Resource Allocation in Cooperative OFDMA Systems with Fairness Constraint

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SUMMARY This letter investigates a subchannel and power allocation (SPA) algorithm which maximizes the throughput of a user under the constraints of total transmit power and fair subchannel occupation among relay nodes. The proposed algorithm reduces computational complexity from exponential to linear in the number of subchannels at the expense of a small performance loss.

**Key words:** OFDMA, cooperative communication, fairness

1. Introduction

OFDM and Cooperative communication have been gaining more and more attention, due to their ability to increase the throughput of wireless systems. In order to utilize radio resource efficiently in cooperative OFDMA systems, resource allocation, such as power control and frequency allocation then is worthy of investigation.

Two classes of optimization problems, rate adaptive (RA) and margin adaptive (MA), are investigated in OFDM systems. The objective of RA is to maximize users’ error free capacity with the constraints of total transmit power and co-channel interference (CCI) avoidance [1] and [2]. While the objective of MA is to achieve the minimum overall transmit power given the constraints of users’ data rates or bit error rates [3].

Resource allocation schemes for two-hop cooperative OFDMA systems were proposed in [5] and [6], whose objective is to maximize the system throughput with fair subchannel allocation to the destination and relay nodes, respectively. However, the algorithm in [5] did not consider the resource allocation in the first hop. In this letter, we investigate joint SPA in two hops.

2. System Model and Problem Formulation

Consider an OFDMA-based half-duplex dual-hop cooperation scenario with one source (S)-destination (D) pair and K relay nodes (Rs). It is assumed that Rs can communicate with both S and D while S and D cannot reach each other. In addition, the selected Rs can fully decode the received information, re-encode it, and then forward it to D. Each subchannel in the SR and RD links can only be used by one relay node. The following notations are used to formulate the optimization problem:

- $\mathbb{S}$ is the set of subchannels, $\mathbb{S} = \{1, 2, \ldots, N\}$.
- $\mathbb{K}$ is the set of relay nodes, $\mathbb{K} = \{1, 2, \ldots, K\}$.
- $P_T$ is the total transmission power.
- $n$ and $j$ denote the subchannel in SR and RD links, respectively, $n, j \in \mathbb{N}$.
- $P_{sr}(n)$ and $P_{rd}(j)$ are the power consumed by subchannels $n$ and $j$, respectively.
- $G_{sr}(n)$ is the channel power gain of subchannel $n$, and $G_{rd}(j)$ for $j$, accordingly.
- $P_{sr}(n, j)$ and $P_{rd}(j)$ are the data rate of the link consisting of subchannel $n$ and $j$, which is defined as follows

$$R_k(n, j) = \frac{1}{2} \min \left\{ \log_2 \left( 1 + \frac{P_{sr}(n)G_{sr}(n)}{N_0} \right), \log_2 \left( 1 + \frac{P_{rd}(j)G_{rd}(j)}{N_0} \right) \right\}$$

(1)

Define the equivalent channel gain $G_{rd}(n, j) = \frac{G_{sr}(n)G_{rd}(j)}{G_{sr}(n)G_{rd}(j)}$ and transmit power $P_{sr}(n, j) = P_{sr}(n) + P_{rd}(j)$ [4]. Equation (1) can be written as

$$R_k = \sum_{n=1}^{N} \sum_{j=1}^{N} \frac{\rho(n, j)}{2} \log_2 \left( 1 + \frac{P_{sr}(n, j)G_{sr}(n, j)}{N_0} \right)$$

(2)

The goal of our joint subchannel and power allocation scheme is to find the optimal $\rho(n, j)$ and $P_k(n, j)$ that maximize the transmission rate while maintain the optimum fairness among all Rs. Consequently, the optimization problem can be formulated as

$$\max_{\rho(n, j), P_k(n, j)} \sum_{k=1}^{K} R_k$$

subject to

$$\sum_{n=1}^{N} \sum_{j=1}^{N} P_{sr}(n, j) \leq P_T, P_{sr}(n, j) \geq 0$$

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where \( N \) is set to be the multiples of \( K \). Thus \( N/K \) is always an integer which denotes that equal number of subchannels are assigned to each relay node. This is a mixed nonlinear integer programming problem which has no general solution. To make the problem more tractable, we use the time-sharing technique [3] which relaxes \( \rho_k(n, j) \) to real numbers within the interval \((0, 1)\). Consequently, \( R_k \) is expressed as

\[
R_k = \sum_{n=1}^{N} \sum_{j=1}^{N} \frac{\rho_k(n, j)}{2} \log_2 \left( 1 + \frac{P_{\text{res}}(n, j) G_{\text{res}}(n, j)}{N \rho_k(n, j)} \right).
\]

(4)

Thus the original optimization problem is transformed into a nonlinear programming problem with concave objective function and convex constraints set.

### 3. Proposed Algorithm

Ideally, subchannels and power should be distributed jointly to achieve the optimal solution of (3). However, it is not viable in practice due to the prohibitive computational complexity. Separating the SPA problem into two subproblems is an effective approach to reduce the complexity, because the number of variables in the objective function is reduced by almost half.

**Subchannel Assignment:**
- Set \( P_{\text{res}}(n, j) = 0, \rho_k(n, j) = 0 \) for all \( k, n, j \).
- Find the maximum \( G_{\text{res}}(n) \) and \( G_{\text{res}}(j) \), calculate \( G_{\text{res}}(n, j) \) for each \( k \).
- Find the maximum \( G_{\text{res}}(n, j) \) of all \( R_k \). Denote the selected \( n \) and \( j \) as \( n^* \) and \( j^* \), respectively. Set \( G_{\text{res}}(n^{*}) \) and \( G_{\text{res}}(j^{*}) \) of all relay nodes to zero.
- Count the number of subchannels assigned to each \( R_k \). If \( R_k \) obtains \( N/K \) subchannels, stop allocating subchannels to it. Otherwise, go to the first step.

**Power distribution:**
- Distribute the total power to each subchannel using waterfilling method.

### 4. Numerical Results and Conclusion

In our simulation setup, the distance from \( R_k \) to \( S \) and to \( D \) are normalized to be 1, and \( R_k \) are located in the middle of the \( SD \) path. The path loss exponent is 3. We simulate 1000 sets of 6-ray frequency selective Rayleigh fading channels and the power delay profile is exponentially decaying with \( e^{-2l} \), where \( l \) is the multipath index. The number of subchannels is 64. The graph labels, Opt-Max, Opt-Fair, and Pro denote the sum rate maximization without fairness constraint, the proposed optimal and suboptimal algorithms, respectively.

Figure 1 illustrates the aggregate data rate of several SPA schemes with different \( SNR \)s. We find that the proposed suboptimal algorithm outperforms [5], and reaches asymptotically close to the optimal one with fairness constraint. In Fig. 2, it can be observed that the rate gain of the proposed algorithm to [5] increase slower when the number of relay nodes is very large. This is because the incremental gains from additional relay nodes will diminish when the number of relay nodes is far more than that of users. We also find in both figures that the capacity gap between the Opt-Max and the other ones becomes larger with the increase of \( SNR \) and the number of relay node.

### 5. Conclusion

In this paper, we studied a joint SPA problem in cooperative OFDMA systems and proposed a suboptimal but efficient solution to it. The proposed algorithm guaranteed equal fairness among all relay nodes while incurring little performance loss.

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**References**


