# Resource Allocation for IRS-Assisted Wireless Powered Communication Networks 

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#### Abstract

This letter studies a multi-user intelligent reflecting surface (IRS)-assisted wireless powered communication networks (WPCN) system. Specifically, a multi-antenna energy station (ES) transmits energy signal to multiple single-antenna users assisted by an IRS. Then users use the harvested energy to transmit respective data to a single-antenna information station (IS) assisted by another IRS. Under this setup, we jointly optimize the energy beamforming, IRS phase shift and time slot allocation to maximize the system throughput by applying the alternating optimization (AO) technique, difference-of-convex (DC) programming, penalty-based method and successive convex approximation (SCA). Simulation results verify that the proposed algorithm can improve the system throughput.


Index Terms-IRS, wireless powered communication networks, alternating optimization, difference-of-convex, penalty-based.

## I. Introduction

THE NUMBER of devices in wireless networks has increased exponentially. According to the forecast in the Cisco report, there will be more than 28 billion devices connected to the network in 2022 [1]. These devices require continuous energy supply, which brings a large energy burden to the existing wireless networks. In order to solve this problem, wireless powered communication networks (WPCN) have attracted more and more attention. This technology is regarded as a promising choice for energy-constrained wireless networks [2]. In WPCN, energy station (ES) uses dedicated energy transfer node to broadcast energy signal for users in downlink (DL), and users use the harvested energy to send data to information station (IS) in uplink (UL). However, the energy transfer efficiency and information transmission rate of WPCN are both very low.

Intelligent reflecting surface (IRS) proposed in recent years has become a promising technology to improve the spectrum efficiency of wireless networks with lower cost [3], [4]. IRS is composed of a large number of low-cost and reconfigurable passive components, each of which can independently reflect signals and apply a certain phase shift. It can be applied to

[^0]WPCN by allowing all elements to adjust the phase shift cooperatively to increase the channel gain, thus improving energy transfer efficiency and information transmission rate.

There have been many studies on IRS-assisted systems (e.g., [5]-[8]). Li et al. studied optimization problem in an IRS-assisted simultaneous wireless information and power transfer (SWIPT) non-orthogonal multiple access (NOMA) system by jointly optimizing beamforming, power spliting and NOMA decoding order [5]. Lyu et al. proposed a hybrid-relaying scheme in IRS-assisted WPCN to improve the downlink energy transfer from a hybrid access point (HAP) to users and uplink information transmission from users to the HAP [6]. Zheng et al. considered an IRSassisted cooperative communication in a two-user WPCN [7]. The main contribution of this letter is to consider a more general model, i.e., the case where ES and IS are separated, compared with most existing researches on HAP. And we propose a feasible algorithm for the IRS-assisted system, which is also applicable to systems deploying one single IRS and systems where ES and IS are regarded as HAP. We consider the coverage of one single IRS is limited due to the far distance. Thus it can be quite practical to consider adopting two IRSs, which can also be extended to systems deployed with more IRSs.

As shown in Fig. 1, we study a novel IRS-assisted WPCN system, where $\mathrm{IRS}_{1}$ and $\mathrm{IRS}_{2}$ are deployed to serve ES and IS, respectively. Specifically, $\mathrm{IRS}_{1}$ is used to assist the energy transfer in DL and $\mathrm{IRS}_{2}$ is used to assist the information transmission in UL. We maximize the throughput via jointly optimizing the energy beamforming, the phase shift of IRS and the time slot allocation, subject to a given signal to noise ratio (SNR) constraint. Simulation results indicate that the proposed algorithm can improve the throughput and reveal which factor has the greatest impact on the system performance.

Notations: Scalars are denoted by lower-case letters, while vectors and matrices are denoted by bold lower-case letters and bold upper-case letters, respectively. $|x|$ denotes the absolute value of a complex-valued scalar $x .\|\mathbf{x}\|$ denotes the Euclidean norm of a complex-valued vector $\mathbf{x}$. For a square matrix $\mathbf{X}$, $\operatorname{tr}(\mathbf{X}), \operatorname{rank}(\mathbf{X}), \mathbf{X}^{H}, \mathbf{X}_{m, n}$ and $\|\mathbf{X}\|$ denote its trace, rank, conjugate transpose, $m$, $n$-th entry and matrix norm, respectively, while $\mathbf{X} \succeq 0$ represents that $\mathbf{X}$ is a positive semidefinite matrix. In addition, $\mathbb{C}^{M \times N}$ denotes the space of $M \times N$ complex matrices. $\mathbb{E}\{\cdot\}$ represents the expectation of random variables and ang $\{\cdot\}$ denotes the phase of the complex value. The distribution of a circularly symmetric complex Gaussian (CSCG) random vector with mean $\mu$ and covariance matrix $\mathbf{C}$ is denoted by $\mathcal{C N}(\mu, \mathbf{C})$, and $\sim$ stands for 'distributed as'.


Fig. 1. IRS-assisted WPCN.

## II. System Model

We consider a WPCN system, where IRS $_{1}$, composed of $M_{1}$ reflecting elements, is deployed between $N$-antenna ES and $K$ single-antenna users, denoted as $u_{k}(k=1,2,3 \cdots K)$, while $\mathrm{IRS}_{2}$, composed of $M_{2}$ reflecting elements, is deployed to assist the information transmission from users to singleantenna IS, as shown in Fig. 1. First, ES broadcasts radio frequency energy to users in DL, and users use their own harvested energy to send private information to IS in UL. The antennas of ES and the array of $\mathrm{IRS}_{1}$ and $\mathrm{IRS}_{2}$ are all distributed in a uniform linear array (ULA). ${ }^{1} \mathbf{e}=$ $\left[e_{1}, e_{2}, \ldots, e_{M_{1}}\right]^{H}$ denotes the phase shift of IRS $_{1}$ in DL, while $\mathbf{q}=\left[q_{1}, q_{2}, \ldots, q_{M_{2}}\right]^{H}$ denotes the phase shift of $\mathrm{IRS}_{2}$ in UL. We assume that all passive elements have an amplitude equal to one, i.e., $\left|e_{m_{1}}\right|^{2}=1, m_{1}=1,2, \ldots, M_{1}$, $\left|q_{m_{2}}\right|^{2}=1, m_{2}=1,2, \ldots, M_{2}$, where $e_{m_{1}}$ denotes the phase shift of the $m_{1}$-th element of $\operatorname{IRS}_{1}$ and $q_{m_{2}}$ denotes the phase shift of the $m_{2}$-th element of $\mathrm{IRS}_{2}$.

The channel gains from ES to $\operatorname{IRS}_{1}$, from $\operatorname{IRS}_{1}$ to $u_{k}$ and from ES to $u_{k}$ can be denoted as $\mathbf{H}_{E, R_{1}} \in \mathbb{C}^{M_{1} \times N}, \mathbf{h}_{R_{1}, k}^{H} \in$ $\mathbb{C}^{1 \times M_{1}}, \mathbf{h}_{E, k}^{H} \in \mathbb{C}^{1 \times N}$, respectively. The channel gains from $\mathrm{IRS}_{2}$ to IS, from $u_{k}$ to $\mathrm{IRS}_{2}$ and from $u_{k}$ to IS can be denoted as $\mathbf{h}_{R_{2}, I}^{H} \in \mathbb{C}^{1 \times M_{2}}, \mathbf{h}_{k, R_{2}} \in \mathbb{C}^{M_{2} \times 1}, h_{k, I} \in \mathbb{C}$. The equivalent channel gain $\mathbf{h}_{k, d l}^{H} \in \mathbb{C}^{1 \times N}$ between ES to $u_{k}$ is

$$
\begin{equation*}
\mathbf{h}_{k, d l}^{H}=\mathbf{h}_{R_{1}, k}^{H} \operatorname{diag}\left(\mathbf{e}^{H}\right) \mathbf{H}_{E, R_{1}}+\mathbf{h}_{E, k}^{H} \tag{1}
\end{equation*}
$$

and the equivalent channel gain between $u_{k}$ to is

$$
\begin{equation*}
h_{k, u l}=\mathbf{h}_{R_{2}, I}^{H} \operatorname{diag}\left(\mathbf{q}^{H}\right) \mathbf{h}_{k, R_{2}}+h_{k, I} \tag{2}
\end{equation*}
$$

We assume that all channels are quasi-static flat fading, i.e., in each transmission time $T$, each channel is constant. We also assume that all channel state information (CSI) is known. ${ }^{2}$

Consider that all users do not have traditional energy supply (such as batteries, etc.), and users can harvest and store energy from the radio frequency signal transmitted by ES in DL. As shown in Fig. 2, each time slot $T$ is divided into $K+1$ parts. During $\tau_{0} T\left(0<\tau_{0}<1\right)$ time, ES broadcasts energy signals to all users. In the remaining $\left(1-\tau_{0}\right) T$ time, all users use the

[^1]

Fig. 2. Time slot allocation.
energy they harvested in DL to send their private data to IS in a time division multiple access (TDMA) manner. For the sake of simplicity, in this letter we will normalize time to $T=1$.

In DL, ES broadcasts $C$ energy beams to all users ( $C$ can be any integer not exceeding $N$ ). The baseband transmission signal can be expressed as

$$
\begin{equation*}
\mathbf{x}_{d l}=\sum_{c=1}^{C} \mathbf{w}_{c} x_{c, d l} \tag{3}
\end{equation*}
$$

where $x_{c, d l}$ represents the energy signal and is assumed to be independent and identically distributed (i.i.d) CSCG random variable with zero mean and unit variance. $\mathbf{w}_{c} \in \mathbb{C}^{N \times 1}$ denotes the $c$-th energy beam. Assuming that there is a maximum transmission power at ES, therefore, we have

$$
\begin{equation*}
\sum_{c=1}^{C}\left\|\mathbf{w}_{c}\right\|^{2}=\sum_{c=1}^{C} \operatorname{tr}\left(\mathbf{w}_{c} \mathbf{w}_{c}^{H}\right) \leq P \tag{4}
\end{equation*}
$$

For the energy receiver, the noise can be ignored in practice. Besides, considering the far distance between ES and $\mathrm{IRS}_{2}$ as well as two path losses, we assume that the cascaded channel, i.e., $\mathrm{ES}^{-\mathrm{IRS}_{2}}$ and $\mathrm{IRS}_{2}-u_{k}$, is negligible. Therefore the energy signal received by $u_{k}$ can be expressed as $y_{k}=\mathbf{h}_{k, d l}^{H} \mathbf{x}_{d l}$. Thus the energy $E_{k}$ received by $u_{k}$ is

$$
\begin{equation*}
E_{k}=\eta \tau_{0} \mathbb{E}\left\{\left|y_{k}\right|^{2}\right\} \tag{5}
\end{equation*}
$$

where $\tau_{0}$ denotes the time split in DL and $\eta$ denotes energy harvested efficiency. This letter does not consider users' own circuit energy consumption and only focuses on the energy consumption for information transmission.

In UL, at $\tau_{k}$ time, $u_{k}$ uses all the energy harvested in DL to send information to IS. The baseband transmission signal of $u_{k}$ can be expressed as

$$
\begin{equation*}
x_{k, u l}=\sqrt{\frac{E_{k}}{\tau_{k}}} s_{k, u l} \tag{6}
\end{equation*}
$$

where $s_{k, u l}$ denotes the information signal of $u_{k}$ and is assumed to be i.i.d CSCG variable with zero mean and unit variance, i.e., $s_{k, u l} \sim \mathcal{C N}(0,1)$.

The signal received by IS at time $\tau_{k}$ can be expressed as $z_{k}=h_{k, u l} x_{k, u l}+n_{k}$, where $n_{k} \in \mathbb{C}$ denotes the additive white Gaussian noise (AWGN) introduced at the receiving end and $n_{k} \sim \mathcal{C N}\left(0, \sigma^{2}\right)$. Similarly, we also assume that the cascaded channel, i.e., $u_{k}-\operatorname{IRS}_{1}$ and $\operatorname{IRS}_{1}-\mathrm{IS}$, can be negligible. So the SNR of IS decoding $u_{k}$ can be expressed as

$$
\begin{equation*}
\gamma_{k}=\frac{\left|h_{k, u l} x_{k, u l}\right|^{2}}{\sigma^{2}} \tag{7}
\end{equation*}
$$

Therefore, the achievable information rate $R_{k}$ of $u_{k}$ is

$$
\begin{equation*}
R_{k}=\log _{2}\left(1+\gamma_{k}\right) \tag{8}
\end{equation*}
$$

## III. Problem Formulation

We define the time slot allocation $\tau=$ $\left[\tau_{0}, \tau_{1}, \tau_{2}, \ldots, \tau_{K}\right]^{T}$, ES energy beamforming $\mathbf{W}_{d l}=\left[\mathbf{w}_{1}, \ldots, \mathbf{w}_{C}\right]$. For the IRS-assisted WPCN system, we jointly optimize ES energy beamforming $\mathbf{W}_{d l}$, $\operatorname{IRS}_{1}$ phase shift $\mathbf{e}, \mathrm{IRS}_{2}$ phase shift $\mathbf{q}$ and time slot allocation $\tau$ to maximize the throughput of the system while ensuring the quality of service (QoS) of each user. The optimization problem can be expressed as follows,

$$
\begin{align*}
(\mathcal{P} 1) \max _{\tau, \mathbf{W}_{d l}, \mathbf{e}, \mathbf{q}} & \sum_{k=1}^{K} \tau_{k} R_{k},  \tag{9a}\\
\text { s.t. } & 0 \leq \tau_{k} \leq 1, k=0,1,2, \ldots, K  \tag{9b}\\
& \sum_{k=0}^{K} \tau_{k}=1  \tag{9c}\\
& \sum_{c=1}^{C}\left\|\mathbf{w}_{c}\right\|^{2} \leq P  \tag{9d}\\
& \gamma_{k} \geq \eta_{k}, k=1,2, \ldots, K  \tag{9e}\\
& \left|e_{m_{1}}\right|^{2}=1, m_{1}=1,2, \ldots, M_{1}  \tag{9f}\\
& \left|q_{m_{2}}\right|^{2}=1, m_{2}=1,2, \ldots, M_{2} \tag{9~g}
\end{align*}
$$

where (9b) and (9c) are time constraints, and (9d) is the maximum transmission power constraint for ES, and (9e) is the SNR constraint for users, and (9f) and ( 9 g ) are unit modulus constraints of the phase shift of IRS. Due to the coupling of variables in the objective function and non-convex constraints of $(9 \mathrm{e})-(9 \mathrm{~g})$, this problem generally can't be solved directly. We propose a feasible algorithm by applying the alternating optimization (AO) technique, i.e., optimizing ES energy beamforming, phase shift of $\mathrm{IRS}_{1}, \mathrm{IRS}_{2}$ and time slot allocation alternately. When optimizing energy beamforming, the problem can be transformed to a semidefinite programming (SDP) problem [9]. When optimizing the phase shift of IRS, in order to solve the non-convex problem, the difference-of-convex (DC) programming, penalty-based method and successive convex approximation (SCA) method are used to transform the non-convex rank-one constraint [10]. When optimizing time slot allocation, this problem is found to be a convex optimization problem with respect to (w.r.t) $\tau$, which can be solved by some standard optimization techniques, e.g., CVX [11].

## IV. Proposed Algorithm

In this section, We show the specific optimization algorithm, which applies AO technique to divide the problem into three sub-problems and these problems are optimized alternately and iteratively until achieving convergence.

## A. Transmitting Beamforming Optimization

First fix $\tau, \mathbf{e}, \mathbf{q}$, and let $\mathbf{S}=\sum_{c=1}^{C} \mathbf{w}_{c} \mathbf{w}_{c}{ }^{H}, \Phi_{k}=\mathbf{h}_{k, d l}$ $\mathbf{h}_{k, d l}^{H}$. We have $\left|h_{k, u l} x_{k, u l}\right|^{2}=\left|h_{k, u l}\right|^{2}\left|\mathbf{h}_{k, d l}{ }^{H} \sum_{c=1}^{C} \mathbf{w}_{c}\right|^{2}=$ $\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\mathbf{h}_{k, d l} \mathbf{h}_{k, d l}{ }^{H} \sum_{c=1}^{C} \mathbf{w}_{c} \mathbf{w}_{c}{ }^{H}\right)=\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\Phi_{k} \mathbf{S}\right)$. The problem ( $\mathcal{P} 1)$ can be transformed into problem $(\mathcal{P} 2)$,
which can be expressed as

$$
\begin{align*}
(\mathcal{P} 2) \max _{\mathbf{S}} & \sum_{k=1}^{K} \tau_{k} \log _{2}\left(1+\frac{\eta \tau_{0}\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\mathbf{\Phi}_{k} \mathbf{S}\right)}{\tau_{k} \sigma^{2}}\right),  \tag{10a}\\
\text { s.t. } & \operatorname{tr}(\mathbf{S}) \leq P,  \tag{10b}\\
& \frac{\eta \tau_{0}\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\mathbf{\Phi}_{k} \mathbf{S}\right)}{\tau_{k} \sigma^{2}} \geq \eta_{k}, k=1,2, \ldots, K,  \tag{10c}\\
& \mathbf{S} \succeq 0 . \tag{10d}
\end{align*}
$$

This problem is an SDP problem and can be solved by CVX toolbox. Let $\mathbf{S}^{*}$ denote the optimal solution of the problem. Then $C=\operatorname{rank}\left(\mathbf{S}^{*}\right)$ is the number of DL energy beams, and $\left\{\mathbf{w}_{1}, \ldots, \mathbf{w}_{C}\right\}=\left\{\cdot \sqrt{\lambda_{1}} \mathbf{p}_{1}, \ldots, \sqrt{\lambda_{C}} \mathbf{p}_{C}\right\}$. is the energy beamforming, where $\lambda_{i}$ is the $i$-th largest eigenvalue of $\mathbf{S}^{*}$ and $\mathbf{p}_{i}$ is the corresponding eigenvector of $\mathbf{S}^{*}$.

## B. IRS Phase Shift Optimization

The second sub-problem can be divided in two parts, optimizing $\mathbf{e}$ and $\mathbf{q}$ separately. When given the remaining variables and optimizing $\mathbf{e}$, the problem $\mathcal{P} 1$ can be transformed into problem ( $\mathcal{P} 3.1$ ), which can be expressed as

$$
\begin{align*}
(\mathcal{P} 3.1) \max _{\mathbf{e}} & \sum_{k=1}^{K} \tau_{k} \log _{2}\left(1+\frac{\eta \tau_{0}\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\boldsymbol{\Phi}_{k} \mathbf{S}\right)}{\tau_{k} \sigma^{2}}\right)  \tag{11a}\\
\text { s.t. } & \gamma_{k} \geq \eta_{k}, k=1,2, \ldots, K  \tag{11b}\\
& \left|e_{m_{1}}\right|^{2}=1, m_{1}=1,2, \ldots, M_{1} \tag{11c}
\end{align*}
$$

where $\operatorname{tr}\left(\Phi_{k} \mathbf{S}\right)=\sum_{c=1}^{C}\left|\mathbf{e}^{H} \operatorname{diag}\left(\mathbf{h}_{R_{1}, k}^{H}\right) \mathbf{H}_{E, R_{1}} \mathbf{w}_{c}+\mathbf{h}_{E, k}^{H} \mathbf{w}_{c}\right|^{2}$. Let $\mathbf{a}_{k, c}=\operatorname{diag}\left(\mathbf{h}_{R_{1}, k}^{H}\right) \mathbf{H}_{E, R_{1}} \mathbf{w}_{c}$ and $b_{k, c}=\mathbf{h}_{E, k}^{H} \mathbf{w}_{c}$. Introduce auxiliary matrix

$$
\mathbf{R}_{k c}=\left[\begin{array}{cc}
\mathbf{a}_{k, c}\left(\mathbf{a}_{k, c}\right)^{H} & \mathbf{a}_{k, c} b_{k, c}^{H} \\
b_{k, c}\left(\mathbf{a}_{k, c}\right)^{H} & 0
\end{array}\right], \overline{\mathbf{e}}=\left[\begin{array}{l}
\mathbf{e} \\
1
\end{array}\right] .
$$

Then we have $\operatorname{tr}\left(\Phi_{k} \mathbf{S}\right)=\sum_{c=1}^{C}\left(\operatorname{tr}\left(\mathbf{R}_{k c} \bar{E}\right)+\left|b_{k, c}\right|^{2}\right)$, where $\bar{E}=\overline{\mathbf{e}} \overline{\mathbf{e}}^{H} \succeq 0$ and $\operatorname{rank}(\overline{\mathbf{e}})=1$. Then the problem can be transformed into ( $\mathcal{P} 3.2$ ) as follows,

$$
\begin{align*}
(\mathcal{P} 3.2) \max _{\overline{\mathbf{E}}} & \sum_{k=1}^{K} \tau_{k} \log _{2}\left(1+\gamma_{k}\right)  \tag{12a}\\
\text { s.t. } & \gamma_{k} \geq \eta_{k}, k=1,2, \ldots, K  \tag{12b}\\
& \overline{\mathbf{E}} \succeq 0  \tag{12c}\\
& \operatorname{rank}(\overline{\mathbf{E}})=1  \tag{12~d}\\
& \overline{\mathbf{E}}_{m_{1}, m_{1}}=1, m_{1}=1,2, \ldots, M_{1}+1 \tag{12e}
\end{align*}
$$

where $\gamma_{k}=\frac{\eta \tau_{0}\left|h_{k, u l}\right|^{2} \sum_{c=1}^{C}\left(\operatorname{tr}\left(\mathbf{R}_{k c} \overline{\mathbf{E}}\right)+\left|b_{k, c}\right|^{2}\right)}{\tau_{k} \sigma^{2}}$. First, we apply DC technique to transform the non-convex rank-one constraint.

Proposition 1: For the positive semidefinite matrix $\mathbf{M} \in$ $\mathbb{C}^{N \times N}, \operatorname{tr}(\mathbf{M}) \geq 0$, the rank-one constraint can be expressed as the difference of two convex functions, i.e.,

$$
\begin{equation*}
\operatorname{rank}(\mathbf{M})=1 \Leftrightarrow \operatorname{tr}(\mathbf{M})-\|\mathbf{M}\|_{2}=0 \tag{13}
\end{equation*}
$$

where $\|\mathbf{M}\|_{2}$ denotes the matrix spectral norm.
According to Proposition 1, the rank-one constraint can be transformed into the difference of two convex functions. Then
this term is added to the objective function as a penalty term. Thus the problem can be transformed as follows,

$$
\begin{align*}
(\mathcal{P} 3.3) \max _{\overline{\mathbf{E}}} & \sum_{k=1}^{K} \tau_{k} \log _{2}\left(1+\gamma_{k}\right)-\lambda\left(\operatorname{tr}(\overline{\mathbf{E}})-\|\overline{\mathbf{E}}\|_{2}\right)  \tag{14a}\\
\text { s.t. } & \gamma_{k} \geq \eta_{k}, k=1,2, \ldots, K  \tag{14b}\\
& \overline{\mathbf{E}} \succeq 0  \tag{14c}\\
& \overline{\mathbf{E}}_{m_{1}, m_{1}}=1, m_{1}=1,2, \ldots, M_{1}+1 \tag{14~d}
\end{align*}
$$

where $\lambda$ is the penalty factor. When $\lambda \rightarrow 0$, the rank-one constraint is equivalent to being relaxed completely. When $\lambda \rightarrow \infty$, the objective function is dominated by the penalty term and the throughput term can be negligible. We can initialize $\lambda$ with a small value and gradually increase $\lambda$ until obtaining a numerical rank-one solution. Considering that $\|\overline{\mathbf{E}}\|_{2}$ is a convex function, the problem is still a non-convex problem. By SCA, the problem can be transformed into a convex optimization problem. In particular, we need to solve the following problem in the $t$-th iteration,

$$
\begin{align*}
(\mathcal{P} 3.4) \max _{\overline{\mathbf{E}}} & \sum_{k=1}^{K} \tau_{k} R_{k}-\lambda\left(\operatorname{tr}(\overline{\mathbf{E}})-\left\langle\partial\left\|\overline{\mathbf{E}}^{t-1}\right\|_{2}, \overline{\mathbf{E}}\right\rangle\right)  \tag{15a}\\
\text { s.t. } & \gamma_{k} \geq \eta_{k}, k=1,2, \ldots, K  \tag{15b}\\
& \overline{\mathbf{E}} \succeq 0  \tag{15c}\\
& \overline{\mathbf{E}}_{m_{1}, m_{1}}=1, \forall m_{1}=1,2, \ldots, M_{1}+1 \tag{15~d}
\end{align*}
$$

where $\overline{\mathbf{E}}^{t-1}$ is the $(t-1)$-th solution and $\partial\left\|\overline{\mathbf{E}}^{t-1}\right\|_{2}$ represents the sub-gradient of the spectral norm, which can be calculated by Proposition 2. Therefore, the problem can be transformed into a convex optimization problem, which can be solved using CVX toolbox. After obtaining the optimal solution $\overline{\mathbf{E}^{*}}$, we have $\mathbf{e}_{m_{1}}^{*}=\operatorname{ang}\left(\mathbf{p}_{m_{1}}\right), m_{1}=1,2, \ldots, M_{1}$, where $\mathbf{p}$ is the eigenvector corresponding to the largest eigenvalue of $\overline{\mathbf{E}^{*}}$. Similarly, $\mathbf{q}^{*}$ can also be obtained in this way.

Proposition 2: For the positive semidefinite matrix M, the sub-gradient $\partial\left\|\overline{\mathbf{E}}^{t-1}\right\|_{2}$ can be calculated by $\mathbf{p}_{1} \mathbf{p}_{1}^{H}$, where $\mathbf{p}_{1}$ is the eigenvector corresponding to the largest singular value of matrix $\mathbf{M}$ [12].

## C. Time Allocation Optimization

For the third sub-problem, given $\mathbf{W}_{d l}, \mathbf{e}, \mathbf{q}$, the problem can be transformed into the problem $(\mathcal{P} 4)$, which can be expressed as follows,

$$
\begin{align*}
(\mathcal{P} 4) \max _{\tau} & \sum_{k=1}^{K} \tau_{k} \log _{2}\left(1+\frac{\eta \tau_{0}\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\mathbf{\Phi}_{\mathrm{k}} \mathbf{S}\right)}{\tau_{k} \sigma^{2}}\right)  \tag{16a}\\
\text { s.t. } & 0 \leq \tau_{k} \leq 1, k=0,1,2, \ldots, K  \tag{16b}\\
& \sum_{k=0}^{K} \tau_{k}=1  \tag{16c}\\
& \frac{\eta \tau_{0}\left|h_{k, u l}\right|^{2} \operatorname{tr}\left(\mathbf{\Phi}_{\mathrm{k}} \mathbf{S}\right)}{\tau_{k} \sigma^{2}} \geq \eta_{k}, k=1,2, \ldots, K \tag{16~d}
\end{align*}
$$

The objective function is the sum of perspective functions of a concave function w.r.t $\tau$ and thus is also a concave function. The constraints $\mathrm{C} 1, \mathrm{C} 2$ and C 4 are all linear functions w.r.t $\tau$. Therefore, the problem is a convex optimization problem, which can be solved by using CVX toolbox. By alternately

```
\(\overline{\text { Algorithm } 1 \text { Joint ES Transmitting Beamforming, IRS Phase }}\)
Shift and Time Allocation Algorithm
    Initialize \(\{\tau\}^{(0)},\{\mathbf{e}\}^{(0)}\) and \(\{\mathbf{q}\}^{(0)}\). Let \(r=0, \varepsilon=10^{-3}\).
    repeat
        Solve problem \((\mathcal{P} 2)\) for given \(\{\tau\}^{(r)},\{\mathbf{e}\}^{(r)},\{\mathbf{q}\}^{(r)}\).
        Solve problem \((\mathcal{P} 3.4)\) for given \(\mathbf{W}_{d l}^{(r+1)},\{\mathbf{q}\}^{(r)}\) (or
    \(\{\mathbf{e}\}^{(r)}\) ) and \(\{\tau\}^{(r)}\).
        Solve problem \((\mathcal{P} 4)\) for given \(\mathbf{W}_{d l}^{(r+1)},\{\mathbf{e}\}^{(r+1)}\) and
    \(\{\mathbf{q}\}^{(r+1)}\).
        Update \(r=r+1\).
    until The increased value of the objective is below the
    threshold \(\varepsilon\).
    return ES energy beamforming, IRS \(_{1}\) phase shift, IRS \(_{2}\)
    phase shift and time slot allocation.
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iterating the three sub-problems, we can obtain the sub-optimal solution when achieving convergence. The overall algorithm can be summarized as Algorithm 1. ${ }^{3}$

## V. Simulation Results

In this section, we show numerical simulation results to prove the effectiveness of the proposed algorithm. In the simulation setting, we consider a three-dimensional coordinate system, where the positions of $\mathrm{ES}, \mathrm{IRS}_{1}, \mathrm{IRS}_{2}$ and IS are $(0 \mathrm{~m}, 0 \mathrm{~m}, 15 \mathrm{~m}),(0 \mathrm{~m}, 20 \mathrm{~m}, 15 \mathrm{~m}),(0 \mathrm{~m}, 30 \mathrm{~m}, 15 \mathrm{~m})$ and ( $0 \mathrm{~m}, 50 \mathrm{~m}, 15 \mathrm{~m}$ ), respectively. $K=6$ users are randomly distributed on a circle with the center $(0 \mathrm{~m}, 25 \mathrm{~m}, 0 \mathrm{~m})$ and the radius 5 m . The maximum power of ES is set to be 43 dBm . ES is equipped with 6 antennas, while IS and users are single-antenna. The space of antenna and IRS element is half wavelength. The SNR threshold of each user is set to 0 dB . The distance path loss model can be expressed as $h(s)=C_{0}\left(\frac{s}{S_{0}}\right)^{\alpha}$, where $C_{0}=-30 \mathrm{~dB}$ is the path loss at the reference distance $S_{0}=1 m$, and $s$ denotes distance and $\alpha$ denotes the path loss exponent. In terms of small-scale fading, we assume that all channels are Rician channels, and the Rician channel model is expressed as follows,

$$
\begin{equation*}
\mathbf{G}=\sqrt{\frac{\beta}{1+\beta}} G^{\mathrm{LoS}}+\sqrt{\frac{1}{1+\beta}} G^{\mathrm{NLoS}} \tag{17}
\end{equation*}
$$

So the total channel gain can be denoted as $h(d) \mathbf{G}$. $\beta$ denotes the Rician factor. $\mathbf{G}^{\mathrm{LoS}}$ denotes the line-of-sight (LoS) component and $\mathbf{G}^{\mathrm{NLoS}}$ represents non-line-of-sight (NLoS) component. $G^{\mathrm{LoS}}$ can be modeled as $G^{\mathrm{LoS}}=$

[^2]

Fig. 3. Sum-throughput versus the number of iterations with different numbers of ES antennas.


Fig. 4. Sum-throughput versus the number of IRS elements.
$\mathrm{a}_{M}{ }^{H}\left(\theta_{\mathrm{AoA}}\right) \mathrm{a}_{N}\left(\theta_{\mathrm{AoD}}\right)$, where $\mathrm{a}_{N}(\theta)$ denotes the array response of $N$ ULA elements. It can be expressed as $\mathrm{a}_{N}(\theta)=$ $\left[1, e^{-j 2 \pi \frac{d}{\lambda} \sin \theta}, \ldots, e^{-j 2 \pi(N-1) \frac{d}{\lambda} \sin \theta}\right]$, where $d$ denotes the space between two adjacent elements. $\theta_{\text {AoA }}$ and $\theta_{\text {AoD }}$ denote the angel of arrival (AoA) and the angel of departure (AoD), respectively. $\mathbf{G}^{\mathrm{NLOS}}$ represents Rayleigh fading component and each element is i.i.d CSCG random variable with zero mean and unit variance. For ES- $u_{k}$ and $u_{k}$-IS direct channels, we set $\alpha_{\mathrm{E}, \mathrm{U}}=\alpha_{\mathrm{I}, \mathrm{U}}=3, \beta_{\mathrm{E}, \mathrm{U}}=\beta_{\mathrm{I}, \mathrm{U}}=0$. For $\mathrm{ES}_{\mathrm{IRS}}^{1} 1$ and $\operatorname{IS}-\mathrm{IRS}_{2}$ channels, we set $\alpha_{\mathrm{E}, \mathrm{R}_{1}}=\alpha_{\mathrm{I}, \mathrm{R}_{2}}=$ $2.2, \beta_{\mathrm{E}, \mathrm{R}_{1}}=\beta_{\mathrm{I}, \mathrm{R}_{2}}=\infty$. For $\mathrm{IRS}_{1}-u_{k}$ and $\operatorname{IRS}_{2}-u_{k}$ channels, we set $\alpha_{\mathrm{U}, \mathrm{R}_{1}}=\alpha_{\mathrm{U}, \mathrm{R}_{1}}=2.5, \beta_{\mathrm{U}, \mathrm{R}_{1}}=\beta_{\mathrm{U}, \mathrm{R}_{2}}=0$. The algorithm convergence threshold is set to $10^{-3}$.

We first evaluate the convergence of the proposed algorithm. Fig. 3 shows the system throughput varies with the number of iterations under different ES antennas. It can be seen that as the number of iteration increases, the throughput increases and can quickly converge to a certain value. In addition, we also study the impact of different ES antennas on the throughput. Specifically, we study the throughput when the number of ES antennas is $4,6,8$ and 10 . As the number of ES antennas increases, the throughput also gradually increases. This also illustrates the importance of multi-antenna beamforming. We can increase the number of ES antennas to improve the system throughput.

Fig. 4 shows the throughput varies with the number of IRS elements under different schemes, i.e., (i) only optimizing IRS phase shift, (ii) only optimizing energy beamforming, (iii) only optimizing time, (iv) overall optimization, i.e., the proposed algorithm. As can be seen, the throughput increases as the number of IRS elements increases under four schemes and
(iv) increases at the fastest pace. It can also be seen that when the number of IRS elements reaches a certain level, optimizing the phase shift of IRS has the greatest impact and achieves a leap forward on system performance compared with time and energy beamforming optimization, which further manifests the importance and necessity of optimizing the phase shift of IRS.

## VI. CONCLUSION

In this letter, we study an IRS-assisted WPCN system, where two IRSs are deployed to assist energy transfer and information transmission. We jointly optimize energy beamforming, the phase shift of IRS and time slot allocation to maximize the throughput. The AO technique, DC technique, the penalty-based method and SCA method are applied to solve the formulated non-convex problem. Simulation results indicate that by increasing the number of ES antennas, increasing the number of IRS elements and using the proposed algorithm, we can all improve the system throughput. Besides, compared with energy beamforming and time slot allocation optimization, optimizing the phase shift of IRS produces the most far-reaching improvement on system performance when the number of IRS is large enough.

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[^1]:    ${ }^{1}$ It can also be extended to the situation of uniform plane array (UPA).
    ${ }^{2}$ CSI can be acquired by the receiving mode of IRS or deploying a small part active elements for channel estimation.

[^2]:    ${ }^{3}$ The computational complexity of the designed algorithm mainly depends on iteratively solving the SDP problem for optimizing beamforming and IRS phase shift as well as the linear programming (LP) problem for optimizing time slot. By using the interior point method for SDP problem [9], the computational complexity of optimizing beamforming and two IRSs is $O\left(N^{3.5}\right), O\left(\left(M_{1}+1\right)^{3.5}\right)$ and $O\left(\left(M_{2}+1\right)^{3.5}\right)$, respectively. For LP problem, the computational complexity of optimizing time slot is $O(K+1)$. Thus, the overall computational complexity is $O\left(N^{3.5}+\left(M_{1}+1\right)^{3.5}+\right.$ $\left.\left(M_{2}+1\right)^{3.5}+(K+1)\right)$. For convergence analysis, the objective function of each sub-problem is monotonically non-decreasing in each iteration. The overall algorithm is non-decreasing after each iteration. As the objective function is upper bounded by a finite value due to the limited transmit power, the proposed algorithm is guaranteed to converge to a certain point.

