ and $\mathbf{c} \triangleq \{c_k\}_{k=1}^K$ are constant and are determined in advance under the constraint of the power budget, i.e., $\sum_{k=1}^K p_k = P_b$ and $\sum_{k=1}^K c_k = P_b$.

Remark 1: Due to the fact that $\zeta_k(c'_k) < \zeta_k(c_k)$ for all $c'_k > c_k$ and fixed \mathbf{p} (or $\zeta_k(p'_k) < \zeta_k(p_k)$ for all $p'_k > p_k$ and fixed \mathbf{c}) with the optimal beamforming structure of the problem in (38), we can adjust the predetermined constant vectors \mathbf{p} and \mathbf{c} to meet the equivalent condition of (20b) using an iterative method.

Based on the relaxed problem in (38) and the results of problem *Tp*, we can directly obtain the general iterative algorithm for problem *Fp*, as shown in Algorithm 2.

This algorithm is also always convergent to a stationary point. The convergence analysis of this algorithm is also referred to as the block coordinate method [18]. The numerical results presented herein will demonstrate the convergence. In addition, we can obtain the computational complexity for this algorithm as $O(N_b^3)$ times the number of iterations [18].

Algorithm 2 : Joint Design Algorithm for Problem Fp

-
- 1: **Initialize:** $p_k = c_k = \frac{P_b}{K}$, $\mathbf{P} = \mathbf{F} = \sqrt{\frac{P_b}{N_b}} \mathbf{I}$, $\mathbf{G} = \rho \mathbf{I}$, and ρ is set under the relay's power constraints, $\mathbf{A}_k/\mathbf{W}_k \leftarrow (15)/(17)$ with $\mathbf{P} = \mathbf{F} = \sqrt{\frac{P_b}{N_b}} \mathbf{I}$ and $\mathbf{G} = \rho \mathbf{I}$, $\forall k \in \mathcal{U}$.
 - 2: **For:**
 - 3: $\mathbf{P}_k \leftarrow (27)$ for fixed \mathbf{F} , \mathbf{G} , \mathbf{W} , and \mathbf{A} based on (38), $\forall k \in \mathcal{U}$;
 - 4: $\mathbf{F}_k \leftarrow (32)$ for fixed \mathbf{P} , \mathbf{G} , \mathbf{W} , and \mathbf{A} based on (38), $\forall k \in \mathcal{U}$;
 - 5: $\mathbf{G} \leftarrow (37)$ for fixed \mathbf{P} , \mathbf{F} , \mathbf{W} , and \mathbf{A} ;
 - 6: $\mathbf{A}_k \leftarrow (15)$ for fixed \mathbf{P} , \mathbf{F} , and \mathbf{G} , $\forall k \in \mathcal{U}$;
 - 7: $\mathbf{W}_k \leftarrow (17)$ for fixed \mathbf{P} , \mathbf{F} , and \mathbf{G} , $\forall k \in \mathcal{U}$;
 - 8: Update \mathbf{p} and \mathbf{c} by the following steps: $p_i \leftarrow (p_i - \Delta)$, $c_i \leftarrow (c_i - \Delta)$, and $p_j \leftarrow (p_j + \Delta)$, $c_j \leftarrow (c_j + \Delta)$, where $i = \arg \min_{i=1, \dots, K} \zeta_i$, $j = \arg \max_{j=1, \dots, K} \zeta_j$, and $\Delta = (1 - \frac{K \zeta_i}{\sum_{k=1}^K \zeta_k}) c_i$.
 - 9: **End:** The convergence criterion is satisfied.
-

7. Numerical Results

In this section, numerical results are obtained in order to verify the performance superiority of the proposed design scheme over 2,000 random channel realizations for an MIMO-RBC with coordinated users (WCU). The proposed design scheme is referred to as WMMSE-WCU and is compared to the following schemes:

1. *WMMSE-MRC&MRT-WCU*. This scheme is derived from that presented in [12]. In [12], only \mathbf{P} is designed for the BS. Here, we have extended the scheme to include \mathbf{F} for fair comparison. As a result, the extended scheme from [12] also considers direct users and relay users. However, the BM design for the RS is based on the principles of the maximum ratio combining (MRC) and maximum ratio transmission (MRT).
2. *GCI-I-WCU*. This scheme also considers direct users and relay users. However, the PM design for the BS is based on the GCI scheme [22], and the BM design for the RS is fixed to be scaled by the identity matrix \mathbf{I} .
3. *WMMSE-NCU* [12]. This scheme considers only relay users. However, as far as we know, this scheme is near-optimal for scenarios that do not consider coordinated users (direct users).

All schemes are compared under the same condition of various network parameters with the perfect CSIT and imperfect CSIT to verify the effectiveness of the proposed scheme in both cases. We jointly consider the configuration of both large-scale and small-scale fading wireless channels, and the matrices of the channel follow i.i.d. $\mathcal{CN}(0, \frac{1}{\ell^\tau})$ entries [22], where ℓ and $\tau = 3$ indicate the normalized distance between two nodes and the path loss exponent, respectively. In the network settings, the BS, the RS, and the relay users (RUs) are arranged along a line, with all of the RUs located at the same position. All of the direct users (DUs) are also deployed at the same position, and distance (ℓ_{ib}) between the BS and the DUs is equal to half of the distance ($\ell_{rb} + \ell_{kr}$) between the BS and the RUs, as depicted in Fig. 2 (In our settings, we choose the same position in order to obtain the same distance. For the same distance,

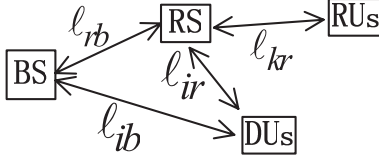


Fig. 2 Distances between the BS and the RS, between the BS and (RUs), and between the BS and DUs, i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$.

different users may experience the same “long-term fading”, such as path loss, for example. For the same distance, different users experience different “short-term fading”. In other words, users at the same position have different multi-path fading. The proposed scheme is based on precoding, which can help us to reduce the interference.). Note that the RUs (or DUs) are deployed at the same position in these simulations only for simulation convenience, and the results can be extended to other deployment setups for RUs (or DUs).

For problem F_p , to provide a fair comparison, all of these schemes are adjusted to be suitable for a max-min achievable rate among all users (For the purposes of performance comparison, we should use the same standard. For each scheme, we have conducted a simulation to maximize the minimum-value of each user, which enables fair comparison.).

7.1 Convergence Property

We first show the convergence properties of the proposed precoding strategy in Figs. 3 and 4 for problems T_p and F_p , respectively. Figure 4 shows that, in the initial rounds, each user has a different minimum rate, and the minimum rates for all of the users eventually converge after 30 iterations (Please note that the convergence property observed in Figs. 3 and 4 is one kind of realization for Algorithms 1 and 2, respectively. Other topologies, such as different channel-gains, can also be used to demonstrate the convergence property of the proposed algorithms. In addition, the input parameters of Algorithm 1 (Algorithm 2) and Fig. 3 (Fig. 4) are accordance with each other.).

7.2 Rate Comparison

Figures 5 through 8 show the results for problem T_p . Figure 5 shows the average sum rate of the network versus the transmit power with perfect CSIT and imperfect CSIT, when the positions of all nodes are fixed. Note that, for fair comparison, the total transmit powers of the base station during two phases are the same for all schemes. Thus the transmitted power in the first phase of WMMSE-NCU is boosted twice as large as that of WMMSE-WCU since WMMSE-WCU allows BS to transmit signal in the second phase. All of the distances are normalized by that between the BS and the RUs. In Fig. 5, e_x is constant. This may not be satisfied under the SNR varying condition as e_x has inverse correlation to SNR if the main occurrence factor of CSIT error is thermal noise on the receiver. To reflect the influence of the

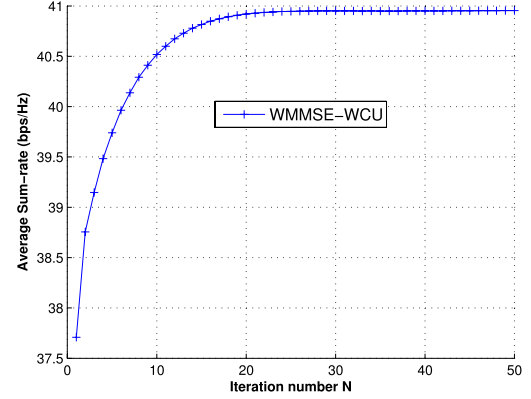


Fig. 3 Convergence properties for one (randomly selected) channel realization with $P_b = P_r$ (signal-to-noise ratio (SNR) = 24 dB, where $N_b = N_r = 4$, $K = 4$, and $e_x = 0.1$). The BS is located at (0, 0), and the RS is located at (0, 0.5). Moreover, all relay users are located at (0, 1.0), and all direct users are located at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$ and $\ell_{kr} = \ell_{ir}$), and $w_k = 1, \forall k \in \mathcal{U}$ (Here, the SNR indicates the value of the transmitter side [21]. In other words, the SNR is equal to P_b or P_r , which has been normalized based on a noise level of 1.).

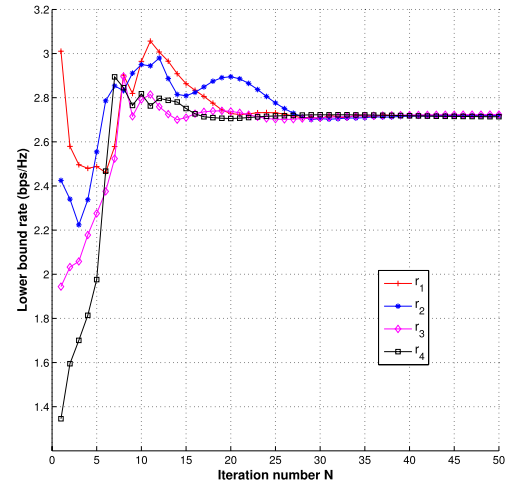


Fig. 4 Convergence properties for one (randomly selected) channel realization with $P_b = P_r$ (SNR = 20 dB) and $r_k = -\log \det |\mathbf{E}_k|$ ($k = 1, 2, 3, 4$), where $N_b = N_r = 4$, $K = 4$, and $e_x = 0.2$. The BS is located at (0, 0), and the RS is located at (0, 0.5). Moreover, all of the relay users are located at (0, 1.0), and all of the direct users are located at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$ and $\ell_{kr} = \ell_{ir}$), and $w_k = 1, \forall k \in \mathcal{U}$.

estimation error, Fig. 6 shows the average sum-rate of the network versus estimation error e_x , when the powers at BS and RS are fixed. The proposed scheme with the designed precoding strategy outperforms the other schemes with different estimation error. Figure 7 shows the average sum rate of the network versus the relay position for the case in which the powers at the BS and the RS are fixed. Figure 8 shows the average sum rate of the network versus K and versus the numbers of antennas of the BS and the RS, where the number of antennas of the BS is the same as that of the RS.

Figures 5 through 8 indicate that the proposed scheme with the designed precoding strategy outperforms the other

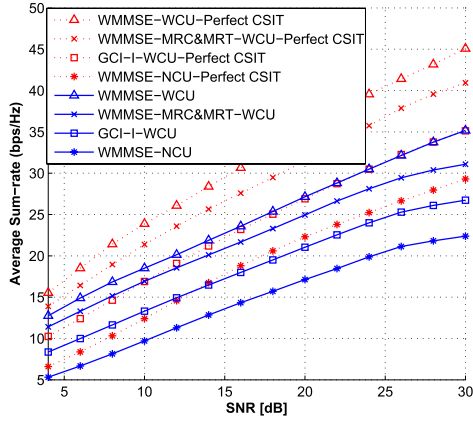


Fig. 5 Average sum rate versus transmit power with $P_b = P_r$ (SNR dB), where $N_b = N_r = 6$, $K = 6$, $e_x = 0.1$, and $e_x = 0$ (i.e., perfect CSIT). The BS is located at $(0, 0)$, and the RS is located at $(0, 0.5)$. Moreover, all relay users are located at $(0, 1.0)$, and all direct users are located at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$ and $\ell_{kr} = \ell_{ir}$), and $w_k = 1, \forall k \in \mathcal{U}$.

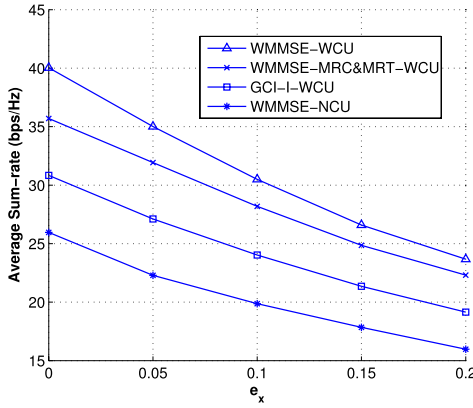


Fig. 6 Average sum-rate versus the estimation error e_x with $P_b = P_r = 24$ (SNR dB), where $N_b = N_r = 6$, $K = 6$. BS is located at $(0, 0)$, RS is at $(0, 0.5)$, all relay-users are at $(0, 1.0)$ and all direct-users are at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$ and $\ell_{kr} = \ell_{ir}$), and $w_k = 1, \forall k \in \mathcal{U}$.

schemes, especially the scheme that does not consider DUs, in the case of either perfect CSIT or imperfect CSIT. This is because schemes that do not consider DUs (i.e., the BS remains silent during the second phase) experience DOF loss, and the maximum number of degrees of freedom is N_b . However, the maximum number of degrees of freedom in the proposed scheme with the designed precoding strategy is $2N_b$. Furthermore, all of the schemes that consider direct users and relay users, i.e., the proposed WMMSE-WCU, WMMSE-MRC&MRT-WCU, and GCI-I-WCU, were applied to the same number of MUs. Figures 5 through 8 indicate that the proposed WMMSE-WCU scheme performs better than WMMSE-MRC&MRT-WCU or GCI-I-WCU. Schemes that consider non-coordinated users (NCU), such as WMMSE-NCU, only support half of mobile-users. Accordingly, based on Eq. (11), we can conclude that the proposed scheme can better deal with multi-user interferences caused by the RUs and DUs.

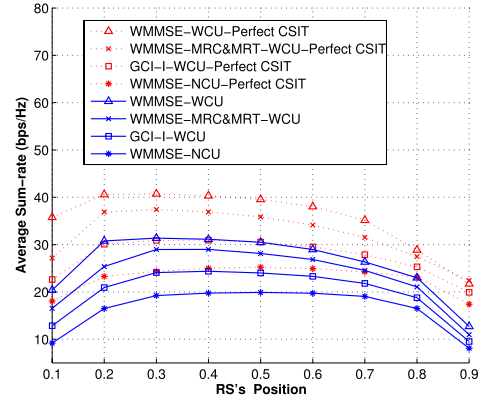


Fig. 7 Average sum rate versus relay position with $P_b = P_r = 24$ (SNR dB), where $N_b = N_r = 6$, $K = 6$, $e_x = 0.1$ and $e_x = 0$ (i.e., perfect CSIT). The BS is located at $(0, 0)$, and the RS is located at $(0, x)$. Moreover, all relay users are located at $(0, 1.0)$, and all direct users are located at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$, where x is in $[0.1, 0.9]$, and $w_k = 1, \forall k \in \mathcal{U}$.

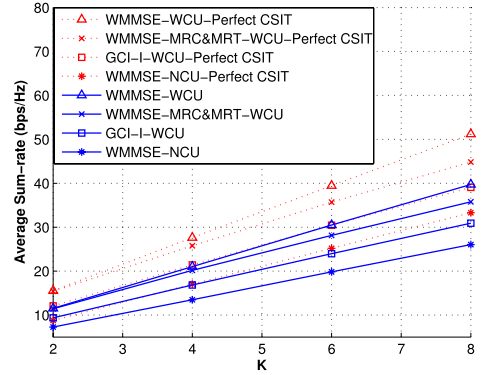


Fig. 8 Average sum rate versus number of BS antennas, where $N_b = N_r = K$, $N_1 = N_2 = \dots = N_K = 2$, $e_x = 0.1$, and $e_x = 0$ (i.e., perfect CSIT), and $P_b = P_r = 24$ (SNR dB). The BS is located at $(0, 0)$, and the RS is located at $(0, 0.5)$. All relay users are located at $(0, 1.0)$, and all direct users are located at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$ and $\ell_{kr} = \ell_{ir}$), and $w_k = 1, \forall k \in \mathcal{U}$.

7.3 Sum Rate and Minimum Rate

The results in Fig. 9 show the achievable sum rates and the achievable minimum rate among all users for different schemes with the perfect CSIT and imperfect CSIT. The sum rate and minimum rate of the proposed scheme are higher than those of the other schemes, for the cases of either perfect CSIT or imperfect CSIT, except for the minimum rate of the WMMSE-NCU scheme, because the number of users in the WMMSE-NCU scheme is half that of other schemes, less inter-user interference is introduced.

8. Conclusion

The half-duplex constraint is a major potential weakness for relay techniques because system bandwidth resources are used inefficiently due to the necessity of dedicated bandwidth for relay retransmissions. In the present study, we

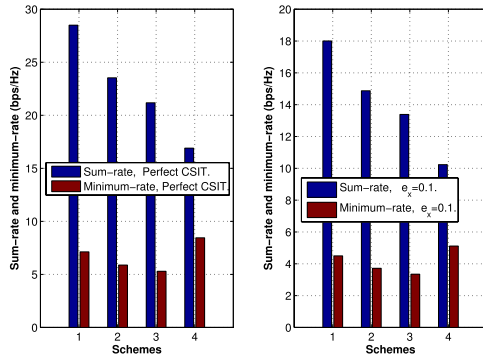


Fig. 9 Sum rate and minimum rate of various schemes for one (randomly selected) channel realization with $e_{rb} = e_{kb} = e_{kr} = 0$ (i.e., perfect CSIT) and $e_{rb} = e_{kb} = e_{kr} = 0.1$, where $P_b = P_r = 20$ (SNR dB), $N_b = N_r = 4$, and $K = 4$ for the 1st, 2nd, and 3rd schemes, and $K = 2$ for the 4th scheme. The BS is located at (0, 0), and the RS is located at (0, 0.5). All relay users are located at (0, 1.0), and all direct users are located at $(0.25, -\sqrt{3}/4)$ on a two-dimensional surface (i.e., $\ell_{ib} = \frac{1}{2}(\ell_{rb} + \ell_{kr})$ and $\ell_{kr} = \ell_{ir}$), and $w_k = 1$, $\forall k \in \mathcal{U}$. (1 \triangleq WMMSE-WCU, 2 \triangleq WMMSE-MRC&MRT-WCU, 3 \triangleq GCI-I-WCU, and 4 \triangleq WMMSE-NCU.)

proposed a general communication scheme with a precoding design strategy for a MIMO-RBC with coordinated users in order to overcome the relay's HDC and achieve full DoF so as to improve the frequency efficiency. Furthermore, since the problems associated with the throughput and fairness criteria for the robust source PM and relay BM designs are non-linear and non-convex, we considered a WMMSE precoding design method by which to jointly design the source PM and relay BM based on an alternating optimization approach. The proposed scheme with the advanced precoding strategy can significantly increase the diversity gain of the MIMO-RBC, and its performance has been verified numerically. As an extension, in the future, we intend to investigate a regenerative, full-duplex scheme that considers coordinated users.

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Appendix

The achievable rate of a MIMO channel is an extension of the mutual information formula for a SISO channel. Specifically, the achievable rate is given in terms of the mutual information between the channel input vector \mathbf{s}_k and output vector \mathbf{y}_k , as follows:

$$\mathcal{R}_k \leq I(\mathbf{s}_k; \mathbf{y}_k) = H(\mathbf{y}_k) - H(\mathbf{y}_k|\mathbf{s}_k) \quad (\text{A} \cdot 1)$$

Since this noise \mathbf{N}_k has fixed entropy independent of the channel input, maximizing mutual information is equivalent to maximizing the entropy in \mathbf{y}_k . The entropy of \mathbf{y}_k is maximized when \mathbf{y}_k is a zero-mean circularly symmetric complex Gaussian (ZMCSCG) random vector [30], but \mathbf{y}_k is only ZMCSCG if the input \mathbf{s} is ZMCSCG. This yields

$$H(\mathbf{y}_k) = \log \det \left| \pi e \left(\sum_{i=1}^K \mathbf{H}_k \tilde{\mathbf{P}}_i \tilde{\mathbf{P}}_i^\dagger \mathbf{H}_k^\dagger + \mathbf{N}_k \mathbf{N}_k^\dagger \right) \right|, \quad (\text{A} \cdot 2)$$

$$H(\mathbf{y}_k|\mathbf{s}_k) = \log \det \left| \pi e \left(\sum_{i=1, i \neq k}^K \mathbf{H}_k \tilde{\mathbf{P}}_i \tilde{\mathbf{P}}_i^\dagger \mathbf{H}_k^\dagger + \mathbf{N}_k \mathbf{N}_k^\dagger \right) \right|, \quad (\text{A} \cdot 3)$$

Then, assuming Gaussian signaling for the source, the k th MU can achieve the following rate during the two phases:

$$\mathcal{R}_k = H(\mathbf{y}_k) - H(\mathbf{y}_k|\mathbf{s}_k) = \log \det \left| \mathbf{I} + \tilde{\mathbf{P}}_k^\dagger \mathbf{H}_k^\dagger \mathbf{R}_{\mathbf{I}_k}^{-1} \mathbf{H}_k \tilde{\mathbf{P}}_k \right|, \quad (\text{A} \cdot 4)$$

where $\mathbf{R}_{\mathbf{I}_k} = \mathbf{N}_k \mathbf{N}_k^\dagger + \sum_{i=1, i \neq k}^K \mathbf{H}_k \tilde{\mathbf{P}}_i \tilde{\mathbf{P}}_i^\dagger \mathbf{H}_k^\dagger$.



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