

# Asymptotic throughput analysis of random beamforming for multi-antenna cognitive broadcast networks

Jianbo Ji<sup>1,2</sup>, Wen Chen<sup>1</sup>, Shanlin Sun<sup>2</sup>

<sup>1</sup>Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China

<sup>2</sup>Department of Electronic Engineering, Guilin University of Aerospace Technology, Guilin 541004, People's Republic of China

E-mail: jijianbo@sjtu.edu.cn

**Abstract:** Random beamforming (RBF) has received much attention recently in downlink beamforming because of its simple structure, low-feedback load and same throughput scaling as that obtained using dirty paper coding at the transmitter. In this study, the authors analyse the performance of RBF for cognitive downlink multi-antenna system in terms of the throughput of the secondary network. The authors consider a secondary broadcast station with multiple antennas utilising the licensed spectrum of each primary receiver to broadcast information to multiple secondary users (SUs) simultaneously, as long as the interference power inflicted at each primary receiver is less than a predefined threshold. Firstly, they derived the secondary network closed-form throughput approximation of a single-beam RBF by exploiting extreme value order statistics. Then, closed-form approximation for secondary network throughput on multiple-beam RBF is presented. Simulation results verify the validity of the authors approximation results analysis even with fewer SUs.

## 1 Introduction

Currently, radio spectrum management is faced with the challenge of accommodating a growing number of wireless applications and services on a limited amount of spectrum. Cognitive radio (CR) technology has been proposed as a promising solution to implement efficient reuse of the licensed spectrum by unlicensed devices [1, 2]. The key idea behind CR is that an unlicensed/secondary user (SU) is allowed to communicate over the frequency band originally licensed to primary user (PU), as long as the transmission of SU does not generate an unfavourable effect on the operation of PU. In other words, three categories of CR network paradigms have been proposed: overlay, interweave and underlay CR networks [3]. The overlay CR allows concurrent transmissions of PU and SU with the help of sophisticated coding techniques, such as dirty paper coding (DPC). In the interweave CR system, SUs are allowed to access the spectrum licensed to PUs only when the PUs do not occupy the spectrum. This approach comes from the idea of opportunistic communication. In an opportunistic spectrum access system, the SUs can only transmit in 'white spaces', that is, the frequency bands or time intervals where the PUs are silent [1]; in the underlay CR, which is the focus of this paper, the SUs are allowed to utilise PU's spectrum only if the interference caused by the SUs is regulated below a predetermined level. This type of CR is also known as 'spectrum sharing' [3, 4]. In a spectrum sharing system, SUs may be allowed to transmit

simultaneously with active PUs, as long as the interference power from the SUs to the PU is less than an acceptable threshold. The maximum allowable interference power is called interference temperature [3, 4], which guarantees the quality of service (QoS) of the primary user regardless of the SU's spectrum utilisation. Such approaches also have great potential to manage interference in future heterogeneous networks [5] or hierarchical network, for example, femtocell. Clearly, the latter can achieve higher spectral efficiency at the expense of additional side information at the SUs and the increased signalling overhead. Spectrum sharing approach has been actively investigated in [4–7].

For next generation wireless systems, significant research efforts have been devoted to multi-antenna techniques, which can attain substantial capacity gains [8–11]. In order to improve the information rate, a variety of precoding schemes have been proposed for multiuser systems including optimal DPC or transmit beamforming [12–14]. However, these methods require the assumption that the transmitter has perfect channel state information (CSI). In practice, it is difficult to meet this assumption, particularly in frequency division duplexing systems. Therefore designing a system with less feedback is of great interest. The random beamforming (RBF) technology presents an excellent way to meet this requirement by feeding back only the signal-to-noise ratio (SNR) or signal-to-interference-plus-noise ratio (SINR) of the overall channel from each user [15–17]. Furthermore, it achieves the same

throughput scaling law obtained with perfect CSI using DPC at the transmitter.

Based on the proportional fair scheduling strategy, the single-beam RBF is proposed in [15, 17] and the throughput scaling law is approximately analysed. Meanwhile, the conceptual idea of an RBF with multi-beam transmission is first introduced in [15]. The RBF with multiple orthogonal transmit beams is further developed in [16, 17], in which the non-asymptotic upper and lower bounds on system throughput as well as the throughput scaling laws are presented, based on the maximum SINR scheduling strategy. In [18], the multi-beam RBF technology is first introduced in the scenario of CR networks, and the lower and upper bounds on the SU throughput are presented with the assumption of perfect interference CSI available at the SBS.

In this paper, we analyse the throughput performance of cognitive RBF systems in the presence of multiple PUs. An exact throughput expression is difficult to compute, since distribution of the SU maximum received SNR or SINR is quite complicated to deal with for a large number of SU  $N$ . Thus, exploiting the theory of extreme order statistics, we first derive an asymptotic closed-form of the throughput for the single-beam RBF. Then, the multi-beam RBF is compared with the single-beam one in terms of throughput. Unlike the previous works [18] where only the approximate throughput scaling of RBF is concerned [18], we focus on the closed-form of throughput evaluation by exploiting the asymptotic theory of extreme order statistics. Moreover, we characterise the throughput for those beamforming, and provide a closed approximation form.

The rest of this paper is organised as follows. Section 2 provides the asymptotic throughput of cognitive single-beam RBF. In Section 3, we discuss the performance of cognitive multiple-beam RBF on asymptotic throughput. Numerical simulation results are provided to validate the theoretical results in Section 4. Finally, the conclusion is drawn in Section 5.

## 2 Cognitive single-beam RBF

### 2.1 System model and scheduling strategy

As shown in Fig. 1, we consider the downlink of a single-cell cellular CR system where the secondary base station (SBS) with  $M$  antennas transmits packets to  $N$  single antenna SUs. The SBS monitors  $I$  primary receivers. Any transmission from the SBS to the SUs is allowed provided that the resulting interference power level at each active primary receiver is below the predefined threshold, which is called the interference temperature constraint [2, 4, 5]. The interference temperature  $Q$  represents the maximum allowable interference power level at each primary receiver. Independent and identically distributed (i.i.d.) Rayleigh flat fading is assumed in each wireless link and the channel coefficients are all zero-mean circularly symmetric complex Gaussian random variables with unit variance. It is worthwhile to mention that, similar to that in [4, 19], the detailed protocol between the primary transmitters and the primary receivers is ignored, and the interference from primary transmitters can be translated into the noise term of the secondary system. We assume that perfect CSI is available between the SBS and the SUs and even between the primary and secondary systems.

Similar to [17], SBS transmission time is divided into consecutive and equal time slots, and each time slot is less

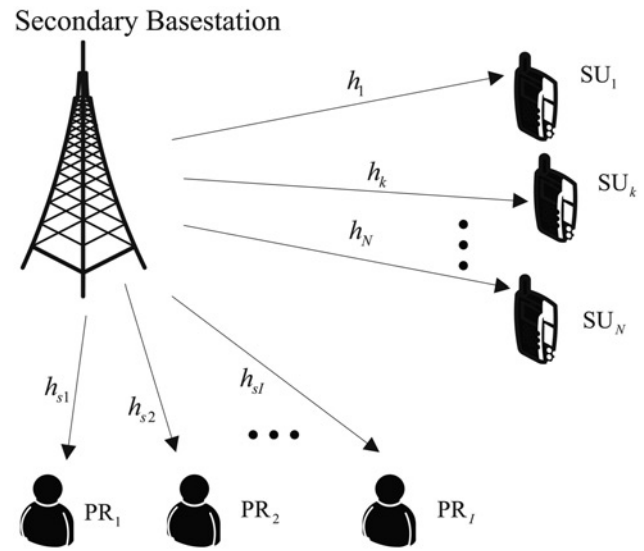


Fig. 1 System model for the secondary network coexisting with  $I$  PRs

than the possible time delay but long enough so that there is a coding strategy available which closely operates to Shannon channel capacity. Moreover, each time slot is divided into a number of mini slots of equal size, and several initial mini slots are used to transmit common pilot symbols so that the SBS can determine which SU should be chosen for data transmission in the rest of the mini slots. In this case, let  $x \in \mathbb{C}^{1 \times 1}$  be the transmit symbol in one time slot. We assume that the SBS satisfies the sum power constraint  $P$ , and at each antenna, uniform power ( $P/M$ ) is allocated across the data streams. Then, the received symbol  $y_k \in \mathbb{C}^{1 \times 1}$  of SU  $k$  is given by

$$y_k = \sqrt{\frac{P}{M}} \mathbf{h}_k^\dagger \mathbf{b}_S x + z_k, \quad k = 1, \dots, N \quad (1)$$

where  $\mathbf{b}_S \in \mathbb{C}^{M \times 1}$  is the beamforming vector,  $E\{\mathbf{b}_S^\dagger \mathbf{b}_S\} = 1$ , the symbol  $\dagger$  refers to conjugate transpose,  $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$  is  $k$ th SU channel and  $|x|^2 = 1$ . In single-beam RBF, the SBS selects the SU with the highest received SNR for each signal transmission. Let the received SNR of  $k$ th SU be  $\gamma_k$ , which is sent back to the SBS by a reliable feedback mechanism. Then, according to (1), the index of a scheduled SU is

$$\hat{k} = \arg \max_{k=1, \dots, N} \gamma_k \quad (2)$$

where  $\gamma_k = (P/M) |\mathbf{h}_k^\dagger \mathbf{b}_S|^2$ , in which  $|x|$  denotes the amplitude of  $x$ . In the considered system, we consider that there are  $I$  active primary receivers and hence the SBS has  $I$  interference channels denoted by  $\mathbf{h}_{si}$ ,  $i \in \{1, \dots, I\}$ . All interference channels are assumed to be known at the SBS through periodic sensing of pilot signal from the primary receivers by the hypothesis of channel reciprocity. To guarantee the interference constraint given by the primary receiver, the SBS transmission power condition is satisfied as

$$\frac{P}{M} |\mathbf{h}_{si}^\dagger \mathbf{b}_S|^2 \leq Q, \quad i = 1, \dots, I \quad (3)$$

In order to maximise the secondary network throughput, the

SBS transmits at maximum allowable power, which can be written according to (3)

$$P = \frac{MQ}{\alpha_{\max}} \quad (4)$$

where  $\alpha_{\max} = \max_{1 \leq i \leq I} |\mathbf{h}_{si}^\dagger \mathbf{b}_s|^2$ . Since  $|\mathbf{h}_{si}^\dagger \mathbf{b}_s|^2 (i = 1, \dots, I)$  follows  $\chi^2(2)$  distribution, the PDF of  $\alpha_{\max}$  is obtained by

$$f_{\alpha_{\max}}(x) = Ie^{-x}(1 - e^{-x})^{I-1} \quad (5)$$

## 2.2 Throughput analysis

In this subsection, we develop the secondary network throughput of a single-beam RBF in conjunction with SU maximum SNR scheduling. The distribution function of the SU maximum received SNR  $\gamma_k$  should first be calculated. We consider the symmetric Rayleigh fading channels, that is, each element of  $\mathbf{h}_k$  is an i.i.d. complex Gaussian random variable with zero mean and unit variance. Therefore  $\gamma_k$  follows  $\chi^2(2)$  distribution for given  $P$ , and the PDF and CDF of  $\gamma_k$  are, respectively,

$$f_{\gamma_k|P}(\gamma) = \frac{M}{P} \exp\left(-\frac{\gamma}{P/M}\right) \quad (6)$$

$$F_{\gamma_k|P}(\gamma) = 1 - \exp\left(-\frac{\gamma}{P/M}\right) \quad (7)$$

Substituting (4) into  $F_{\gamma_k|P}(\gamma)$  in (7), and taking the average over  $\alpha_{\max}$ , we can further obtain the following CDF of  $\gamma_k$

$$\begin{aligned} F_{\gamma_k}(\gamma) &= \int_0^\infty F_{\gamma_k|P}(\gamma) |_{P=(MQ)/(\alpha_{\max})} \times f_{\alpha_{\max}}(x) dx \\ &= \int_0^\infty \left(1 - \exp\left(-\frac{\gamma \alpha_{\max}}{Q}\right)\right) \times f_{\alpha_{\max}}(x) dx \\ &= 1 - \sum_{n=0}^{I-1} \binom{I-1}{n} (-1)^n \frac{IQ}{(n+1)Q + \gamma} \end{aligned} \quad (8)$$

By taking the derivation on (8), we can obtain the PDF of  $\gamma_k$

$$f_{\gamma_k}(\gamma) = \sum_{n=0}^{I-1} \binom{I-1}{n} (-1)^n \frac{IQ}{((n+1)Q + \gamma)^2} \quad (9)$$

According to extreme value theory, the CDF and PDF of  $\gamma_k$  can be respectively given by

$$F_{\gamma_k}(\gamma) = [F_{\gamma_k}(\gamma)]^N, \quad f_{\gamma_k}(\gamma) = N f_{\gamma_k}(\gamma) [F_{\gamma_k}(\gamma)]^{N-1}$$

Therefore we have

$$F_{\gamma_k}(\gamma) = \left[1 - \sum_{n=0}^{I-1} \binom{I-1}{n} (-1)^n \frac{IQ}{(n+1)Q + \gamma}\right]^N \quad (10)$$

$$\begin{aligned} f_{\gamma_k}(\gamma) &= \sum_{n=0}^{I-1} \binom{I-1}{n} (-1)^n \frac{IQ}{((n+1)Q + \gamma)^2} \\ &\times \left[1 - \sum_{n=0}^{I-1} \binom{I-1}{n} (-1)^n \frac{IQ}{(n+1)Q + \gamma}\right]^{N-1} \end{aligned} \quad (11)$$

Then, the average throughput of the secondary network with single-beam RBF is obtained by

$$R = E_{\gamma_k} \{\ln(1 + \gamma_k)\} = \int_0^\infty \ln(1 + \gamma) dF_{\gamma_k}(\gamma) \quad (12)$$

However, deriving an explicit closed form of the above average throughput is difficult, and even if we obtain a closed form, the complicated result hardly provides insights. To obtain some insights into the average throughput of the secondary networks, we asymptotically analyse the average throughput of the secondary network through extreme value theory [20]. We need the following lemma to identify the throughput.

**Lemma 1:** Let  $z_1, \dots, z_N$  be a sequence of i.i.d. positive random variables with continuous PDF  $f_Z(z)$  and CDF of  $F_Z(z)$ . Define the grow function as  $g_Z(z) = ((1 - F_Z(z))/(f_Z(z)))$ . If  $\lim_{z \rightarrow \infty} (d/dz)g_Z(z) = 0$ , then  $F_Z(z)$  belongs to the domain of attraction of Gumbel distribution. In other words, standardised limiting distribution converges to Gumbel distribution as

$$\frac{\max_{1 \leq k \leq N} z_k - b_N}{a_N} \sim e^{-e^{-z}} \quad (13)$$

where

$$a_N = F_Z^{-1}\left(1 - \frac{1}{Ne}\right) - F_Z^{-1}\left(1 - \frac{1}{N}\right) \quad (14)$$

$$b_N = F_Z^{-1}\left(1 - \frac{1}{N}\right) \quad (15)$$

**Theorem 1:** The secondary network average throughput of single-beam RBF follows

$$\begin{aligned} R &= (1 - E_0) \ln\left(1 - \frac{5Q}{2} + \frac{Q}{2} \sqrt{(5 + 4\sqrt{1 + 24N})}\right) \\ &+ E_0 \ln\left(1 - \frac{5Q}{2} + \frac{Q}{2} \sqrt{(5 + 4\sqrt{1 + 24Ne})}\right) \end{aligned} \quad (16)$$

where  $E_0 = 0.5772\dots$  is the Euler constant.

**Proof:** It is known in the context of extreme value theory that the limiting distribution belongs to one of the three domains of attraction: Frechet, Weibull and Gumbel distribution [20]. In order to investigate the performance of multiple

primary receivers while keeping the analytical complexity tractable, we focus on the analysis with the assumption that there are four primary receivers, that is,  $I=4$ . The results can be extended to other values of  $I$  in a straightforward fashion. From (8) and (9), it is straightforward to show  $\lim_{\gamma \rightarrow \infty} (d/d\gamma) \left[ \left( (1 - F_{\gamma_k}(\gamma)) / (f_{\gamma_k}(\gamma)) \right) \right] \neq 0$ . Therefore  $F_{\gamma_k}(\gamma)$  in (8) is not in the domain of attraction of Gumbel distribution. Now, we address the following observation, let a new random variable  $z_k = \ln(1 + \gamma_k)$ . Since the logarithm function monotonically increases, we have

$$F_{z_k}(z) = \Pr(z_k \leq z) = \Pr(\ln(1 + Y_k) \leq z) \\ = \Pr(Y \leq e^z - 1). \quad (17)$$

Substituting  $\gamma = e^z - 1$  into (8) yields

$$F_Z(z) = 1 - \sum_{n=0}^3 \binom{3}{n} (-1)^n \frac{4Q}{(n+1)Q + e^z} \quad (18)$$

Furthermore, it is straightforward to show that the PDF of  $Z$  is

$$f_Z(z) = \sum_{n=0}^3 \binom{3}{n} (-1)^n \frac{4Qe^z}{((n+1)Q + e^z - 1)^2} \quad (19)$$

Substituting (18) and (19) into the growth function  $g_Z(z)$ , we have  $\lim_{N \rightarrow \infty} (d/dz)g_Z(z) = 0$ . According to Lemma 1, the  $F_Z(z)$  belongs to the domain of attraction of Gumbel distribution. From Pickands [21], it has been shown that convergence in distribution for maximum of non-negative random variables results in moment convergence, that is, we have  $\lim_{N \rightarrow \infty} E[(\max_{1 \leq k \leq N} z_k - b_N)/(a_N)]^p = E_0^p$  for any positive real number  $p$ . We consider  $p=1$ . Hence, the average of  $\max_K z_k$  can be evaluated by its normalising constants as follows:  $\lim_{N \rightarrow \infty} E[\max_{1 \leq k \leq N} z_k] = b_N + E_0 a_N$ , that is, the average throughput of the secondary network is obtained by

$$R = b_N + E_0 a_N \quad (20)$$

In the following, we will determine the value of  $a_N$  in (14) and  $b_N$  in (15). The closed forms of  $a_N$  and  $b_N$  are obtained by exploiting 'solve' function in MATLAB, respectively,

$$a_N = \ln \left( 1 - \frac{5Q}{2} + \frac{Q}{2} \sqrt{(5 + 4\sqrt{1 + 24Ne})} \right) \\ - \ln \left( 1 - \frac{5Q}{2} + \frac{Q}{2} \sqrt{(5 + 4\sqrt{1 + 24N})} \right) \quad (21)$$

$$b_N = \ln \left( 1 - \frac{5Q}{2} + \frac{Q}{2} \sqrt{(5 + 4\sqrt{1 + 24N})} \right) \quad (22)$$

Substituting (21) and (22) into (20), Theorem 1 is proved.  $\square$

From (21) and (22), we can see that the secondary network throughput of single-beam RBF is independent of  $M$ , which is different to that of single-beam RBF in [17], that is, multiple antenna technology does not provide any benefit to the throughput of secondary network in single-beam RBF scheme. This is because the distribution in (8) and (9) of single-beam RBF based on SNR criterion has nothing to do

with  $M$ . When the number of SU  $N$  is very large,  $a_N$ ,  $b_N$  scale as  $\ln N^{(1/4)}$ , respectively. Therefore the secondary network throughput of single-beam RBF scales as  $\ln N^{(1/4)}$ . We can extend the above results to the other special  $I$ . In the case that  $I=1, 2$  and  $3$ , the  $a_N$ ,  $b_N$  scale as  $\ln N$ ,  $\ln N^{(1/2)}$  and  $\ln N^{(1/3)}$ , respectively, from which, we can conclude that the secondary network throughput of single-beam RBF scales as  $\ln N^{(1/I)}$  in term on the number of primary receivers  $I$ .

### 3 Cognitive multi-beam RBF

In the subsection, we consider a cognitive multi-beam RBF technique where the SBS opportunistically transmits to  $M$  SUs simultaneously [16]. Specifically, the SBS constructs  $M$  orthogonal beams, denoted by  $\{\mathbf{b}_k\}_{k=1}^M$ , and assigns each beam to an SU. Then, the SBS broadcasts to  $M$  selected SUs. Considering an equal power allocation among  $M$  SUs, the transmitted signal from the SBS is given by

$$\mathbf{x}_s = \sqrt{\frac{P}{M}} \sum_{k=1}^M \mathbf{b}_k x_k \quad (23)$$

where  $\mathbf{b}_k$  is the  $k$ th beamforming vector with dimension  $M \times 1$ , and  $x_k \in \mathbb{C}^{1 \times 1}$  is the signal transmitted along the  $k$ th beam assuming that the  $k$ th beam is assigned to the  $i$ th SU. From (23), the received signal at  $i$ th SU is given by

$$y_i = \sqrt{\frac{P}{M}} \mathbf{h}_i^\dagger \mathbf{b}_k x_k + \sum_{j \neq k} \sqrt{\frac{P}{M}} \mathbf{h}_i^\dagger \mathbf{b}_j x_j + z_i \quad (24)$$

where  $\mathbf{h}_i^\dagger$  is the  $1 \times M$  vector of channel coefficient from the SBS to the  $i$ th SU. The received SINR at the  $i$ th SU with respect to beam  $k$  is

$$\text{SINR}_{i,k} = \frac{(P/M) |\mathbf{h}_i^\dagger \mathbf{b}_k|^2}{1 + (P/M) \sum_{j \neq k} |\mathbf{h}_i^\dagger \mathbf{b}_j|^2} \quad (25) \\ = \frac{|\mathbf{h}_i^\dagger \mathbf{b}_k|^2}{(M/P) + \sum_{j \neq k} |\mathbf{h}_i^\dagger \mathbf{b}_j|^2} \triangleq \frac{t}{(M/P) + y}$$

where  $t = |\mathbf{h}_i^\dagger \mathbf{b}_k|^2$  and  $y = \sum_{j \neq k} |\mathbf{h}_i^\dagger \mathbf{b}_j|^2$ . Note that  $t$  is i.i.d. with  $x^2(2)$  distribution. Since  $\mathbf{b}_1, \dots, \mathbf{b}_M$  are orthogonal [16],  $y$  follows  $x^2(2M-2)$  distribution. The random beam technique assigns each beam to the SU which results in the highest SINR. Since the probability of more than two beams assigned to the same SU is negligible [16], we have

$$R \simeq E \left[ \sum_{k=1}^M \ln \left( 1 + \max_{1 \leq i \leq N} \text{SINR}_{i,k} \right) \right] \quad (26) \\ = ME \left[ \ln \left( 1 + \max_{1 \leq i \leq N} \text{SINR}_{i,k} \right) \right]$$

Since the SINR is symmetric across all beams, the subscript  $k$  will be omitted in the following analysis. In order to evaluate the throughput, we have to obtain the distribution of variable

SINR<sub>i</sub>. The CDF of SINR<sub>i</sub> can be written as for given  $P$

$$\begin{aligned} F_{\gamma_k|P}(\gamma) &= \Pr(\text{SINR}_i < \gamma) \\ &= \int_0^\infty \Pr(t < \gamma(y + M/P)) f_Y(y) dy \\ &= \int_0^\infty \left(1 - e^{-\gamma(y + M/P)}\right) \frac{y^{M-2} e^{-y}}{(M-2)!} dy = 1 - \frac{e^{-M\gamma/P}}{(1+\gamma)^{M-1}} \end{aligned} \quad (27)$$

In order to protect each primary receiver from harmful interference, the transmit power of the SBS should satisfy the interference power requirement of each primary receiver, which follows

$$\frac{P}{M} \left| \mathbf{h}_{si}^\dagger \sum_{k=1}^M \mathbf{b}_k x_k \right|^2 \leq Q, \quad i = 1, \dots, I \quad (28)$$

Similar to that in Section 2.2 of Section 2, maximum transmit power of the SBS is

$$P = \frac{MQ}{\alpha_{\max}} \quad (29)$$

where  $\alpha_{\max} = \max_{1 \leq i \leq I} |\mathbf{h}_{si}^\dagger \sum_{k=1}^M \mathbf{b}_k x_k|^2$ . Denote  $\alpha_{si} = |\mathbf{h}_{si}^\dagger \sum_{k=1}^M \mathbf{b}_k x_k|^2$ . Since  $\mathbf{b}_1, \dots, \mathbf{b}_M$  are orthogonal to each other, and  $|x_k|^2 = 1$ , we can obtain that  $\alpha_{si} = \sum_{k=1}^M |\mathbf{h}_{si}^\dagger \mathbf{b}_k|^2$ . For each  $k$ , the variable  $|\mathbf{h}_{si}^\dagger \mathbf{b}_k|^2$  follows  $x^2(2)$  distribution. Then, we have  $\alpha_{si}$  follows  $x^2(2M)$  distribution. Therefore the PDF of  $\alpha_{\max}$  is obtained by

$$f_{\alpha_{\max}}(x) = I \left( 1 - e^{-x} \sum_{j=0}^{M-1} \frac{x^j}{j!} \right)^{I-1} \frac{x^{M-1} e^{-x}}{(M-1)!} \quad (30)$$

In order to investigate the performance of multiple primary receivers while keeping the analytical complexity tractable, we focus on the following analysis with the assumption that there are two primary receivers, that is,  $I=2$ . The results can be extended to other values of  $I$  in a straightforward fashion. When  $I=2$ , (30) is simplified as

$$f_{\gamma_{\max}}(x) = \frac{2x^{M-1} e^{-x}}{(M-1)!} - 2e^{-2x} \sum_{j=0}^M \frac{x^{M+j-1}}{j!(M-1)!} \quad (31)$$

Substituting (31) into  $F_{\gamma_k|P}(\gamma)$  in (27) and taking average over  $\alpha_{\max}$ , we can further obtain the following PDF of  $\gamma_k$

$$\begin{aligned} F_{\gamma_k}(\gamma) &= 1 - \frac{2Q^M}{(Q+\gamma)^M(1+\gamma)^{M-1}} \\ &\quad + \frac{2}{(1+\gamma)^{M-1}} \sum_{j=0}^M \left( \frac{Q}{\gamma+2Q} \right)^{M+j} \frac{(M+j-1)!}{j!(M-1)!} \end{aligned} \quad (32)$$

After derivation of (32), one obtains the corresponding PDF

$$f_{\gamma_k}(x)$$

$$\begin{aligned} f_{\gamma_k}(\gamma) &= \frac{2Q^M}{(Q+\gamma)^M(1+\gamma)^{M-1}} \left( \frac{M}{Q+\gamma} + \frac{M-1}{1+\gamma} \right) \\ &\quad + \frac{2}{(1+\gamma)^M} \sum_{j=0}^M \left( \frac{Q}{\gamma+2Q} \right)^{M+j} \frac{(M+j-1)!}{j!(M-1)!} \\ &\quad \times \left( 1 - M - \frac{(1+\gamma)(M+j)}{\gamma+2Q} \right) \end{aligned} \quad (33)$$

Following the steps in Section 2.2 of Section 2 let  $z_k = \ln(1+\gamma_k)$ , then, the CDF of  $z_k$  is

$$\begin{aligned} F_Z(z) &= \Pr(Z \leq z) = \Pr(\ln(1+\gamma) \leq z) \\ &= \Pr(\gamma \leq e^z - 1) \end{aligned} \quad (34)$$

After some operations, the CDF of  $z_k$  is

$$\begin{aligned} F_Z(z) &= 1 - \frac{2Q^M}{(e^z + Q - 1)^M e^{(M-1)z}} \\ &\quad + \frac{2}{e^{(M-1)z}} \sum_{j=0}^M \left( \frac{Q}{e^z + 2Q - 1} \right)^{M+j} \frac{(M+j-1)!}{j!(M-1)!} \end{aligned} \quad (35)$$

After derivation of (35), the corresponding PDF  $f_Z(z)$  can be obtained as

$$\begin{aligned} f_Z(z) &= \frac{2Q^M}{(e^z + Q - 1)^M e^{(M-2)z}} \left( \frac{M}{e^z + Q - 1} + \frac{M-1}{e^z} \right) \\ &\quad + \frac{2}{e^{(M-1)z}} \sum_{j=0}^M \left( \frac{Q}{e^z + 2Q - 1} \right)^{M+j} \frac{(M+j-1)!}{j!(M-1)!} \\ &\quad \times \left( 1 - M - \frac{e^z(M+j)}{e^z + 2Q - 1} \right) \end{aligned} \quad (36)$$

From (35) and (36), we have  $\lim_{z \rightarrow \infty} (d/dz)((1 - F_Z(z))/(f_Z(z))) = 0 + O(e^{-z})$ . Therefore  $F_Z(z)$  also belongs to the domain of attraction of Gumbel distribution. Similar to the analysis in Section 2, the average throughput of the secondary network with multi-beam RBF is

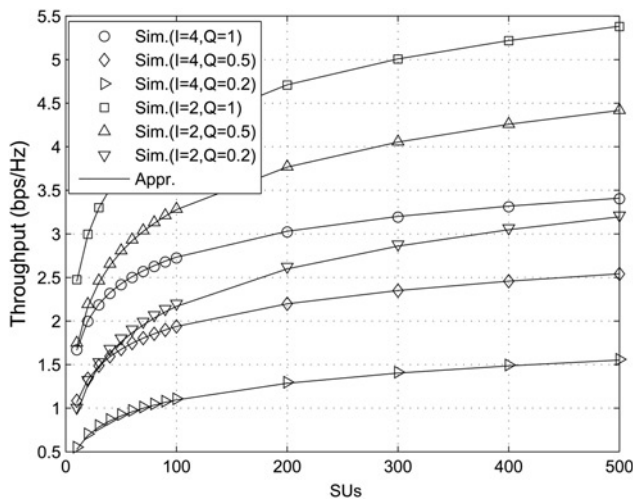
$$R = M(b_N + E_0 a_N) \quad (37)$$

where  $a_N, b_N$  are the solutions of  $a_N = F_Z^{-1}(1 - (1/Ne)) - F_Z^{-1}(1 - (1/N))$  and  $b_N = F_Z^{-1}(1 - (1/N))$ , respectively. The closed forms of  $a_N$  and  $b_N$  cannot be obtained, and their values can be calculated by providing initial values to the non-linear equation solver, such as  $f$  solve function in MATLAB.

## 4 Numerical results

In this section, we compare our throughput analysis for cognitive multiple-antenna input single-antenna output (MISO) downlink RBF systems with numerical simulations to confirm the validity of our analysis. These results are



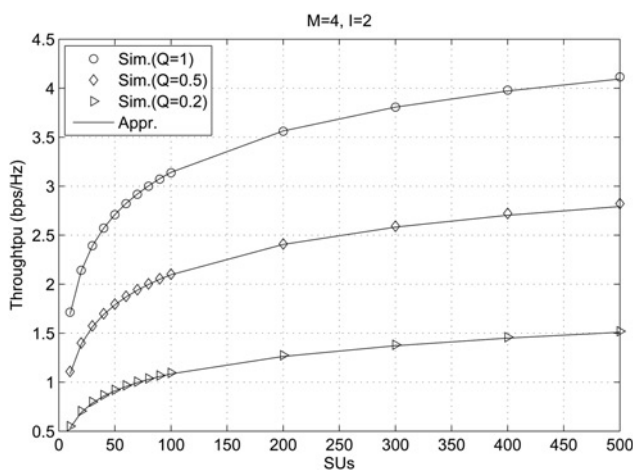


**Fig. 2** Comparison of approximation and simulation results of secondary network throughput for single-beam RBF

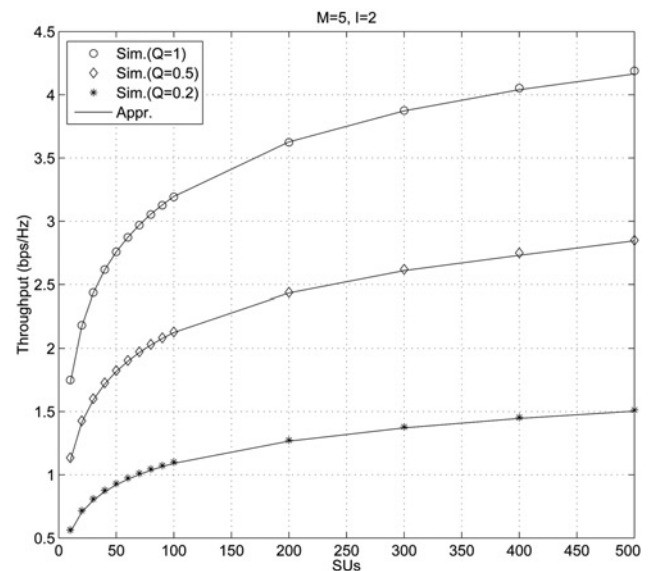
obtained through Monte Carlo simulation results over Rayleigh fading channels.

In Fig. 2, we compare our closed approximation and simulation with the Monte Carlo method in cognitive single-beam RBF, with  $I=4, 2$  and the number  $N$  of SUs ranged over  $[10, 500]$ . From Fig. 2, we observe that the approximated result in (16) agrees perfectly with the simulation result in (12) for various number of SUs, even with a small number of SUs  $N$ . It is verified that our asymptotic approximation result exactly characterises the performance of the throughput. This is because the SU maximum instant throughput belongs to the domain of attraction of Gumbel distribution, and the convergence in distribution for maximum of non-negative random variables results in moment convergence. The simulation curves also show that the throughput increases with the number of SUs, which is termed as multi-user diversity, and increases with the interference temperature  $Q$ . These results also demonstrate that the throughput decreases as the number of primary receivers increases, since more constraints reduce the available degree of freedom at the SBS.

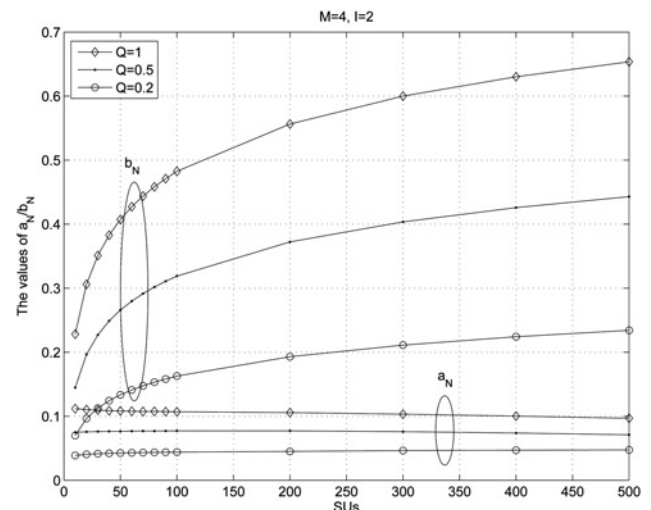
In Figs. 3 and 4, we compare our throughput approximation (37) and Monte Carlo simulation results in multi-beam RBF scheme, with  $M=4, 5$ ,  $I=2$  and the numbers  $N$  of SUs



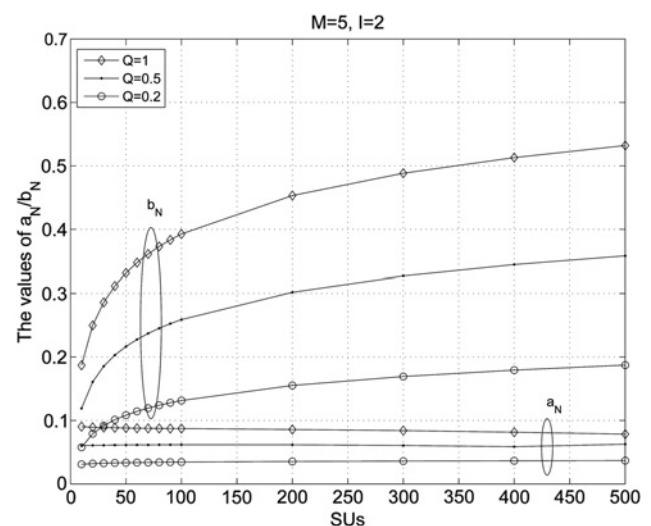
**Fig. 3** Comparison of approximation and simulation results of secondary network throughput for multiple-beam RBF  $M=4$



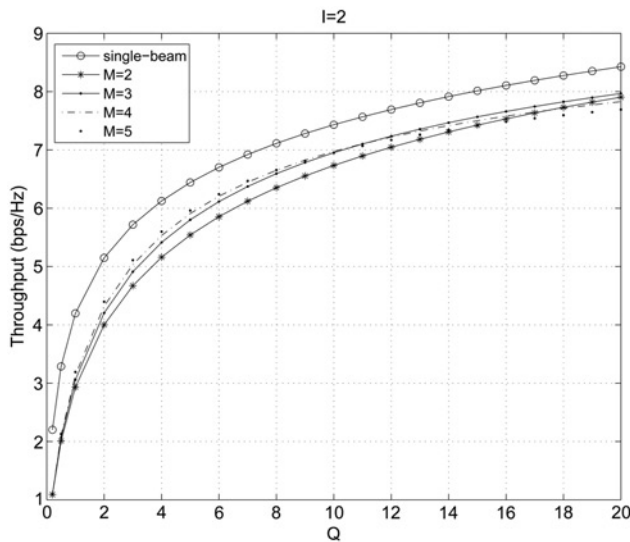
**Fig. 4** Comparison of approximation and simulation results of secondary network throughput for multiple-beam RBF  $M=5$



**Fig. 5** Values of  $a_N$  and  $b_N$  by exploiting numerical methods



**Fig. 6** Values of  $a_N$  and  $b_N$  by exploiting numerical methods



**Fig. 7** Cognitive multi-beam RBF against single-beam RBF in system throughput

ranged over [10, 500]. In (37), because of the complicated forms, we have not obtained the  $a_N$  and  $b_N$  closed solutions. The values of  $a_N$  and  $b_N$  in Figs. 5 and 6 are obtained by exploiting numerical methods, such as  $f$  solve function in MATLAB. After several iterations, we can obtain their values by setting both of their initial values as 0. From Figs. 3 and 4, we also observe that our approximation result in (37) seems quite well matched to the simulations. It is testified that the method of  $a_N$  and  $b_N$  by  $f$  solve mean is correct. Unlike the single-beam RBF, we find that the system throughput of multi-beam RBF improves by increasing the transmit beams  $M$ , by comparing Figs. 3 and 4. For example, when  $Q=1$ , and  $N=50$ , the throughput grows from 2.7097 to 2.7581 bps/Hz, whereas transmit beam  $M$  increases from 4 to 5. This is because of the spatial multiplexing gain introduced by multiple simultaneously transmitted beams. Similar to the single-beam RBF, the simulation curves also show that the capacity increases with the number of SUs, and increases with the interference temperature  $Q$ .

In Fig. 7, we compare cognitive multi-beam RBF ( $M=2, 3, 4$  and 5) and single-beam RBF in system throughput in terms of the number of SUs  $N=100$  and primary receivers  $I=2$ , and ranges over [0.2, 20]. First, we can observe that the single-beam RBF outperforms the multi-beam RBF on system throughput. This is because the interference power requirement of each PU in multi-beam RBF is much stringent than the one in the single-beam RBF, which is verified by the PDF in (33) and (5). More stringent requirement will result in decreasing the SBS transmit power in multi-beam RBF.

## 5 Conclusion

The throughputs of single-beam and multi-beam RBF in cognitive MISO systems are intensively investigated, based on the asymptotic theory of extreme order statistics. Our closed-form throughput approximations are very tight with the simulation results even with fewer SUs. Therefore they can be used to effectively evaluate the system throughput of RBF. In future work, we will consider a more practical

scenario. The interference from PU can be taken into consideration, which is an interesting topic for future work.

## 6 Acknowledgment

This work is supported by the National 973 project #2012CB316106 and #2009CB824900, by NSFC #60972031 and #61161130529, by National Key Laboratory Project #ISN11-01, by Scientific Research of Guangxi Educational Committee Project #200103YB149, by Talents Project of Guangxi Educational Committee and Guilin University of Aerospace Technology Research Project #X10Z003.

## 7 References

- 1 Mitola, J.: 'Cognitive radio: an integrated agent architecture for software defined radio'. PhD dissertation, KTH, Stockholm, Sweden, December 2000
- 2 Haykin, S.: 'Cognitive radio: brain-empowered wireless communications', *IEEE J. Sel. Areas Commun.*, 2005, **23**, (2), pp. 201–220
- 3 Srinivasa, S., Jafar, S.A.: 'The throughput potential of cognitive radio: a theoretical perspective', *IEEE Commun. Mag.*, 2007, **45**, (5), pp. 73–79
- 4 Ban, T.W., Choi, W., Jung, B.C., Sung, D.K.: 'Multi-user diversity in a spectrum sharing system', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (1), pp. 102–106
- 5 Ghasemi, A., Sousa, E.S.: 'Fundamental limits of spectrum-sharing in fading environments', *IEEE Trans. Wirel. Commun.*, 2007, **6**, (2), pp. 649–658
- 6 Viswanath, P., Tse, D.N.C., Laroia, R.L.: 'Opportunistic beamforming using dumb antennas', *IEEE Trans. Inf. Theory*, 2002, **48**, (6), pp. 1277–1294
- 7 Website of FCC, available at <http://www.fcc.gov/sptf/resports.html>
- 8 Telatar, I.E.: 'Capacity of multi-antenna Gaussian channels', *Eur. Trans. Telecommun.*, 1999, **10**, pp. 585–595
- 9 Lee, I., Chan, A., Sundberg, C.-E.W.: 'Space-time bit-interleaved coded modulation for OFDM systems', *IEEE Trans. Signal Process.*, 2004, **52**, (6), pp. 820–825
- 10 Lee, H., Lee, B., Lee, I.: 'Iterative detection and decoding with an improved V-BLAST for MIMO-OFDM systems', *IEEE J. Sel. Areas Commun.*, 2006, **24**, pp. 504–513
- 11 Lee, H., Lee, I.: 'New approach for error compensation in coded V-BLAST OFDM systems', *IEEE Trans. Commun.*, 2007, **55**, pp. 345–355
- 12 Caire, G., Shamai, S.: 'On the achievable throughput of a multi-antenna Gaussian broadcast channel', *IEEE Trans. Inf. Theory*, 2003, **49**, pp. 1691–1706
- 13 Yoo, T., Goldsmith, A.: 'On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming', *Proc. IEEE J. Sel. Areas Commun.*, 2006, **24**, (3), pp. 528–541
- 14 Sung, H., Lee, S.-R., Lee, I.: 'Generalized channel inversion methods for multiuser MIMO systems', *IEEE Trans. Commun.*, 2009, **57**, (11), pp. 3489–3499
- 15 Viswanath, P., Tse, D.N.C., Laroia, R.: 'Opportunistic beamforming using dumb antennas', *IEEE Trans. Inf. Theory*, 2002, **48**, (6), pp. 1277–1294
- 16 Sharif, M., Hassibi, B.: 'On the capacity of MIMO broadcast channels with partial side information', *IEEE Trans. Inf. Theory*, 2005, **51**, (2), pp. 506–522
- 17 Xia, M., Zhou, Y., Ha, J., Chung, H.K.: 'Opportunistic beamforming communication with throughput analysis using asymptotic approach', *IEEE Trans. Veh. Technol.*, 2009, **58**, (5), pp. 506–522
- 18 Li, Y., Nosratinia, A.: 'Capacity limits of multiuser multiantenna cognitive networks', *IEEE Trans. Inf. Theory*, 2012, **58**, (7), pp. 4493–4508, available at <http://www.arxiv.org/pdf/1008.3641>
- 19 Ding, H., Ge, J., da Costa, D.B., Jiang, Z.: 'Asymptotic analysis of cooperative diversity systems with relay selection in a spectrum-sharing scenario', *IEEE Trans. Veh. Technol.*, 2011, **60**, (2), pp. 457–472
- 20 Arnold, B.C., Nagaraja, H.N.: 'A first course in order statistics' (John Wiley Sons, Inc., New York, 1992)
- 21 Pickands, J.: 'Moment convergence of sample extremes', *Ann. Math. Stat.*, 1968, **39**, (3), pp. 881–889