Capacity analysis of multicast transmission schemes in a spectrum-sharing scenario

J. Ji\(^1,2\) W. Chen\(^1\)

\(^1\)Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
\(^2\)Department of Electronic Engineering, Guilin University of Aerospace Technology, Guilin 541004, People’s Republic of China
E-mail: jijianbo@sjtu.edu.cn

Abstract: In this study, the authors consider the asymptotic capacity of wireless multicast and unicast transmission schemes in a spectrum-sharing system. In these schemes, a secondary access point (SAP) utilises the licensed spectrum of an active primary user (PU) to send a common information to multiple secondary users simultaneously, as long as the interference power inflicted on the PU is less than a predefined threshold. At the SAP, interference channel-state information between the SAP and the PU is used to calculate the maximum allowable SAP transmit power to limit the interference. The authors derive the average capacity of these schemes based on extreme value theory. From the derived asymptotic capacities, the insights to the capacity behaviour can be drawn.

1 Introduction

Currently, modern radio spectrum management is faced with the challenge of accommodating a growing number of wireless applications and services on a limited amount of spectrum. Cognitive radio (CR) technology has been proposed as a promising solution to implement efficient reuse of the licensed spectrum by unlicensed devices [1, 2]. In general, CR may be implemented by means of opportunistic spectrum access or spectrum sharing. In an opportunistic system, the secondary users (SUs) can only transmit in ‘white spaces’, that is, the frequency bands or time intervals where the primary users (PUs) are silent [1]; in a spectrum-sharing system, SUs may be allowed to transmit simultaneously with active PUs, as long as the interference power from the SUs to the PUs is less than an acceptable threshold. The maximum allowable interference power is called interference temperature \(Q\) [3, 4], which guarantees the quality of service of the PU regardless of the SU’s spectrum utilisation. Such approaches also have great potential to manage interference in future heterogeneous networks [5] or hierarchical network, for example, femtocell. Clearly, the latter can achieve higher spectral efficiency at the expense of additional side-information at the SUs and the increased signalling overhead. Spectrum-sharing approach has been actively researched [4–7].

Over the last few years, opportunistic scheme has drawn much attention as an effective means of exploiting multiuser diversity inherent in wireless networks. However, most existing work on opportunistic scheme focus on applications that the base station (BS) schedules different data to multiple users. In this paper, we consider a broadcast channel in a single cell system where the same information is sent by the BS to multiple users with different instantaneous channel conditions. Two conventional schemes have been extensively studied for this scenario in the literature; namely, the multicast and the unicast scheme. The transmission of common information to a set of multiple users is referred to as physical layer multicast herein and is opposed to the transmission to a single user, which is called unicast. Typical applications of such a multicast scenario include the streaming of video or audio content to users. Multicast has been considered under the name Multimedia Broadcast/Multicast Service [8] as a feature of Universal Mobile Telecommunications System (UMTS) and under the name Multicast and Broadcast Service as a feature of WiMAX [9]. The Evolved Multimedia/Multicast service in the context of 3GPP/UMTS-long-term evolution includes explicit provisions for point-to-multipoint physical layer multicast [10–13]. In the multicast transmission, the multicast gain resulting from the fact that any information transmitted is decoded by all users, which capacity performance is limited by the worst user in the group at any time. In [11], Gopala and Gamal indicate that the throughput performance of the multicast transmission is limited by the worst channel user, which capacity limit scales as \(\ln(1 + \Theta(1))\) [In this paper, we denote \(f(n) = O(g(n))\) if and only if there are constant \(c\) and \(n_0\) such that \(f(n) \leq cg(n)\) for any \(n > n_0\). \(f(n) = \Omega(g(n))\) if and only if there are constant \(c\) and \(n_0\) such that \(f(n) \geq cg(n)\) for any \(n > n_0\). \(f(n) = \Theta(g(n))\) if and only if there are constants \(c_1\), \(c_2\) and \(n_0\) such that \(c_1g(n) \leq f(n) \leq c_2g(n)\) for any \(n > n_0\). We use \(E(\cdot)\) to denote the expectation. We also use \(\ln\) to denote natural logarithm, \(Pr(\cdot)\) denotes probability, the symbol \(\sim\) denotes...
in distribution], where \( P \) is the transmit power. However, the capacity of multicast transmission scheme in spectrum-sharing systems is different from that of non-spectrum sharing, because interference regulation affects the transmit power of the secondary access point (SAP). To the best of our knowledge, the capacity of multicast transmission scheme in a spectrum-sharing environment has not been investigated. Therefore it is desirable to address the effects on the capacity of this scheme in the spectrum-sharing system where the SAP restrictively utilises a licensed spectrum. In this paper, we derive the asymptotic capacity of the multicast scheme which is shown to approximate as \( Q \).

The unicast scheme serves the best instantaneous user at the highest supportable data rate by exploiting the multiuser gain. Research results indicate that the asymptotic capacity in the unicast scheme scales as the number of users of multiuser gain. Recently, ideas from opportunistic communication were used in spectrum-sharing CRs by selectively activating one or more SUs to maximise the SU throughput while satisfying interference constraints [4, 6]. Some of the related works are summarised as follows. The multiuser diversity gain in cognitive networks is studied in [4, 14–17], by selecting the SU with the highest signal to interference and noise ratio under the PU interference constraints. In [14], Hong and Choi have investigated the multiuser diversity gain in an opportunistic CR system, which has been shown to grow like \( \ln \ln (N) \), where \( N \) is the number of SU. Jamal et al. [17] and Shen et al. [18] found that the SU throughput can be increased by simultaneously activating as many secondary transmitters as possible. However, these asymptotic analysis only propose scaling laws for asymptotic signal-to-noise power ratio (SNR), rather than providing exact results. In this paper, we give a closed-form of the asymptotic capacity under full channel state information (CSI) knowledge at the SAP in the unicast scheme.

2 System and channel model

As shown in Fig. 1 [19], a spectrum-sharing homogeneous network in a single cell system is considered, where an SAP utilises the licensed spectrum of a PU to send a common information to a set of \( N \) SUs. There are two special transmission schemes; in the first scheme the SAP can exploit the wireless multicast advantage so that all SUs in the system can hear the transmission. Therefore the capacity performance of such a scheme is limited by the worst SU in the network at any time. Another case is unicasting scheme that serves the best instantaneous SU at the highest supportable information rate by exploiting the multiuser gain. All users in the network are assumed to be equipped with a single antenna. In the system, any transmission from the SAP to the SUs is allowed provided that the resulting interference power level at the PU is below the predefined threshold, which is called the interference temperature constraint [2, 7, 20]. The interference temperature \( Q \) represents the maximum allowable interference power level at the PU. The channel from the SAP to the PU and the \( j \)th SU are denoted by \( h_{sp} \) and \( h_{j} \), respectively, where \( j \in \{1, \ldots, N\} \), which channel gains are denoted by \( \alpha_{sp} \) and \( \beta_{j} \), respectively. The channel gains \( \alpha_{sp} \) and \( \beta_{j} \) are assumed to be independent and identically distributed (i.i.d.) exponential random variables. Utilising the feedback scheme, the SAP can obtain the interference CSI through periodic sensing of pilot signal from the PU by the hypothesis of channel reciprocity [21]. Then the SAP computes the maximum allowable transmit power depending on \( \alpha_{sp} \) so as to satisfy the interference temperature constraint at the PU. The SAP allocates its peak power for transmission provided that the interference temperature is satisfied with its peak power. Otherwise, it adaptively adjusts its transmit power to the allowable level so that the interference perceived at the PU is maintained as a given interference temperature level \( Q \). Correspondingly, the transmit power of the SAP \( P_{t} \) is

\[
P_{t} = \min \left( P, \frac{Q}{\alpha_{sp}} \right)
\]

where \( P \) represents the peak power of the SAP transmission.

It is worthwhile to mention that, similar to that in [4, 22], the detailed protocol between the primary transmitter and the primary receiver is ignored, and the interference from primary transmitters can be translated into the noise term of the secondary system.

3 Asymptotic capacity of secondary multicast/unicast scheme

In this case, the SAP can obtain the perfect estimation of interference channel gains \( \alpha_{sp} \) by direct feedback from the PU [5]. Accordingly, the received SNR \( \gamma \) at the \( j \)th SU is

\[
\gamma = \frac{P_{t} \beta_{j}}{\alpha_{sp}^{2}} = \begin{cases} 
\frac{P_{t} \beta_{j}}{\alpha_{sp}^{2}}, & \alpha_{sp} \leq \frac{Q}{P} \\
\frac{Q \beta_{j}}{\alpha_{sp}}, & \alpha_{sp} > \frac{Q}{P}
\end{cases}
\]

where the variance of white Gaussian noise is normalised to be one. To simplify mathematical analysis, \( \alpha_{sp} \) and \( \beta_{j} \) are both assumed to be i.i.d. exponential random variables with unit mean. The cumulative density function (cdf) of the received SNR \( \gamma \) at the \( j \)th SU is

\[
F_{\gamma}(\gamma) = \begin{cases} 
\Pr \left[ \alpha_{sp} \leq \frac{Q}{P} (1 - e^{-\gamma/P}) \right] \\
+ \Pr \left[ \alpha_{sp} > \frac{Q}{P} \right] \Pr \left[ \alpha_{sp} > \frac{Q}{P} \right]
\end{cases}
\]

Fig. 1 System model for the SU network coexisting with a PU
Take the distributions of $\alpha_g$ and $\beta_j$ into consideration, we have

\[
F_{\gamma}(\gamma) = (1 - e^{-(Q/P)}) (1 - e^{-(\gamma/P)}) + e^{-(Q/P)} \left( 1 - \frac{Q}{Q + \gamma} e^{-(\gamma/P)} \right)
\]

by which, we derive the probability density function (pdf) $f_{\gamma}(\gamma)$ as

\[
f_{\gamma}(\gamma) = \frac{1}{P} (1 - e^{-(Q/P)}) e^{-(\gamma/P)} + \frac{Q(P + \gamma)}{P(Q + \gamma)^2} e^{-(\gamma/Q/P)}
\]

(5)

### 3.1 SU asymptotic capacity of multicast scheme

In this multicast scheme, the SAP always transmits to all SUs at the information rate decodable by the SU with the worst channel. This scheme maximally exploits the multicast gain by always transmitting to the SUs with the least instantaneous SNR. Therefore the SU average capacity of the scheme is given by [11]

\[
C_M \triangleq NE[\ln(1 + \gamma_{\text{min}})] = N \int_0^\infty \ln(1 + \gamma) f_{\gamma_{\text{min}}}(\gamma) d\gamma
\]

(6)

where $\gamma_{\text{min}} \triangleq \min_{1 \leq i \leq N} \gamma_i$, whose pdf is $f_{\gamma_{\text{min}}}(\gamma) = N f_{\gamma}(\gamma) (1 - F_{\gamma}(\gamma))^{N-1}$. Unfortunately, a closed-form solution of (6) is difficult to obtain with large $N$, and even if we obtain a closed-form, the complicated result hardly provides insights. To obtain some insights into (6), we will use extreme value theory to analyse the asymptotic behaviour of (6) for large $N$.

**Theorem 1:** When $P \gg Q$, the SU asymptotic capacity of the multicast scheme in spectrum-sharing systems approximates as $C_M = \Theta(Q)$ for the large number $N$ of SU.

**Proof:** Using the results from extreme value theory in [23], the distribution of the SU received minimum SNR $\gamma_{\text{min}}$ satisfies $\Pr(\gamma_{\text{min}}/b_N) \leq \gamma \sim W(\gamma)$ as $N \to \infty$, where $W(\gamma)$ is a Weibull-type distribution with cdf $F(\gamma) = 1 - \exp(-\gamma^\alpha)$ for $\gamma > 0$, and $\alpha$ satisfies $\lim_{\gamma \to \infty} (F_{\gamma}(\gamma - 1/\gamma)) = \gamma^\alpha$. Using L'Hospital rule, it is easy to show that $\alpha = 1$. The variable $b_N$ satisfying $F_{\gamma}(b_N) = (1/N)$.

Consider the fact that (4) can be rewritten as

\[
(1 - e^{-(Q/P)}) (1 - e^{-(b_N/P)}) + e^{-(Q/P)} \left( 1 - \frac{Q}{Q + b_N} e^{-(b_N/P)} \right)
\]

\[
= \frac{1}{N}
\]

(7)

For analysis simplicity, we assume $P \gg Q$. Then (7) becomes

\[
\frac{Q}{Q + b_N} e^{-(b_N/P)} = 1 - \frac{1}{N}
\]

(8)

Taking $\ln(\cdot)$ on both sides of (8), we have

\[
\ln(Q) - \ln(Q + b_N) - \frac{b_N}{P} = \ln \left( 1 - \frac{1}{N} \right)
\]

(9)

From (8), $b_N$ approaches to zero as $N$ increases. Thus, the terms $\ln(Q + b_N)$ and $\ln(Q)$ in the left-hand side of (9) become dominant. From the results in [11, 13], we can see $b_N$ scales as $b_N = \Theta(Q/N)$. This means that the distribution of $(N/Q)\gamma_{\text{min}}$ approaches to that of a Weibull random variable $W$ as $N$ increases. In other words, for some constant $l > 0$, we have $\Pr((N/Q)\gamma_{\text{min}} \leq \gamma) \approx \Pr(W \leq \gamma)$. From the result in Theorem 2.1 of [11, 24] and [13], it is concluded that $(N/Q)E[\gamma_{\text{min}}] = \Theta(l/N)$.

Using Jensen's inequality, the average capacity of the multicast scheme can be upper bounded as $C_M = NE[\ln(1 + \gamma_{\text{min}})] \leq N \ln(1 + E[\gamma_{\text{min}}])$. Owing to $\ln(1 + (c/x)) = \Theta(1/x)$ for large $x$ and constant $c$, it is concluded that $C_M = NO(Q/N) = O(Q)$. Now, we lower bound the average capacity as

\[
C_M = N \int_0^\infty (1 + \gamma) dF_{\gamma_{\text{min}}}(\gamma) \geq N \int_{b_N}^\infty (1 + \gamma) dF_{\gamma_{\text{min}}}(\gamma)
\]

\[
= C_M \geq N \ln(1 + b_N)(1 - F_{\gamma_{\text{min}}}(b_N))
\]

where $F_{\gamma_{\text{min}}}(x) = 1 - (1 - F_{\gamma}(x))^N$. Then using the fact that $F_{\gamma}(b_N) = 1/N$, we obtain

\[
F_{\gamma_{\text{min}}}(b_N) = 1 - \left( 1 - \frac{1}{N} \right)^N = 1 - e^{N\ln(1-(1/N))}
\]

\[
= 1 - e^{-\left( 1 + O(1/N) \right)}
\]

Therefore we have $C_M \geq N \ln(1 + b_N) = \Theta(Q)$. We can, therefore obtain $C_M = \Theta(Q)$. Combining this with the upper bound, we have $C_M = \Theta(Q)$.

**Remark 1:** From Theorem 1, the capacity saturation in the spectrum-sharing system is different to that in the non-spectrum-sharing system presented as $\ln(1 + \Theta(1/P))$ in [11]. Here, the SU capacity only depends on the interference temperature $Q$ and has nothing to do with the SAP transmission peak power $P$. This is because that the worst SU average received SNR is only restricted by the $Q$. Furthermore, it increases linearly with $Q$, which is because that the SAP transmission power increases with $Q$ as shown in (1).

### 3.2 SU asymptotic capacity of unicast scheme

Now we consider the unicast scheme, where the SAP chooses the SU that has maximal received SNR from $N$ SUs at each transmission. The same message will have to be repeatedly transmitted until all SUs are served. The SU average capacity of the unicast scheme is given by [11],

\[
C_U \triangleq E[\ln(1 + \gamma_{\text{max}})] = \int_0^\infty \ln(1 + \gamma) f_{\gamma_{\text{max}}}(\gamma) d\gamma
\]

where $\gamma_{\text{max}} \triangleq \max_{1 \leq i \leq N} \gamma_i$, whose pdf is denoted as $f_{\gamma_{\text{max}}}(\gamma) = N f_{\gamma}(\gamma) F_{\gamma}(\gamma)^{N-1}$, based on which, the asymptotic capacity is shown in the following theorem.

**Theorem 2:** For $P \gg Q$, the SU asymptotic capacity of the unicast scheme in the spectrum-sharing system...
where \( N \) is the number of SU, \( x \) is the interference temperature, \( \gamma \) is the average SNR, \( C_U \) is the unicast capacity, \( h_u \) is the fading gain, \( N_0 \) is the noise power, and \( W \) is the bandwidth of the channel.

**Proof:** From the results on extreme order statistics in [23], the distribution of the SU maximum received SNR satisfies

\[
\Pr((\gamma_{\max} - a_N)/(b_N)) \leq y \sim \Pr(W \leq y) \quad \text{as} \ N \to \infty,
\]

where \( a_N, b_N > 0 \) and \( W \) belongs to one of the three standard extreme value distributions: Frechet, Weibull and Gumbel distributions. It is well known that there are only three possible non-degenerate limiting distributions for maximum. The distribution function of \( \gamma, \ F_\gamma(\gamma) \), determines the exact limiting distribution. The following lemma indicates a sufficient condition for a distribution function \( F_\gamma(\gamma) \) belonging to the domain of attraction of the Gumbel distribution.

**Lemma 1:** [25] Let \( \gamma_1, \gamma_2, \ldots, \gamma_N \) be i.i.d. positive random variables with continuous and strictly positive pdf \( f_\gamma(\gamma) \) for \( \gamma > 0 \) and cdf of \( F_\gamma(\gamma) \) be the growth function. If \( \lim_{\gamma \to \infty} (d/d\gamma) f_\gamma(\gamma) = 0 \), then there exist constants \( a_N \) and \( b_N > 0 \) such that \( (\gamma_{\max} - a_N)/(b_N) \) uniformly converges in distribution to a normalised Gumbel random variables as \( N \to \infty \).

Now, we adopt Lemma 1 to obtain the asymptotic behaviour of \( \gamma_{\max} \). For \( P \gg Q \), the cdf and pdf in (4) and (5) can be simplified as \( F_\gamma(\gamma) \sim 1 - (Q(1 + \gamma)/P) \) and \( f_\gamma(\gamma) \sim (P(Q + \gamma)/P^2) e^{-(Q/P)\gamma} \), respectively. Thus, we have the limit of the function

\[
\lim_{\gamma \to \infty} \frac{d}{d\gamma} g_\gamma(\gamma) = \lim_{\gamma \to \infty} \frac{d}{d\gamma} F_\gamma(\gamma) = \frac{d}{d\gamma} \frac{P(Q + \gamma)}{P + Q + \gamma} = 0
\]

Therefore \( F_\gamma(\gamma) \) belongs to the domain of attraction of the Gumbel distribution. According to the limited throughput distribution (LTD) theorem in [25], if the SU’s \( \gamma_{\max} \) belongs to the distribution of the Gumbel, the instant capacity \( \ln(1 + \gamma_{\max}) \) also belongs to the domain of attraction of the Gumbel distribution, and the normalising constants are

\[
a_N = \ln \left(1 + \frac{1 - (1/N)}{1 - (1/N)}\right) \\
b_N = \ln \left(1 + \frac{1 - (1/N)}{1 - (1/N)}\right)
\]

random variable results in moment convergence, and the SU average capacity of unicast scheme \( C_U = E[\ln(1 + \gamma_{\max})] \) has \((C_U - a_N)/(b_N) \sim E_0 \). For large \( N \), the SU capacity of this scheme can be evaluated by using the following expression \( C_U \sim a_N + E_0 b_N [25] \), where \( E_0 = 0.5772 \ldots \) is the Euler constant. In the following, we need to seek the value of \( a_N \) and \( b_N \). From \( 1 - F_\gamma(\gamma) = (1/N) \), we can obtain

\[
\frac{Q}{Q + \gamma} e^{-(Q/P)\gamma} = \frac{1}{N}
\]

**Solution of (13) can be obtained by Matlab software**

\[
\gamma = PW \frac{(Q/P)}{e^{(Q/P)}} - Q \approx PW \frac{(Q/P)}{P} - Q
\]

**Remark 2:** Equation (10) is different to the asymptotic capacity derived in [4], and the simulation in Fig. 4 will demonstrate that (10) much outperforms that in [4], and closely approximates to the ergodic capacity. In addition, from [27], we have \( \gamma \sim \ln z \) for large \( z \). Therefore \( C_U \) can be approximated as \((1 - E_0)\ln(1 + P(\ln NQ - \ln P) - Q) + E_0 \ln(1 + P(\ln NQ - \ln P) - Q)\) for large \( N \). We can see that the average capacity of the unicast scheme in the spectrum-sharing system grows like \( \Theta(\ln(N)) \), which discloses the scaling feature of the asymptotic capacity and reinforces the result in [4, 14–17].

**4 Numerical results**

Here we present simulation results to validate our theoretical claims. These results are obtained through Monte Carlo simulations. In Fig. 2 and Fig. 4, the symbol ‘Approx.’ refers to asymptotic result. Fig. 2 shows the SU average capacity of the unicast scheme against the number \( N \) of SU in spectrum-sharing systems, when all perfect CSI are available at the SAP. The SAP transmission peak power \( P \) is 20 dB. There are three different values of interference temperature \( Q \). When \( Q = 0 \) dB (1 W), the capacity approximates \( \Theta(1) \). When \( Q = -3 \) dB (0.5 W) and \( -5 \) dB (0.3 W), the capacity approximates as \( \Theta(0.5) \) and \( \Theta(0.3) \), respectively. It is also shown in Fig. 2 that the asymptotic results are still accurate even if the number of SUs is small. The capacity only depends on \( Q \), which increases linearly with the interference temperature \( Q \). Fig. 3 shows that the capacity has almost nothing to do with the SAP transmission peak power.

Fig. 4 shows the average capacity of the unicast scheme against the number \( N \) of SU for two different transmission peak power \( P = 20, 30 \) dB and \( Q = 0 \) dB. It is verified that the asymptotic approximation results exactly characterise the performance of the capacity. The performance of our
approximation results is superior to that in [4]. The simulation curves show that the capacity increases with the number of SUs, which grow like \( \ln(W(N)) \).

5 Conclusion

The SU average capacities of two schemes in spectrum sharing have been investigated, based on the asymptotic theory of extreme order statistics. Especially, we have derived the SU average capacity of the multicast/unicast schemes with perfect CSIs available in closed-form. Our closed-form capacities are very tight with the simulation results even with fewer SUs.

6 Acknowledgment

This work was supported by NSF China (no. 60972031), by SEU SKL project (no. W200907), by Huawei Funding YJCB (no. 2009024WL), by National 973 project (no. 2009CB824900), by GuiJiaoKeYan Funding (no. 200103YB149), by Programme for Excellent Talents in Guangxi Higher Education Institutions and Funding (no. X10Z003).

7 References

8 3GPP: ‘Multimedia broadcast/multicast service (MBMS); architecture and functional description,’ TS 23.246, V8.1.0, 2007